Math 205, Summer I 2016

Week 1b (continued):

An $m \times n$ **matrix** A is an array

with m horizontal rows; n vertical columns

i, jth entry $a_{i,j}$ in the ith row, and

jth column.

row vector or row n-vector, $\vec{a}$, is a $1 \times n$

matrix, just one row.

column vector or column n-vector, $\vec{b}$, is a

$n \times 1$ matrix, just one column.

Example 1. Give the rows and columns of

$$A = \begin{pmatrix} 2 & 10 & 6 \\ 5 & -1 & 3 \end{pmatrix}.$$

What are the entries $a_{1,2}, a_{2,1}, a_{3,1}, a_{1,3}$?

The matrix sum, $A + B$, is defined only when A and B have the same shape; and then the i,jth entry of $A + B$ is $a_{i,j} + b_{i,j}$, the sum of the i,jth entries of A and B.

The scalar multiple of the matrix A by the scalar (number!) c is the matrix with the same shape as A, but with i,jth entry of $ca_{i,j}$.

Matrix multiplication: 1. row n-vector by column m-vector, only when $n = m$, is the number

$a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$,

where the a_i are the entries of $\vec{a}$ and the b_i are the entries of $\vec{b}$.

2. $m \times n$ matrix A by p-column vector $\vec{b}$, only when $n = p$, is the m-column vector with ith entry (ith row of A) $\cdot$ $\vec{b}$.

Problem 2. Multiply $A = \begin{pmatrix} -1 & 2 \\ 4 & 7 \\ 5 & -4 \end{pmatrix}$ by

$$c = \begin{pmatrix} 5 \\ -1 \end{pmatrix}.$$

Solution:

$$\begin{aligned} Ac &= \begin{pmatrix} -1 & 2 \\ 4 & 7 \\ 5 & -4 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} (-1)(5) + 2(-1) \\ 4(5) + 7(-1) \\ 5(5) + (-4)(-1) \end{pmatrix} = \begin{pmatrix} -7 \\ 13 \\ 29 \end{pmatrix}. \end{aligned}$$

Linear Systems

If A is an $m \times n$ matrix with entries $a_{i,j}$,
$\vec{x}$ is the n-column vector with entries $x_1, \ldots x_n$,
and $\vec{b}$ is the m-column vector with entries $b_1, \ldots b_m$,
the matrix equation $A\vec{x} = \vec{b}$ gives m equations,
each of the form (ith row of A) $\cdot \vec{x} = b_i$,
which is called a **linear system of m equations in n variables.**

In the equation $A\vec{x} = \vec{b}$, the matrix A is called the **coefficient matrix** of the system, and $\vec{x}$ and $\vec{b}$ are called the vector of unknowns and the right-hand side vector, respectively.

Our method of solution involves one more matrix associated with the system of equations, the **Augmented Matrix**

$$A^{\#} = (A|\vec{b}).$$

Problem 3. Find the augmented matrix of the following system of equations.

$$\begin{aligned}
2x_1 + 2x_2 \qquad - 3x_4 \qquad &= 0 \\
-x_1 - \ x_2 + x_3 + x_4 - x_5 &= 2 \\
x_1 + \ x_2 + x_3 - 2x_4 - x_5 &= 2
\end{aligned}$$