Homework #6

Math 205

Due Thursday, Mar 24 (TuTh) or Friday, Mar 25 (MWF).

You must show your work in order to receive full credit; correct answers with no, or insufficient work, may not receive credit.

1. (6 points) Let $B = \{(3,5), (2,-1)\}$ be a (ordered) basis for $\mathbb{R}^2$, and let $E = \{(1,0), (0,1)\}$ be the standard basis.

   (a) Find the change-of-basis matrix $P_{E \leftarrow B}$.

   (b) Find the change-of-basis matrix $P_{B \leftarrow E}$.

   (c) If $\vec{v} = (3, -1)$, find $[\vec{v}]_B$.

2. (6 points) The nullspace of the matrix

$$A = \begin{bmatrix} 5 & -6 & 8 & -8 \\ -3 & 8 & 4 & -4 \\ 2 & 2 & 12 & -12 \end{bmatrix}$$

   has basis $B = \{(-4, -2, 1, 0), (4, 2, 0, 1)\}$

   (a) Is $\vec{v} = (3, -1, 3, 2)$ in the nullspace of A? If so, find $[\vec{v}]_B$.

   (b) Is $\vec{w} = (-12, -6, 4, -1)$ in the nullspace of A? If so, find $[\vec{w}]_B$.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & -3 & 1 & -1 & 0 & -1 \\ -1 & 3 & 0 & -1 & 1 & 3 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 2 & -6 & 0 & 2 & 1 & 0 \end{bmatrix}.$$

   (a) (6 points) Find a basis for the nullspace of A.

   (b) (3 points) Find a basis for the rowspace of A.

   (c) (3 points) Find a basis for the columnspace of A. Explain how you know that the vectors you give are a basis. Did you compute something? If not, how did you know which vectors to pick?

4. (6 points) The following problems are to be solved using the Rank-Nullity Theorem.

   (a) If A is a 6-by-4 matrix with $\text{Rank}(A) = 1$, what is the dimension of the null space of A? Justify your answer.

   (b) If A is a 6-by-7 matrix, is it possible that $\text{Rank}(A) = 4$ and the dimension of the null space of A is 3? Justify your answer.