Problem 5, exam 2, part (a):

Given that the characteristic polynomial of A is $(\lambda - 2)^2$, the only eigenvalue is $\lambda_1 = 2$, with (algebraic) multiplicity 2.

For the eigenspace, we reduce the coef matrix of the system $(A - 2I)\vec{x} = \vec{0}$,

$$A - 2I = \begin{pmatrix} -3 & 9 \\ -1 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -3 \\ 0 & 0 \end{pmatrix},$$

so for $x_2 = a$ we get $x_1 = 3a$, and the eigenvectors for $\lambda_1 = 2$ are the

vectors $(x_1, x_2) = a(3, 1)$ with $a \neq 0$, from the space with basis $(3, 1)$.

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part (b): The matrix A is not diagonalizable. The most straightforward explanation is that A is 2-by-2, so we need two linearly independent eigenvectors, but we only have one. The easiest explanation to state correctly is that the geometric multiplicity is 1, but that does not match the algebraic multiplicity of 2 (for the eigenvalue $\lambda_1 = 2$). Alternatively, the dimension of the eigenspace (which is the geom. multiplicity) is 1, and does not match the alg. multiplicity.