Problem 5, make-up exam 2, part (a):

Given that the characteristic polynomial of A is $(\lambda-2)(\lambda-1)$, eigenvalues are $\lambda_1 = 2$, and $\lambda_2 = 1$.

For the eigenspace for $\lambda_1 = 2$, we reduce the coef matrix of the system $(A-2I)\vec{x} = \vec{0}$,

$$A - 2I = \begin{pmatrix} -9 & 36 \\ -2 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & -4 \\ 0 & 0 \end{pmatrix},$$

so for $x_2 = a$ we get $x_1 = 4a$, and the eigenvectors for $\lambda_1 = 2$ are the

vectors $(x_1, x_2) = a(4,1)$ with $a \neq 0$, from the space with basis $(4,1)$.

For the eigenspace for $\lambda_2 = 1$,

$$A - 1I = \begin{pmatrix} -8 & 36 \\ -2 & 9 \end{pmatrix} \to \begin{pmatrix} 2 & -9 \\ 0 & 0 \end{pmatrix},$$

so for $x_2 = 2b$ we get $x_1 = 9b$, and the eigenvectors for $\lambda_2 = 1$ are $(x_1, x_2) = b(9,2)$ with $b \neq 0$, from the space with basis $(9,2)$.

---

part (b):

The matrix A is diagonalizable, with $S = \begin{pmatrix} 4 & 9 \\ 1 & 2 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.