Math 205, Spring 2011

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Text - Goode-Annin, 3rd edt.

Office hours: M,F: 2:20-3:30, Tue: 11:30-12:30
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1. Course Info

2. Homework 1:

   Chapter 1, Sections 1, 2, 3, 4, 5, 6

   Due Monday, Jan 24th
Chapter 1: Intro, Slope Fields, verify solution (Monday, 1/17)

1.4, 1.5: Separable DE, Population/logistic growth (start 1.6; Wednesday, 1/19)

1.6 Linear Equations (Friday, 1/21)
Problem 1:

Verify that the function \( y = c_1 \sqrt{x} \) is a solution of \( y' = \frac{y}{2x} \)

Solution:

Compute \( y' \) and check.

\[
y' = c_1 \left( \frac{1}{2} \right) x^{-\frac{1}{2}}.
\]

\[
\frac{y}{2x} = \frac{c_1 \sqrt{x}}{2x}
\]

\[
= c_1 \left( \frac{1}{2} \right) \frac{\sqrt{x}}{(\sqrt{x})^2}
\]

\[
= c_1 \left( \frac{1}{2} \right) \frac{1}{\sqrt{x}}
\]

\[
= y'.
\]
Problem 2:

Determine all values $r$ so $y = e^{rx}$ is a solution to $y'' - 4y' + 3y = 0$. 
Linear DE, Main Step: to solve linear DE
\[ \frac{dy}{dx} + p(x)y = q(x) \]
multiply by the integral factor \( f = e^{\int p \, dx} \)
and use
\[ \frac{d}{dx}(fy) = f(y' + py) \]
on the left to get \( \frac{d}{dx}(fy) = f \, q \).

Problem:
Solve
\[ \frac{dy}{dx} + \frac{2x}{(1 - x^2)}y = 4x, \quad -1 \leq x \leq 1. \]

Solution:
Find integral factor, inside integral first:
\[ \int \frac{2x}{(1 - x^2)} \, dx \]
\[ = - \ln(1 - x^2) = \ln \left( (1 - x^2)^{-1} \right) \quad \text{(simplify!)} \].
So \( e^{\int \frac{2x}{(1-x^2)} \, dx} = e^{\ln((1-x^2)^{-1})} \)

\[ = (1 - x^2)^{-1} = \frac{1}{1 - x^2} = f. \]

Multiply by \( f = \frac{1}{1 - x^2} \) and use Main Property:

\[ \frac{1}{1 - x^2} \left( \frac{dy}{dx} + \frac{2x}{(1 - x^2)}y \right) = \frac{4x}{1 - x^2}, \]

\[ \frac{d}{dx} \left( \frac{y}{1 - x^2} \right) = \frac{4x}{1 - x^2}. \]

Now integration gives \( \frac{y}{1 - x^2} = (-2 \ln (1 - x^2)) + c, \)

so \( y = (1 - x^2) \left(- \ln ((1 - x^2)^2) + c \right). \)

Notice that we can check this instance of the Main Property directly (using the product rule).