

Week 2b, Homework 7:

Chapter 3, Sections 1, 2, 3

Homework 8:

Chapter 3, Section 4 (review); Chapter 4, Section 1

Section 3.1 Intro

Section 3.2 Determinants using row operations

Section 3.3 Determinates using expansion by minors

We compute $\det \begin{pmatrix} 2 & 1 & 5 \\ 4 & 2 & 3 \\ 9 & 5 & 1 \end{pmatrix}$ using the

(first) row expansion (by minors):

$$\begin{aligned} \det(A) &= 2 \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 3 \\ 9 & 1 \end{vmatrix} + 5 \begin{vmatrix} 4 & 2 \\ 9 & 5 \end{vmatrix} \\ &= 2(2 - 15) - (4 - 27) + 5(20 - 18) \\ &= 2(-13) - (-23) + 5(2) = -26 + 33 = 7. \end{aligned}$$

Problem

$$\text{Reduce } A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{pmatrix}$$

to an upper triangular matrix and use the reduction to find $\det(A)$.

Solution:

$$A \rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{pmatrix} (r_1 \rightarrow r_1 - r_2)$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 3 & 7 & 11 \\ 0 & 5 & 12 & 15 \\ 0 & 7 & 15 & 18 \end{pmatrix}$$

$$(r_2 \rightarrow r_2 + 3r_1, r_3 \rightarrow r_3 + 4r_1, r_4 \rightarrow r_4 + 5r_1)$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 3 & 7 & 11 \\ 0 & 5 & 12 & 15 \\ 0 & 1 & 1 & -4 \end{pmatrix} (r_4 \rightarrow r_4 - 2r_2)$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 5 & 12 & 15 \\ 0 & 3 & 7 & 11 \end{pmatrix} \quad (r_2 \leftrightarrow r_4)$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 7 & 35 \\ 0 & 0 & 4 & 23 \end{pmatrix}$$

$$(r_3 \rightarrow r_3 - 5r_2, r_4 \rightarrow r_4 - 3r_2)$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & -1 & -11 \\ 0 & 0 & 4 & 23 \end{pmatrix} \quad (r_3 \rightarrow r_3 - 2r_4)$$

$$\rightarrow \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & -1 & -11 \\ 0 & 0 & 0 & -21 \end{pmatrix} = U \quad (r_4 \rightarrow r_4 + 4r_3).$$

Now we have $\det U = (-1)(1)(-1)(-21) = -21$,

and $\det A = (-1)\det U = 21$, since all of the 3rd elementary operations change the determinant by a factor of (1) – so no change – there are no 2nd EROs and exactly one 1st ERO, with each 1st ERO changing the determinant by a factor of (-1).