Exam 2, Math 205, Spring 2013

Problem 3: Let A be the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 5 & 4 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

(a) **Find a basis for the nullspace of A.**

Solution: $A \to \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

The free variables are x_3 and x_4, and $x_1 = -2x_3 + x_4, x_2 = -x_4$.

So $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_3 + x_4 \\ -x_4 \\ x_3 \\ x_4 \end{bmatrix}$ with basis $\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(b) **Find a basis for the column space of A.**

Solution: Since the columns of A that have leading 1's in the reduced matrix are a basis of the column space of A, we use the first and second columns of A:

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \right\}$.