**Math 205, Spring 2011**

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Text - Goode-Annin, 3rd edt.

**Office hours:** M,F: 2:20-3:30, Tue: 11:30-12:30

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## 1. Course Info

## 2. Homework 1:

Chapter 1, Sections 1, 2, 3, 4, 5, 6

Due Monday, Jan 24th

Chapter 1: Intro, Slope Fields, verify solution (Monday, 1/17)

1.4, 1.5: Separable DE, Population/logistic growth (start 1.6; Wednesday, 1/19)

1.6 Linear Equations (Friday, 1/21

**Problem 1:**

Verify that the function $y = c_1\sqrt{x}$ is a

solution of $y\prime = \dfrac{y}{2x}$

**Solution:**

Compute $y\prime$ and check.

$$y\prime = c_1(\tfrac{1}{2})x^{-\frac{1}{2}}.$$

$$\frac{y}{2x} = \frac{c_1\sqrt{x}}{2x}$$

$$= c_1(\tfrac{1}{2})\frac{\sqrt{x}}{(\sqrt{x})^2}$$

$$= c_1(\tfrac{1}{2})\frac{1}{\sqrt{x}}$$

$$= y\prime.$$

**Problem 2:**

Determine all values $r$ so $y = e^{rx}$ is a solution to $y'' - 4y' + 3y = 0$.

**Linear DE, Main Step:** to solve linear DE

$$\frac{dy}{dx} + p(x)y = q(x)$$

multiply by the integral factor $f = e^{\int p\,dx}$

and use

$$\frac{d}{dx}(fy) = f\,(y' + py)$$

on the left to get $\frac{d}{dx}(fy) = fq$.

**Problem:**

Solve $\frac{dy}{dx} + \frac{2x}{(1-x^2)}y = 4x,\ -1 \le x \le 1.$

**Solution:**

Find integral factor, inside integral first:

$$\int \frac{2x}{(1-x^2)}\,dx$$

$$= -\ln(1-x^2) = \ln\left((1-x^2)^{-1}\right) \quad \text{(simplify!)}.$$

So $e^{\int \frac{2x}{(1-x^2)}\,dx} = e^{\ln\left((1-x^2)^{-1}\right)}$

$$= \left(1-x^2\right)^{-1} = \frac{1}{1-x^2} = f.$$

Multiply by $f = \dfrac{1}{1-x^2}$ and use Main Property:

$$\frac{1}{1-x^2}\left(\frac{dy}{dx} + \frac{2x}{(1-x^2)}y\right) = \frac{4x}{1-x^2},$$

$$\frac{d}{dx}\left(\frac{y}{1-x^2}\right) = \frac{4x}{1-x^2}.$$

Now integration gives $\dfrac{y}{1-x^2} = (-2\ln(1-x^2)) + c,$

so $y = (1-x^2)\left(-\ln((1-x^2)^2) + c\right).$

Notice that we can check this instance of the Main Property directly (using the product rule).