ABSTRACTS OF TALKS, 2019 LEHIGH GEOMETRY/TOPOLOGY CONFERENCE

Ian Adelstein [NV III, 10:20-11:00, Friday, June 21]. Yale University

Characterizing Round Spheres Using Half-Geodesics.

Abstract: A half-geodesic is a closed geodesic realizing the distance between any pair of its points. All geodesics in a round sphere are half-geodesics. Conversely, this talk will establish that Riemannian spheres with all geodesics closed and sufficiently many half-geodesics are round. This work is joint with Ben Schmidt.

David Afrifa, Maziar Farahzad, Julinda Pillati Mujo, Ajmain Yamin [NV I, 11:20-12:00, Friday, June 21]. Lehman College, Stony Brook University, Lehman College, Stony Brook University

Smocked Metric Spaces and their Tangent Cones.

Abstract: We introduce the notion of a smocked metric spaces, explore the balls and geodesics in a collection of different smocked spaces, find their rescaled Gromov-Hausdorff limits and prove these tangent cones at infinity exist and are unique and are normed spaces. This is joint work with Prof Sormani, Dr Kazaras and a team of fellow students.

Sergio Almaraz [NV III, 9:10-9:50, Saturday, June 22]. Princeton University

The mass of asymptotically hyperbolic manifolds with non-compact boundary.

Abstract: We define a mass-type geometric invariant for Riemannian manifolds asymptotic to the hyperbolic half-space and prove a positive mass theorem for spin manifolds. This is joint work with Levi Lima (UFC-Brazil).

Martin Bendersky [NV II, 2:40-3:20, Friday, June 21]. Hunter College

The embedding genus of a Knot.

Abstract: I will talk on the relation between the group of a knot and the minimal genus of a surface into which the knot embeds. Joint with Alina Vdovina.

Ethan Berkove [NV III, 3:20-4:00, Thursday, June 20]. Lafayette College

Cohomology of SL_2 groups over quadratic number fields.

Abstract: The Bianchi groups are the matrix groups $SL_2(\mathcal{O})$ (or PSL_2), where $\mathcal{O}$ is an imaginary quadratic extension of the rational numbers. Bianchi groups are a natural generalization of the Modular Group $SL_2(\mathbb{Z})$, and show up in fields as varied as group theory, number theory, and topology.
One area of study of the Bianchi groups involves the calculation of their (co)homology groups. Early calculations of this type were often done a case-by-case basis. In contrast, the recent use of Alexander Rahm's *p-torsion subcomplex* has allowed for a much broader understanding of how the cohomology of the Bianchi groups, and other families, arises. In this talk we will define the p-torsion subcomplex and describe some recent results of Alexander Rahm and the speaker which use it.

Jacob Bernstein [NV I, 1:30-2:30, Friday, June 21]. Johns Hopkins University

Expanders of low entropy.

Abstract: Expanders are special solutions of mean curvature flow that model how the flow resolves a conical singularity. I will discuss uniqueness properties of these flows when they have low entropy. This is joint work with Lu Wang.

Bourzik Charif [NV III, 10:05-10:45, Saturday, June 22]. Cadi Ayyad University, Marrakech

Generalized pseudo-Hessian manifolds.

Abstract: We give an introduction on the generalized pseudo-Hessian manifold which are a natural generalization of pseudo-Hessian manifold. This notion has been introduced in a paper of M. Boucetta and S. Benyyadi.

Li Chen [NV I, 2:40-3:20, Friday, June 21]. University of the District of Columbia

Contraction of 3D-manifolds and the Jordan Separation Property.

Abstract: In this paper, we use the algorithmic procedure (a contraction process) to prove the following theorem: For a finite triangulation of a simply-connected closed and orientable 3-manifold M in Euclidean space, if a (simply-connected closed) 2-cycle which was made by 2-cells of this triangulation separates M into two connected components, then each of the components will also be simply-connected. In addition, we can algorithmically make M to be homeomorphic to a 3-sphere. The relationship of the theorem to a generalized Jordan separation problem, the general Jordan-Schoenflies theorem, and other important problems are also discussed.

Don Davis [NV II, 10:20-11:00, Friday, June 21]. Lehigh University

On the unordered configuration space $C(RP^n, 2)$.

Abstract: We discuss aspects of the unordered configuration space $C(RP^n, 2)$ such as its immersion dimension and topological complexity.

Shubham Dwivedi [NV III, 1:30-2:10, Thursday, June 20]. University of Waterloo

A gradient flow of isometric G_2 structures.

Abstract: We will talk about a flow of isometric G_2 structures. We consider the negative gradient flow of the energy functional restricted to the class of G_2 structures inducing a given Riemannian metric. We will discuss various analytic aspects of the flow including global and local derivative estimates, a compactness theorem and a local monotonicity formula for the solutions. We also study the evolution equation of the torsion and show that under a modification of the gauge and of the relevant connection, it satisfies a nice reaction-diffusion equation. After defining an entropy functional we will prove that low entropy initial data lead to solutions that exist for all time and converge smoothly to a G_2 structure with divergence free torsion. We will also discuss finite time singularities and show that at the singular time the flow converges to a smooth G_2 structure outside a closed set of finite 5-dimensional Hausdorff measure. Finally, we will prove that if the sin gularity is Type I then a sequence of blow-ups of a solution has a subsequence which converges to a shrinking soliton of the flow. This is a joint work with Panagiotis Gianniotis (University of Athens) and Spiro Karigiannis (University of Waterloo).

Gregory Edwards [NV I, 10:20-11:00, Friday, June 21]. University of Notre Dame

The Chern-Ricci flow on primitive Hopf surfaces.

Abstract: The Chern-Ricci flow is a flow of Hermitian metrics which generalizes the Kähler-Ricci flow to non-Kähler metrics. While solutions of the flow have been classified on some families of complex non-Kähler surfaces, the Hopf surfaces provide a family of non-Kähler surfaces on which little is known about the Chern-Ricci flow. We use a construction of locally conformally Kähler metrics of Gauduchon-Ornea to study solutions of the Chern-Ricci flow on primitive Hopf surfaces of class 1. These solutions reach a volume collapsing singularity in finite time, and we show that the metric tensor satisfies a uniform upper bound, supporting the conjecture that the Gromov-Hausdorff limit is isometric to a round S^1. Uniform $C^{1+\beta}$ estimates are also established for the potential. Previous results had only been known for the simplest examples of Hopf surfaces.

John Harper [NV II, 10:05-10:45, Saturday, June 22]. University of Rochester

Cogroups, suspensions, and the EHP-sequence.

Abstract: The existence of co-H or cogroup structures on 2-cell complexes can be read off from birth/death certificate data of the attaching map. In the case of a co-H space which is not a co-group, a simple construction produces a 3-cell cogroup, and numerical conditions will guarantee that it is not a suspension.

Michael Harrison [NV I, 2:25-3:05, Thursday, June 20]. Lehigh University

Fibrations of $\mathbb{R}^3$ *by lines.*

Abstract: Is it possible to cover 3-dimensional space by a collection of lines, such that no two lines intersect and no two lines are parallel? More precisely, does there exist a fibration of $\mathbb{R}^3$ by pairwise skew lines? We discuss a topological classification of these skew fibrations, closely related to the classification of great circle fibrations/Hopf fibrations. We briefly summarize some recent results regarding contact structures on $\mathbb{R}^3$ which are naturally induced by skew fibrations and also briefly discuss the problem without the skew condition, and the problem in higher dimensional space.

Robert Haslhofer [NV I, 4:25-5:25, Friday, June 21]. University of Toronto

Ancient low entropy flows and the mean convex neighborhood conjecture.

Abstract: In this talk, I will explain our recent proof of the mean convex neighborhood conjecture for the mean curvature flow of surfaces in $\mathbb{R}^3$. Namely, if the flow has a cylindrical singularity at a space-time point $X = (x, t)$, then there exists a positive $\epsilon = \epsilon(X) > 0$ such that the flow is mean convex in a space-time neighborhood of size ϵ around X. The major difficulty is to promote the infinitesimal information captured by the tangent flow to a conclusion of macroscopic size. In fact, we prove a more general classification result for all ancient low entropy flows that arise as potential limit flows near X. As an application, we prove the uniqueness conjecture for mean curvature flow through cylindrical singularities. In particular, assuming Ilmanen's multiplicity one conjecture, we conclude that for embedded two-spheres the mean curvature flow through singularities is well-posed. This is joint work with Kyeongsu Choi and Or Hershkovits.

Surena Hozoori [NV II, 2:25-3:05, Thursday, June 20]. Georgia Tech

Ricci Curvature and Contact Topology in Dimension 3.

Abstract: It is well known that in Riemannian geometry, local information can lead to global phenomena. On the other hand, in the category of contact manifolds, we can naturally focus on so called *compatible* Riemannian structures. However, it is very little known about how to use compatible global geometry to achieve contact topological information. We will discuss the problem of Ricci curvature realization for Reeb vector fields associated to a contact 3-manifold. These vector fields have significantly helped understanding contact topology since early 90s. We will use topological tools, namely open book decompositions, to show that every admissible function can be realized as such Ricci curvature for a singular metric which is an honest compatible metric away from a measure zero codimension one set. However, we will see that resolving such singularities depends on contact topological data and is yet to be fully understood.

Yuhao Hu [NV I, 3:30-4:10, Friday, June 21]. University of Colorado Boulder

Geometry of Bäcklund Transformations.

Abstract: In this talk I want to present some recent progress in the study of Bäcklund transformations. The talk will be organized as follows. First, as a motivation, I'll discuss some classical examples. In particular, I'll present Bianchi's theorem on pseudo-spherical line congruences between surfaces in $\mathbb{E}^3$. Then I'll present a geometric (coordinate-free) formulation of Bäcklund transformations. Using this formulation, we can view a Bäcklund transformation of certain types as a manifold equipped with a differential ideal. This allows one to analyze the associated G-structures and to obtain local invariants. Finally, I'll discuss some new classification results and examples obtained from this geometric point of view, using Cartan's method of equivalence. (This talk is associated to the writings: arXiv:1902.04658, arxiv: 1904.02827)

Lan-Hsuan Huang [NV I, 11:00-12:00, Saturday, June 22]. University of Connecticut

Mass rigidity and spacetime symmetry.

Abstract: The study of scalar curvature deformation and rigidity has been one of the central topics in geometric analysis. The seminal work of Schoen and Yau on the Riemannian positive mass theorem gives striking connections between the ADM mass and positive scalar curvature. In particular, it is shown that Euclidean space is uniquely characterized by having zero ADM mass among a large class of manifolds with nonnegative scalar curvature.

In general relativity, those intriguing phenomena extend to a broader setting. It has been conjectured (in various situations) that a manifold minimizing the ADM mass in certain classes of manifolds must have symmetry, in the sense that the corresponding spacetime possesses a Killing vector field. We will overview some recent progress along this direction, with the focus on the spacetime positive mass theorem and Bartnik's stationary conjecture.

Dan Isaksen [NV I, 11:00-12:00, Thursday, June 20]. Wayne State University

The stable homotopy groups of spheres in dimensions 60-90.

Abstract: The computation of the stable homotopy groups of spheres is a central problem of algebraic topology. I will describe the problem, discuss its history, and explain some applications to the theory of high-dimensional manifolds. Inspired by algebraic geometry, motivic homotopy theory is a variant on classical homotopy theory. Surprisingly, this rich motivic structure leads to a new technique for computing stable homotopy groups, allowing us to reach a previously inaccessible range of dimensions.

Sadok Kallel [NV II, 9:10-9:50, Saturday, June 22]. American University of Sharjah and Université de Lille

Thin loop groups.

Abstract: Physicists and geometers introduce the "thin fundamental group" of a smooth manifold to be the space of based smooth loops modulo a homotopy that doesn't "sweep area". This space plays an important role in studying holonomy, connections and transport. We verify that for a finite simplicial complex X and for piecewise linear loops on X, the "thin" loop space is a topological group of the same homotopy type as the space of continuous loops. This turns out not to be the case for the higher loops.

Demetre Kazaras [NV II, 3:20-4:00, Thursday, June 20]. Stony Brook University

Desingularizing positive scalar curvature 4-manifolds.

Abstract: We show that the bordism group of closed 3-manifolds with positive scalar curvature (psc) metrics is trivial by explicit methods. Our constructions are derived from scalar-flat Kähler ALE surfaces discovered by Lock-Viaclovsky. Next, we study psc 4-manifolds with metric singularities along points and embedded circles. Our psc null-bordisms are essential tools in a desingularization process developed by Li-Montoulidis. This allows us to prove a non-existence result for singular psc metrics on enlargeable 4-manifolds with uniformly Euclidean geometry. As a consequence, we obtain a Riemannian positive mass theorem for asymptotically flat 4-manifolds with non-negative scalar curvature and low regularity.

Elahe Khalili Samani [NV III, 2:40-3:20, Friday, June 21]. Syracuse University

Positive curvature and fundamental group.

Abstract: A 1960s question of Chern asks if every abelian subgroup of the fundamental group of a Riemannian manifold with positive sectional curvature is cyclic. While this is not true in general, there are some positive results in the presence of symmetry. I will discuss some new structural results along these lines. I will also discuss an application to an infinite family of positively curved Riemannian manifolds.

Rob Kusner [NV I, 10:05-10:45, Saturday, June 22]. U. of Massachusetts

Willmore Stability and Conformal Rigidity of Minimal Surfaces in S^n.

Abstract: A minimal surface M in the round sphere S^n is critical for area, as well as for the Willmore bending energy $W = \iint (1 + H^2) da$. Willmore stability of M is equivalent to a gap between -2 and 0 in its area-Jacobi operator spectrum.

In recent joint work with Peng Wang, we show the W-stability of M persists in all higher dimensional spheres if and only if the Laplacian of M has first eigenvalue 2. The square Clifford 2-torus in S^3 and the equilateral minimal 2-torus in S^5 have this spectral gap, and each is embedded by first eigenfunctions, so both are "persistently" W-stable. On the other hand, we discovered the equilateral torus has nontrivial third variation (with vanishing second variation) of W, and thus is not a W-minimizer (though it is the W-minimizer if we fix the conformal type)! This is evidence the Willmore Conjecture holds in every codimension.

Another recent result of ours concerns higher genus minimal surfaces (such as those – constructed long ago – by Lawson and also those by Karcher-Pinkall-Sterling) in S^3 which Choe-Soret showed are embedded by first eigenfunctions: we show their first eigenspaces are always 4-dimensional, and that this implies each is (up to Möbius transformations of S^n) the unique W-minimizer in its conformal class. (Some analogous results hold for free boundary minimal surfaces in the unit ball B^n....)

Alice Lim [NV III, 3:30-4:10, Friday, June 21]. Syracuse University

Loops to Infinity and Beyond: $H_{n-1}(M, \mathbb{Z})$ of Noncompact Manifolds with $\mathrm{ric}_f^m \neq 0$.

Abstract: In 1999, Shen and Sormani proved that if $\mathrm{ric} \geq 0$, then $H_{n-1}(M, \mathbb{Z}) = 0$. I will give a sketch of this proof, and then I will give a generalization of this to the $\mathrm{ric}_f^m \geq 0$ case, which I will define in this talk.

Yuhang Liu [NV II, 1:30-2:10, Thursday, June 20]. University of Pennsylvania

On positively curved 6-manifolds.

Abstract: Closed Riemannian manifolds with positive sectional curvature are a class of fundamental objects in Riemannian geometry. There are few known examples of such manifolds, each of which possesses interesting geometric properties. In this talk, I will review a few classical results in positive curvature, including Bonnet-Myers theorem and Synge theorem. Then I will move to the special case of dim 6, talking about the classification problem of positively curved 6-manifolds with certain non-Abelian symmetry conditions. I showed that such manifolds have Euler characteristic 2, 4, or 6, and obtained topological classification after imposing certain restrictions on the action. This is ongoing research on my thesis topic.

Frank Morgan [NV I, 9:10-9:50, Saturday, June 22]. Williams College

Optimal Tiles.

Abstract: A regular hexagon is the least-perimeter unit-area tile of the plane. What is the best pentagon? What about the hyperbolic plane? What about 3D? The talk will include open questions and some recent results, some by undergraduates.

Tony Pantev [NV I, 4:15-5:15, Thursday, June 20]. University of Pennsylvania

Twisted spectral correspondence and torus knots.

Abstract: I will explain how the cohomological invariants of twisted wild character varieties can be derived from enumerative Calabi-Yau geometry and refined Chern-Simons invariants of torus knots. I will describe a geometric approach for computing such topologies which is based on a spectral correspondence for meromorphic Higgs bundles with fixed conjugacy classes at the marked points. This construction is carried out for twisted wild character varieties associated to $(l, lk-1)$ torus knots, providing a colored generalization of theorems of Hausel, Mereb and Wong and of Shende, Treumann and Zaslow. The talk is based on recent joint works with Chuang, Diaconescu, Donagi, and Nawata.

David Recio Mitter [NV II, 11:20-12:00, Friday, June 21]. Lehigh University

Geodesic complexity.

Abstract: The topological complexity $TC(X)$ was introduced in 2003 by Farber to measure the instability of robot motion planning for a robot moving in a space X. There is no efficiency condition on the paths of the robot in this setting. We define a new version of topological complexity in which we require the robots to move along shortest paths (minimizing geodesics), which we call geodesic complexity. We show that the geodesic complexity is sensitive to the metric and in general differs from the topological complexity, which only depends on the homotopy type of the space.

Steve Scheirer [NV I, 1:30-2:10, Thursday, June 20]. Ashland University

Discrete Morse theory and topological complexity of unordered graph configuration spaces.

Abstract: The topological complexity of a space X, denoted by $TC(X)$, is related to the problem of motion planning within the space X. More specifically, $TC(X)$ can be viewed as the minimum number of “continuous rules" that are required to describe how to move between any two points in X. In this talk, we will be interested*infinity* in the topological complexity of unordered graph configuration spaces. We will discuss an approach to studying these spaces using discrete Morse theory and use this approach to determine the topological complexity for certain classes of graphs.

Nicholas Scoville [NV II, 3:30-4:10, Friday, June 21]. Ursinus College

A new digital homotopy theory.

Abstract: We present recent progress with collaborators Greg Lupton and John Oprea towards developing a digital version of homotopy theory. An n-dimensional digital image is a finite subset of the integer lattice along with an adjacency relation. Although there are many papers on digital homotopy theory, many of the notions do not seem satisfactory from a homotopy point of view. Indeed, some of the constructs most useful in homotopy theory, such as cofibrations and path spaces, are absent from the literature or completely trivial. Working in the digital setting, we develop some basic ideas of homotopy theory, including cofibrations and path fibrations, in a way that seems more suited to homotopy theory. We will indicate how our approach may be used, for example, to study Lusternik-Schnirelmann category in a digital setting. One future goal is to develop a characterization of a "homotopy circle" (in the digital setting) using the notion of topological complexity. This is with a view towards recognizing circles, and perhaps other features, using these ideas. This talk will introduce some of the basics of digital topology and will not require any specialized background.

Christina Sormani [NV I, 3:20-4:00, Thursday, June 20]. Lehman College and CUNYGC

Convergence of Riemannian Manifolds and Metric Spaces.

Abstract: Sequences of Riemannian manifolds and metric spaces can be seen to have a variety of limits using various notions of convergence. I will present some new examples and discuss when Gromov-Hausdorff and Intrinsic Flat limits agree and disagree.

Gabor Szekelyhidi [NV I, 9:00-10:00, Friday, June 21]. University of Notre Dame

Gromov-Hausdorff limits of Kähler manifolds.

Abstract: Through the work of Cheeger, Colding, Naber and others we have a deep understanding of the structure of Gromov-Hausdorff limits of Riemannian manifolds with Ricci curvature lower bounds. For polarized Kähler manifolds, this was taken further by Donaldson-Sun, who showed that under two-sided Ricci curvature bounds, non-collapsed limit spaces are projective varieties, leading to major progress in Kähler geometry. I will discuss joint work with Gang Liu giving an extension of this result to the case when the Ricci curvature is only bounded from below.

Yury Ustinovskiy [NV III, 11:20-12:00, Friday, June 21]. New York University

On Hermitian Manifolds of Semipositive Griffiths Curvature.

Abstract: In this talk we will discuss various uniformization problems for complex and algebraic manifolds with 'semipositive curvature'. We will focus on Hermitian manifolds with semipositive Griffiths curvature and explain how metric flows in complex geometry could be applied to study such manifolds.
We show that after arbitrary small deformation of the initial metric along Hermitian Curvature Flow, the null spaces of the Chern-Ricci two-form generate a holomorphic, integrable distribution. This distribution induces an isometric, holomorphic, almost free action of a complex Lie group on the universal cover of the manifold. Our result can be thought of as a Hermitian analogue of the Cheeger-Gromoll/Howard-Smyth-Wu splitting theorem.

David White [NV III, 2:25-3:05, Thursday, June 20]. Denison University

On the existence of N_∞-operads in equivariant homotopy theory.

Abstract: In a 2016 Annals paper, Hill, Hopkins, and Ravenel resolved the Kervaire Invariant One problem using tools from equivariant stable homotopy theory. Of particular importance were equivariant commutative ring spectra and their multiplicative norms. A more thorough investigation of multiplicative norms, using the language of operads, was recently conducted by Blumberg and Hill, though the general existence of their new N-*infinity* operads was left as a conjecture. In this talk, I will prove the Blumberg-Hill conjecture using a new model structure on the category of equivariant operads. This is joint work with Javier Gutierrez.