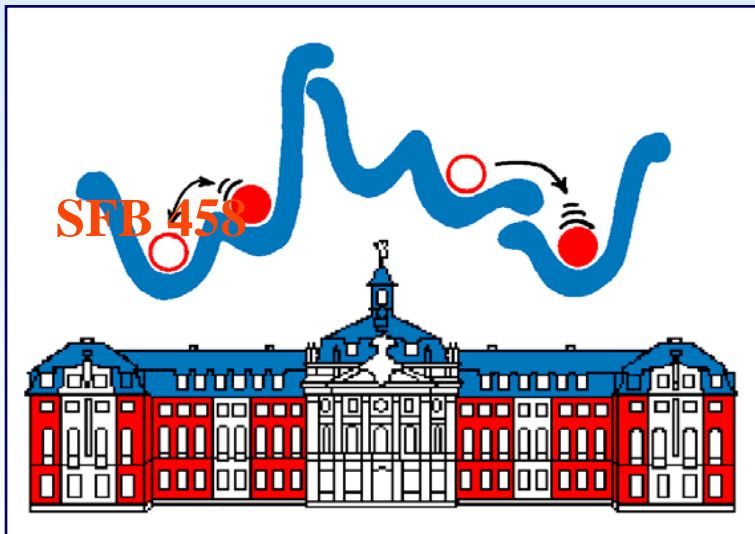


# Ionic motion in materials with disordered structures

## Module 1 (includes Parts 1&2)

Klaus Funke, Münster



Many-particle dynamics  
studied from picoseconds  
to minutes and hours to  
find rules for ionic motion  
in disordered materials

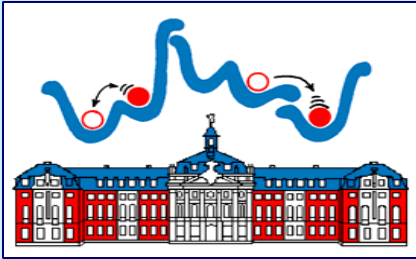
**Glass Tutorial Series:** prepared for and produced by the  
**International Material Institute for New Functionality in Glass**  
An NSF sponsored program – material herein not for sale  
Available at [www.lehigh.edu/imi](http://www.lehigh.edu/imi)

Delivered 5/5/06 at Lehigh University

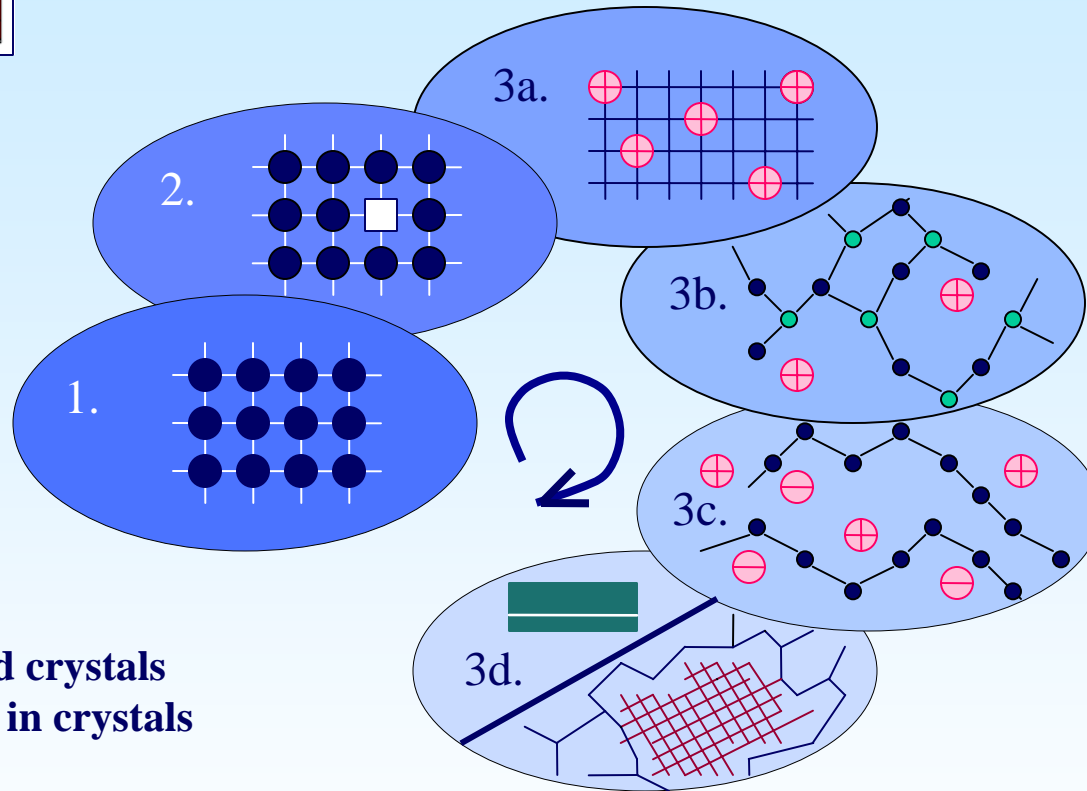


## Video Module 1 (includes Parts 1 &2)

1. Materials and phenomena : the rôle of disorder
2. Microscopy in time :  $\sigma(\omega)$ ,  $\varepsilon(\omega)$  and scaling properties
3. Dynamics of ions in motion : searching for simple rules



# An evolving scheme of materials science



- 1. Ideally ordered crystals
- 2. Point disorder in crystals

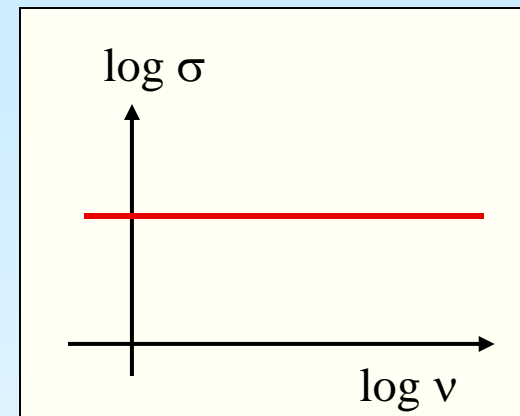
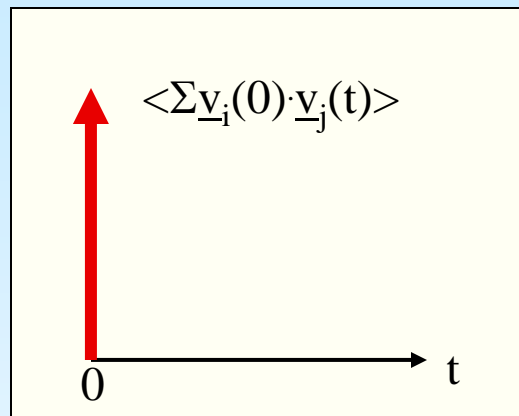
3a. Crystals with structural disorder

3b. Ion-conducting glasses

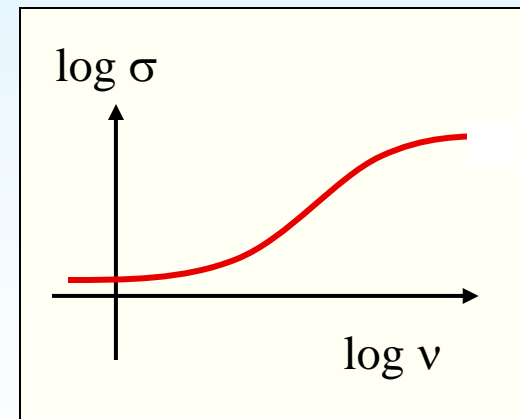
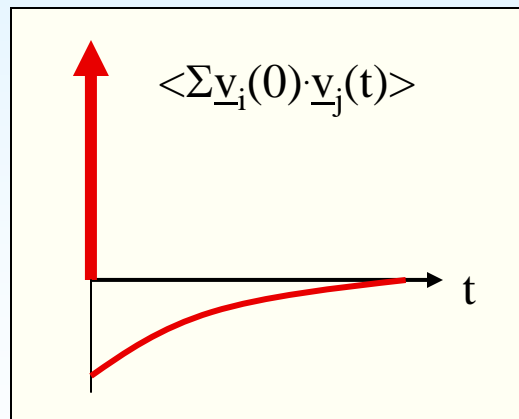
3c. Polymer electrolytes

3d. Thin-film systems, composite systems

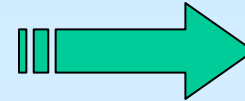
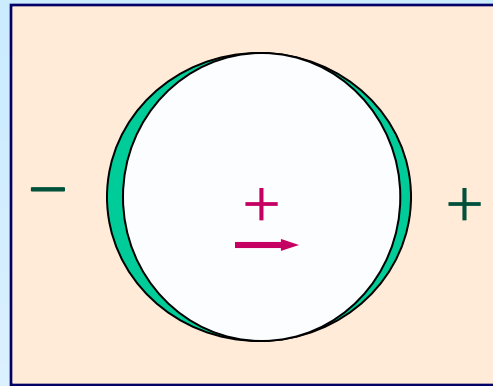
level two,  
random hopping:



level three,  
correlated hopping:

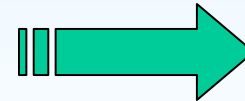
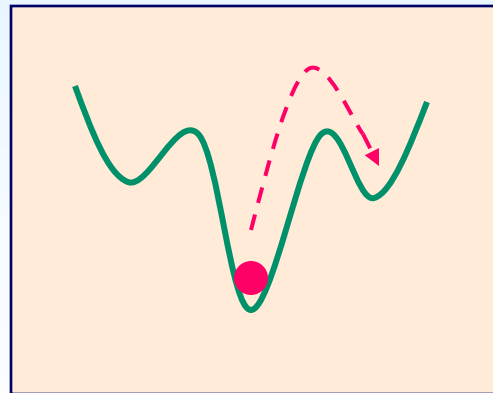


dilute strong  
liquid electrolyte:



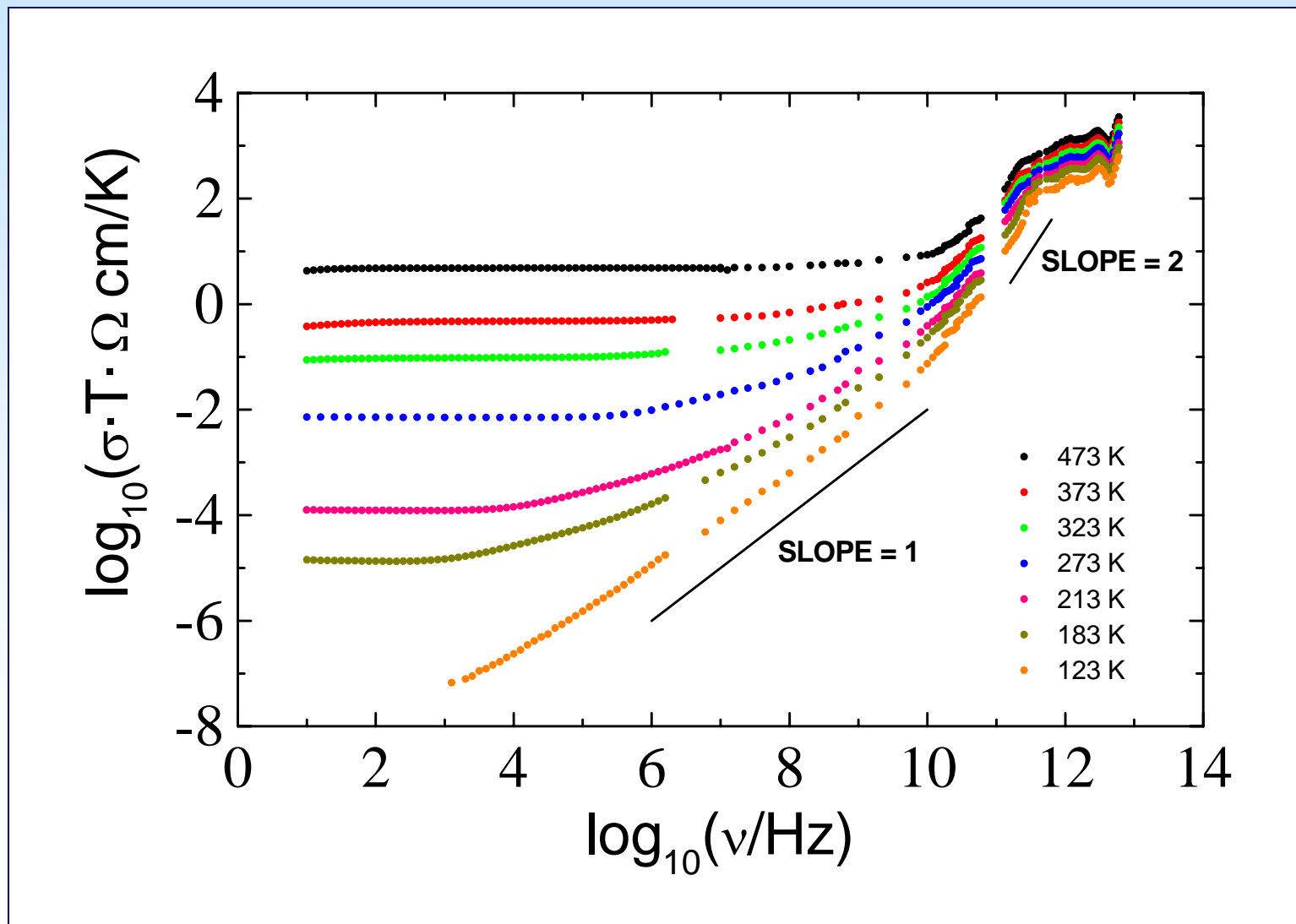
dispersion  
in  $\sigma(\omega)$   
hardly  
visible

solid electrolyte  
with structural  
disorder:



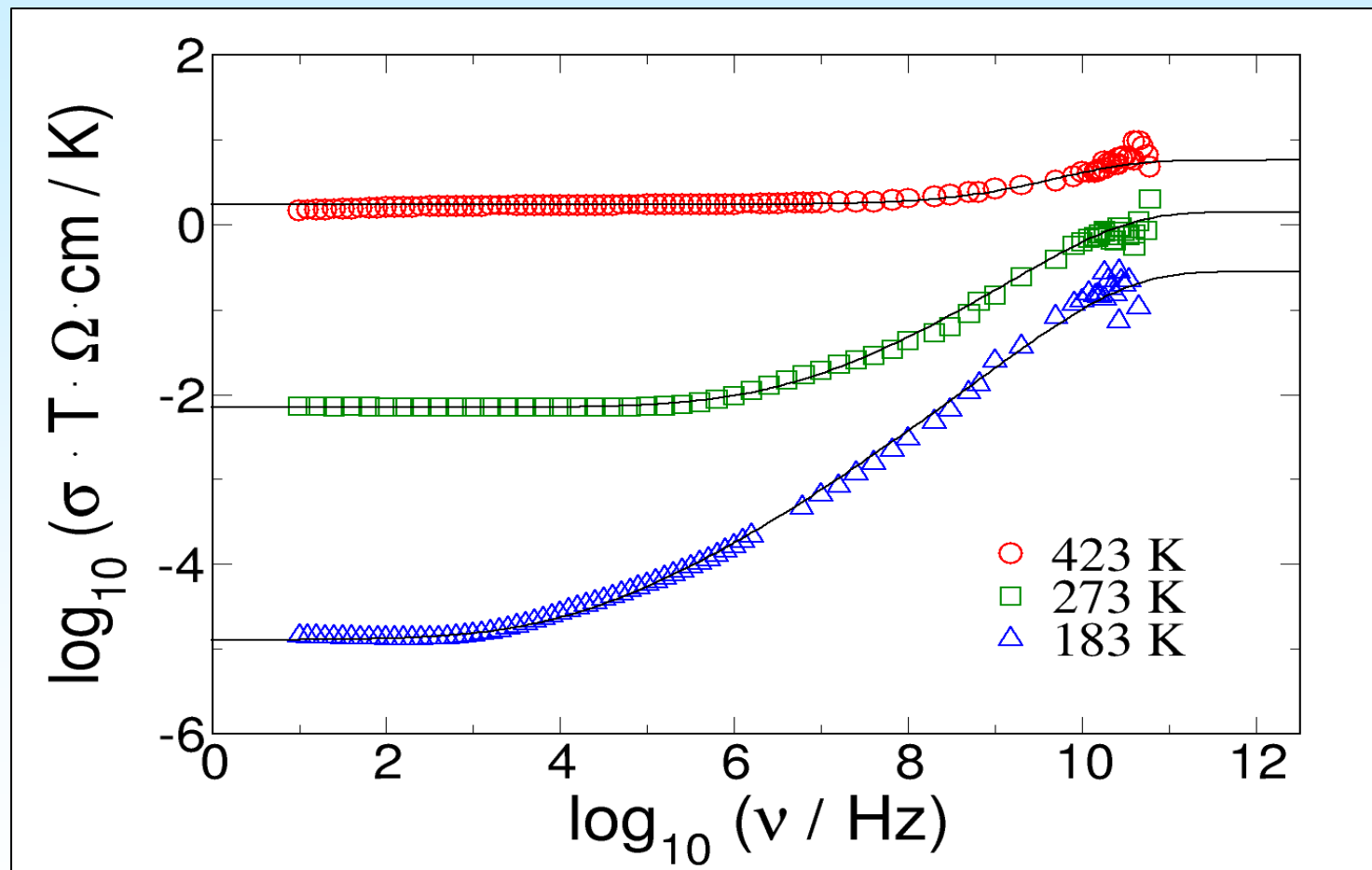
dispersion  
over many  
decades in  
conductivity

$\text{Ag}_2\text{S} \cdot \text{GeS}_2$  glassy electrolyte, from 123 K to 473 K



# $\text{Ag}_2\text{S} \cdot \text{GeS}_2$ glassy electrolyte:

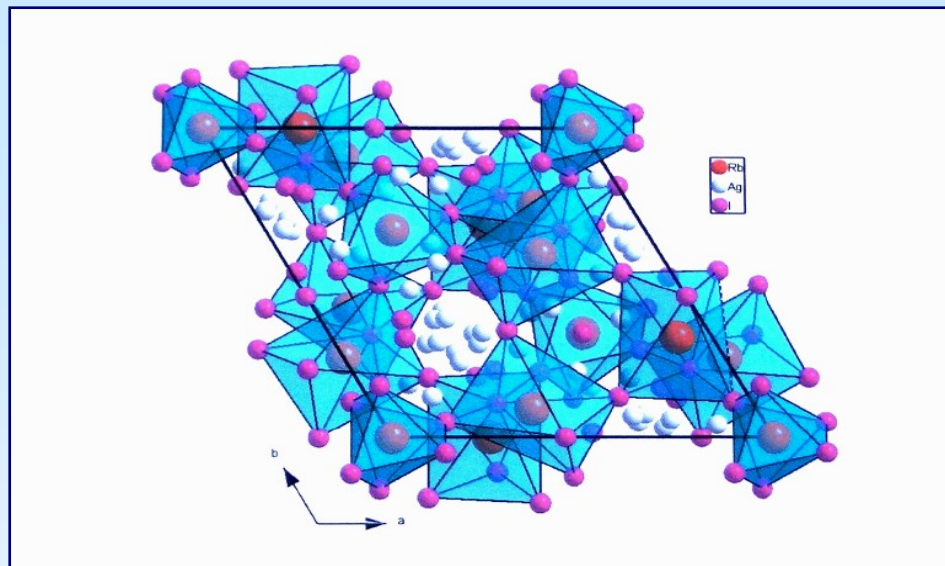
vibrational contribution removed, set of model spectra included



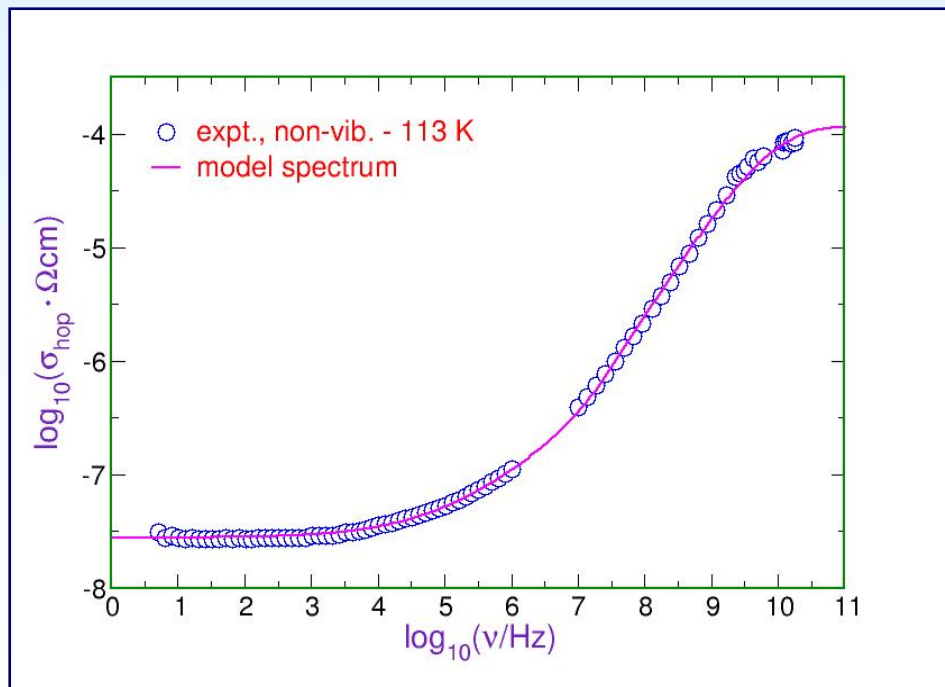
$\sigma(0) \cdot T$  and  $\sigma(\infty) \cdot T$  both Arrhenius activated



crystal structure .....

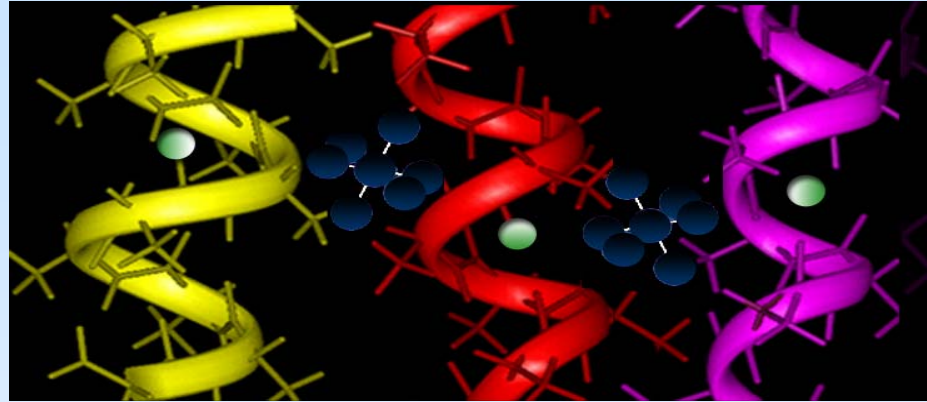


conductivity spectrum  
(vibrations removed)  
at 113 K .....

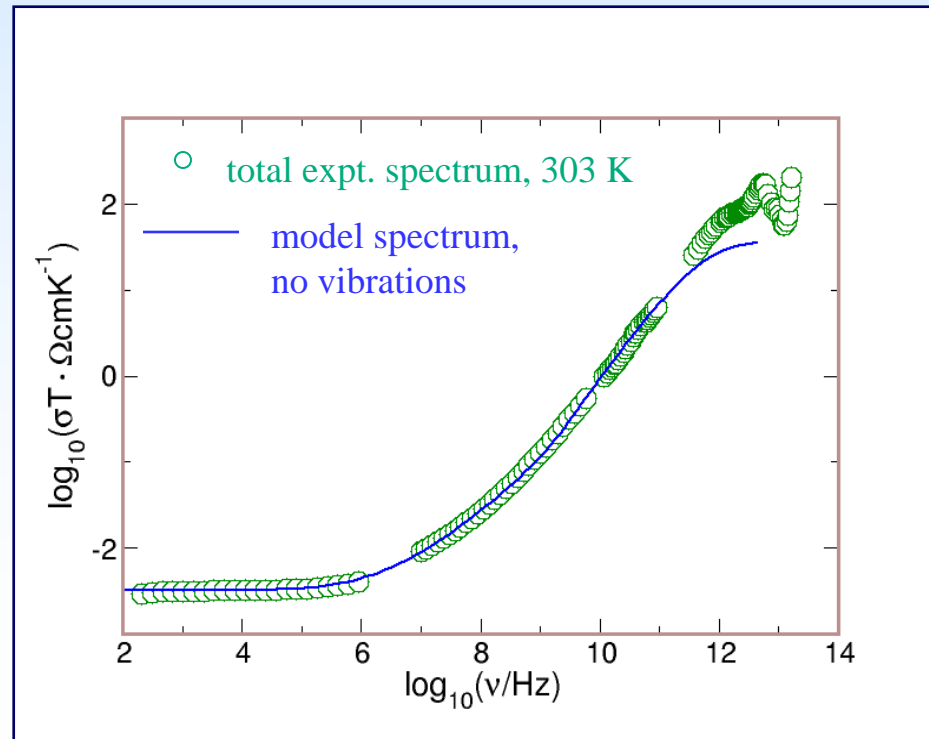




Salt-in-polymer  
electrolyte  
(1 molal  $\text{NaPF}_6$  in  
a polyurethane)



conductivity spectrum  
(including vibrations)  
at 303 K .....

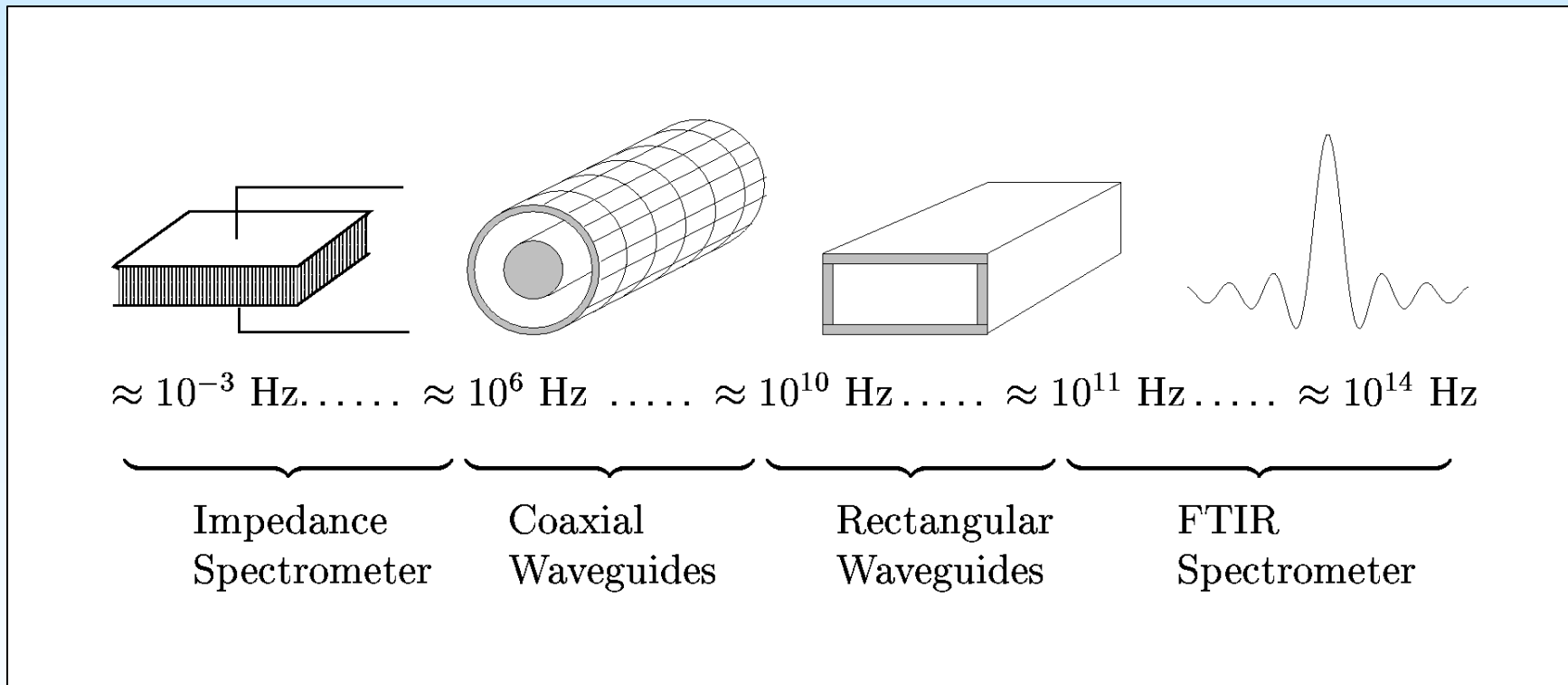


1. Materials and phenomena : the rôle of disorder

2. Microscopy in time :  $\sigma(\omega)$ ,  $\varepsilon(\omega)$  and scaling properties

3. Dynamics of ions in motion : searching for simple rules

## Measuring conductivity spectra, $\sigma(\omega)$ , and permittivity spectra, $\varepsilon(\omega)$ :

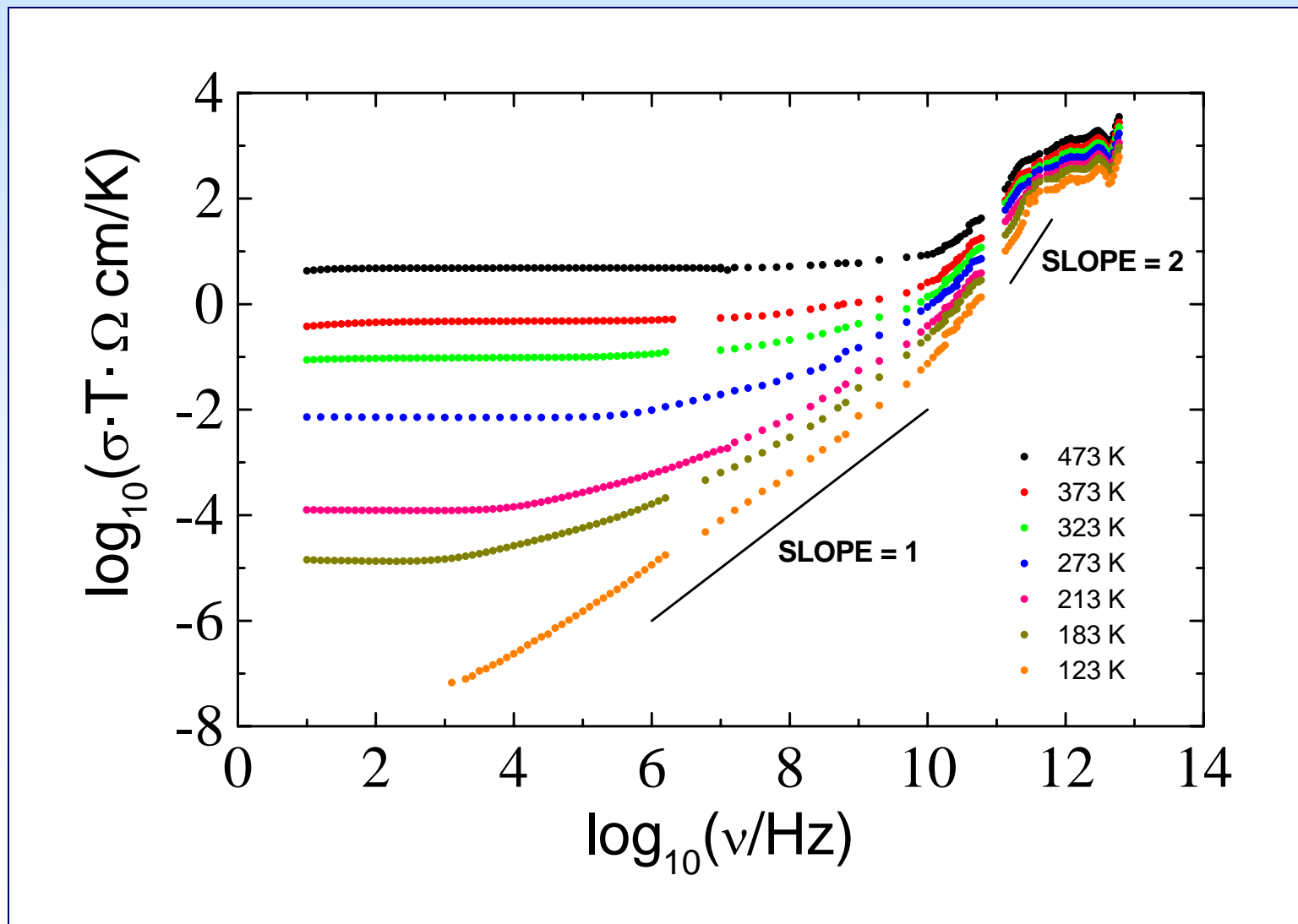


Measured quantities: amplitudes and phases of transmitted or reflected waves

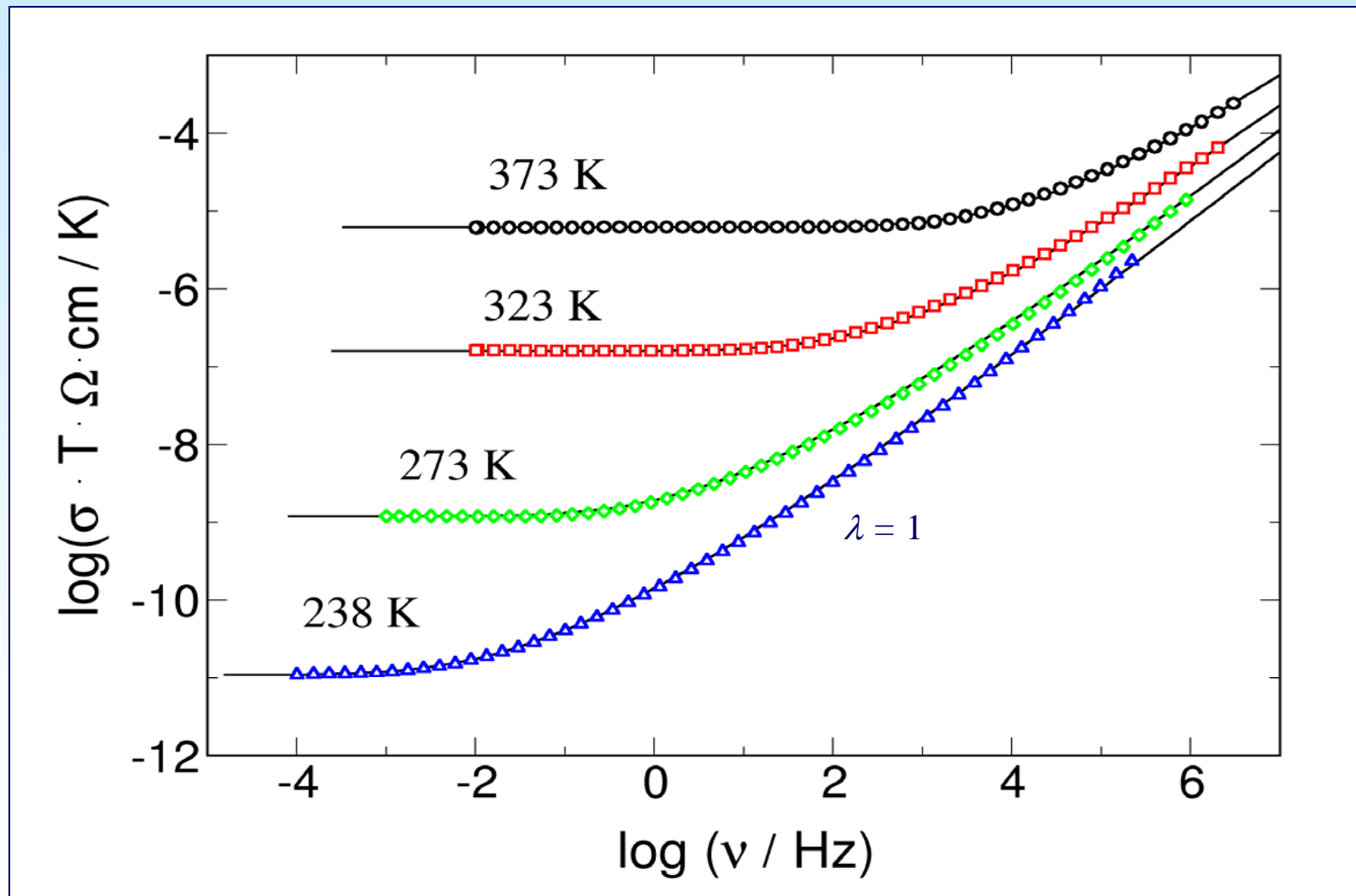
Basis for evaluation: Maxwell's equations plus boundary conditions

$$\rightarrow \boxed{\hat{\sigma}(\omega) = i\omega\varepsilon_0\hat{\varepsilon}(\omega)} \rightarrow \sigma(\omega) = \text{Re}\hat{\sigma}(\omega) \quad \text{and} \quad \varepsilon(\omega) = \text{Re}\hat{\varepsilon}(\omega)$$

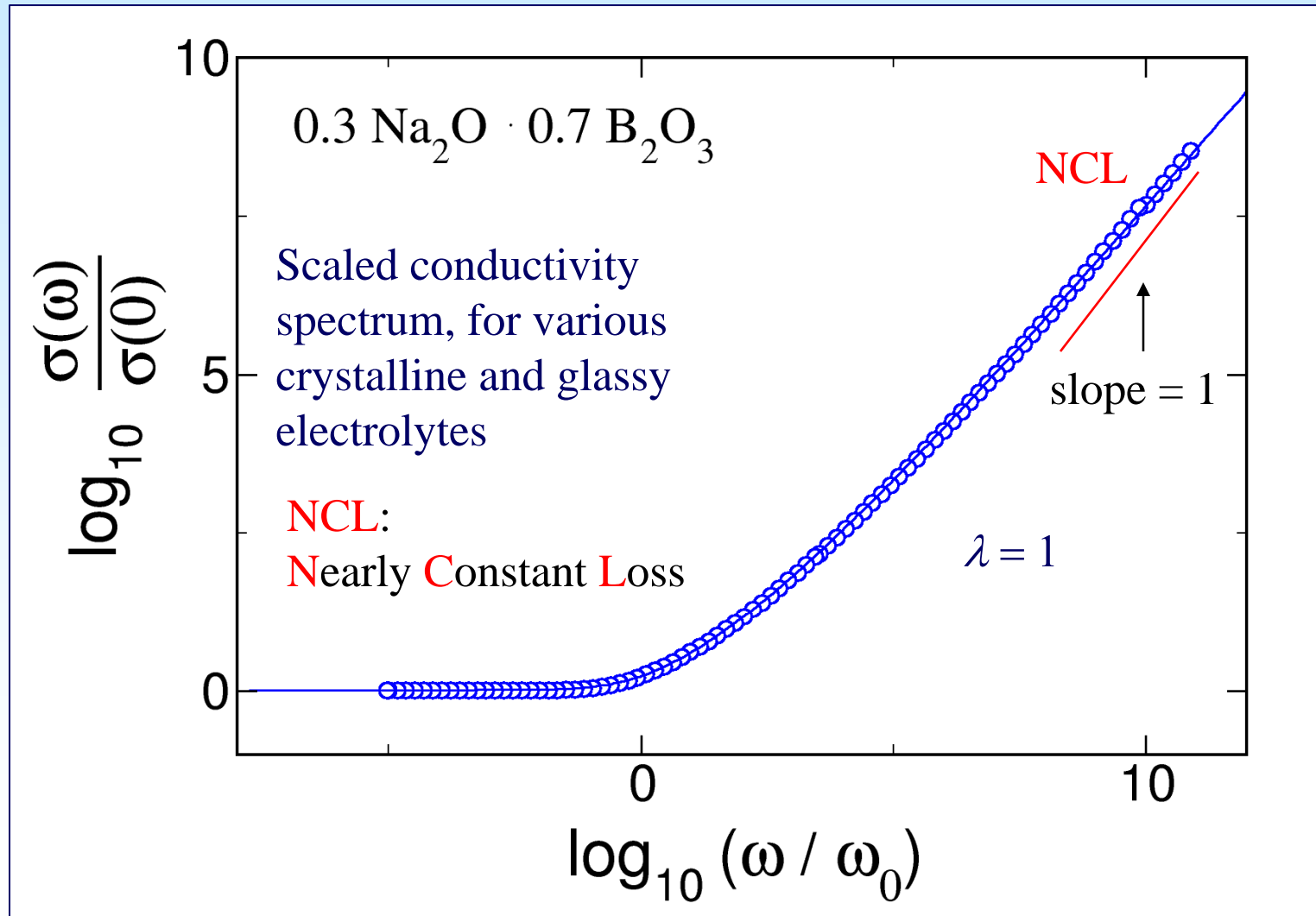
$\text{Ag}_2\text{S} \cdot \text{GeS}_2$  glassy electrolyte, from 123 K to 473 K



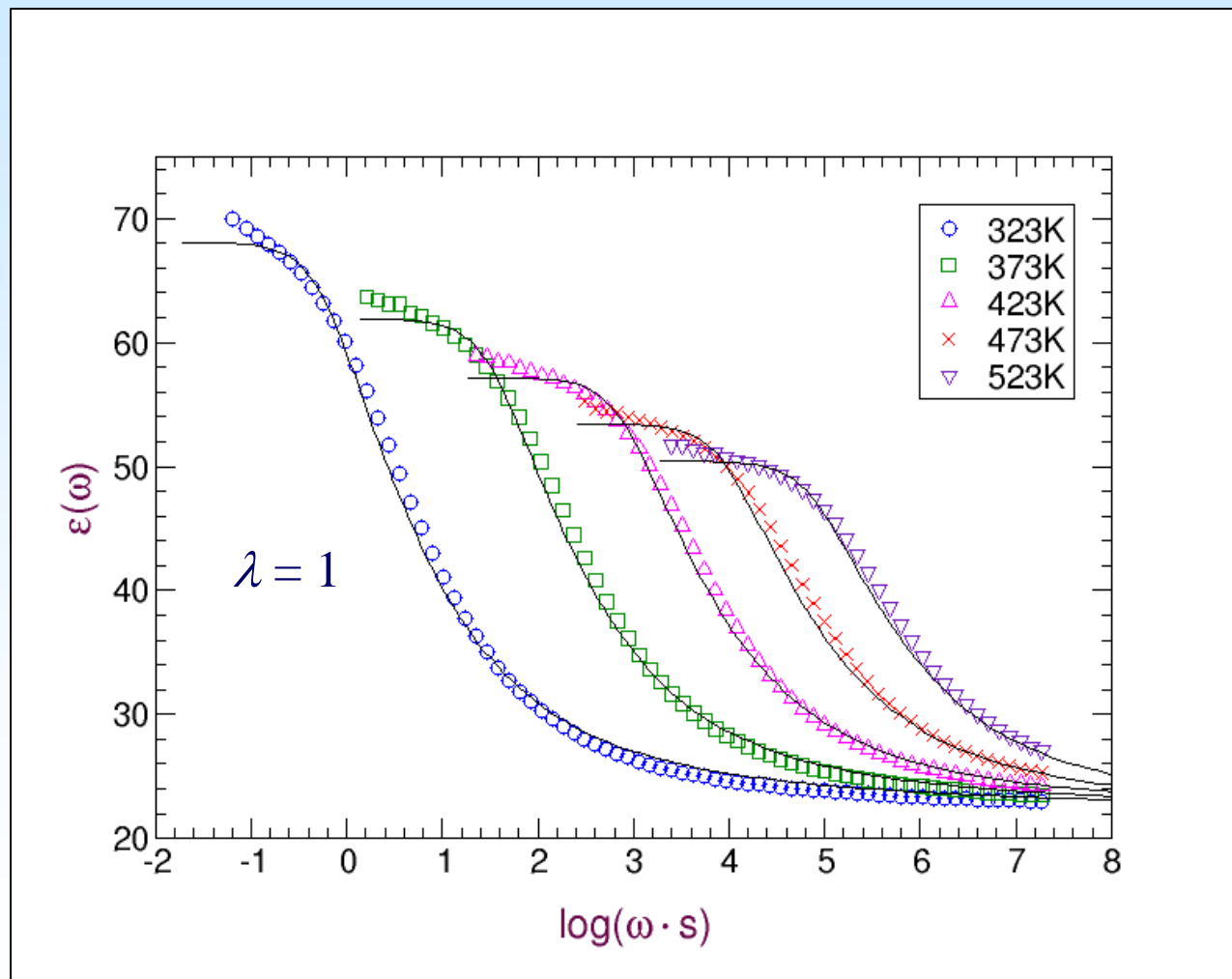
# Conductivity isotherms for glassy $0.3 \text{ Li}_2\text{O} \cdot 0.7 \text{ B}_2\text{O}_3$



## Gradual transition into NCL behavior :

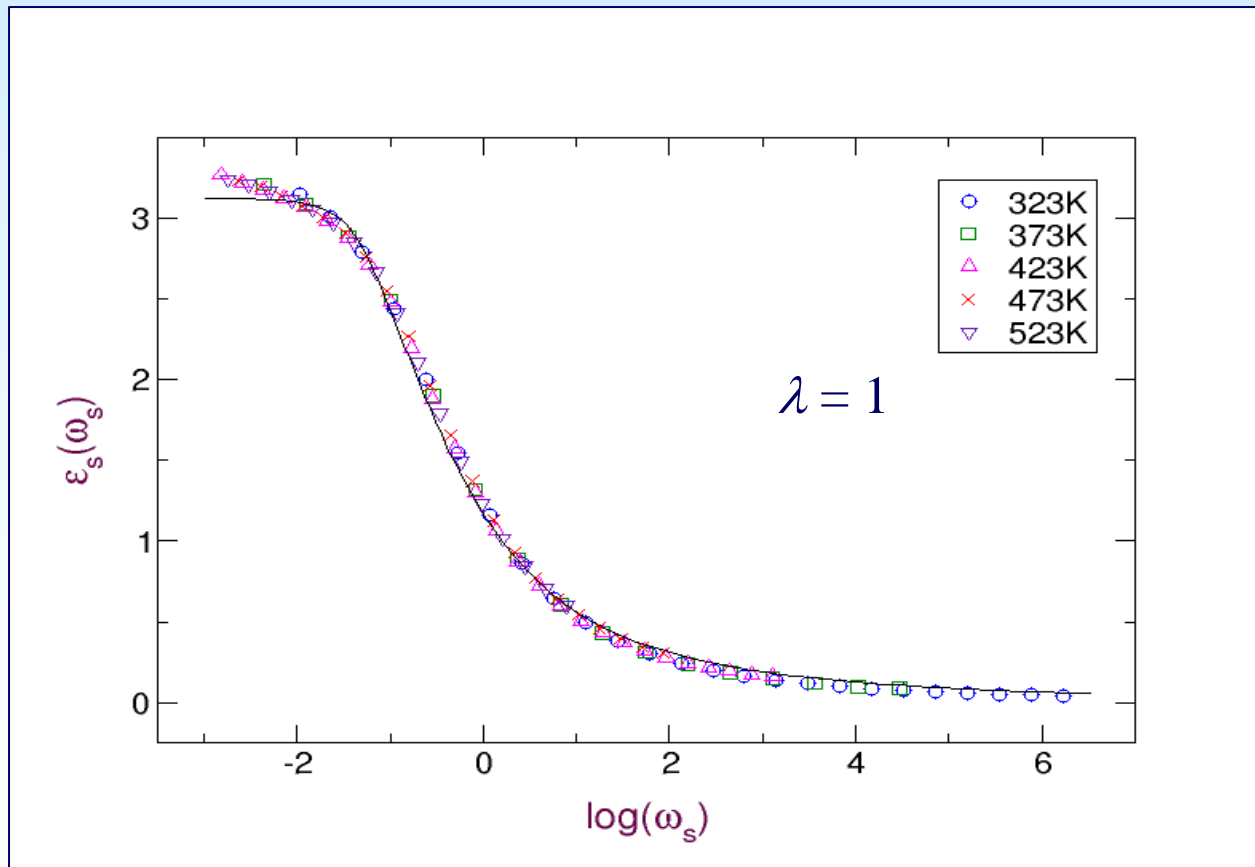


## 0.2 Na<sub>2</sub>O · 0.8 GeO<sub>2</sub> glass, permittivity isotherms



## Scaled permittivity for 0.2 Na<sub>2</sub>O · 0.8 GeO<sub>2</sub> glass

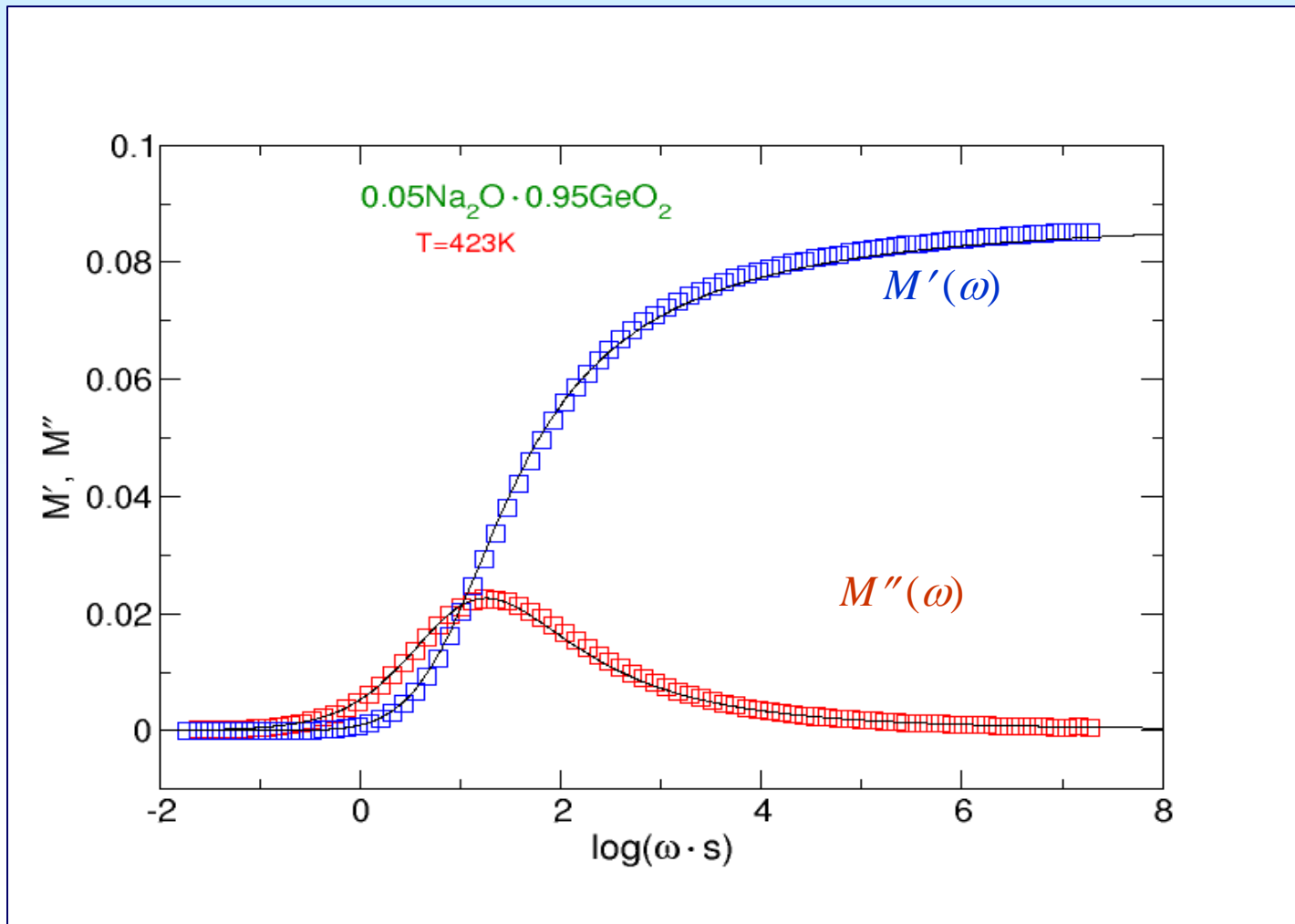
$$\varepsilon_s(\omega_s) = \frac{\omega_0 \cdot \varepsilon_0 \cdot (\varepsilon(\omega_s) - \varepsilon(\infty))}{\sigma(0)} = \frac{1}{\omega_s} \cdot \text{Im} \hat{\sigma}_s(\omega_s)$$





Real and imaginary parts of dielectric modulus do NOT scale

$$\hat{M}(\omega) = M'(\omega) + iM''(\omega) = \frac{1}{\hat{\varepsilon}(\omega)}$$



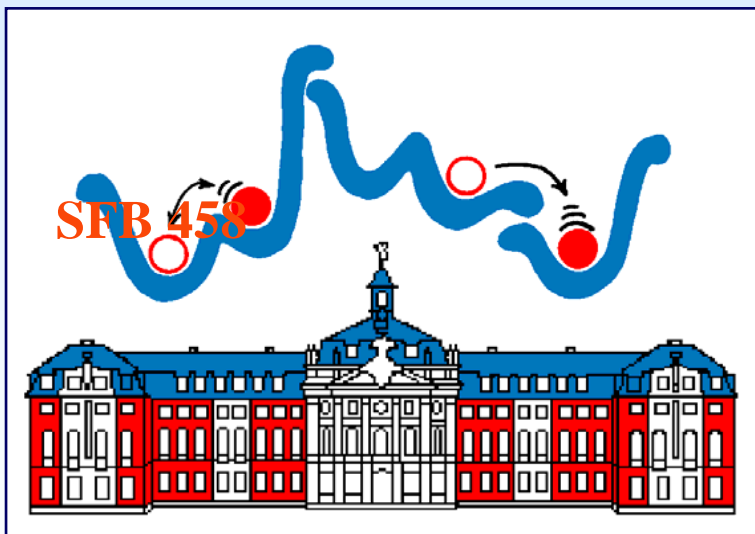
This lecture continues on a 2<sup>nd</sup> module -

Video Module 2 : Dynamics of Ions (Part 3)

# Ionic motion in materials with disordered structures

## Video Module 2 (Part 3- Dynamics of Ions)

Klaus Funke, Münster



Many-particle dynamics  
studied from picoseconds  
to minutes and hours to  
find rules for ionic motion  
in disordered materials

**Glass Tutorial Series:** prepared for and produced by the  
**International Material Institute for New Functionality in Glass**  
An NSF sponsored program – material herein not for sale  
Available at [www.lehigh.edu/imi](http://www.lehigh.edu/imi)

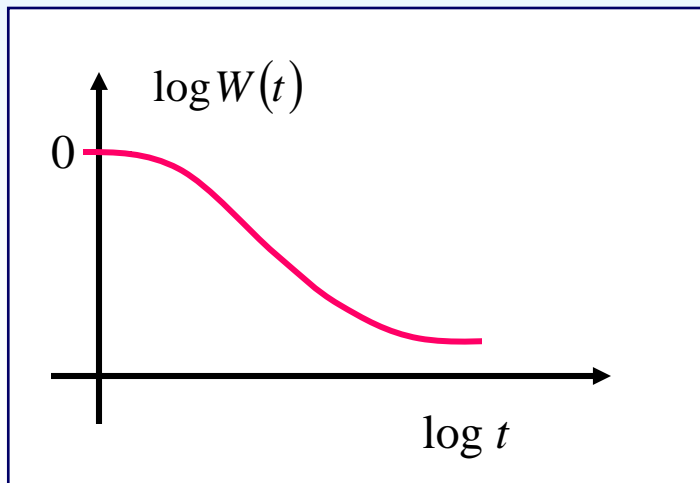
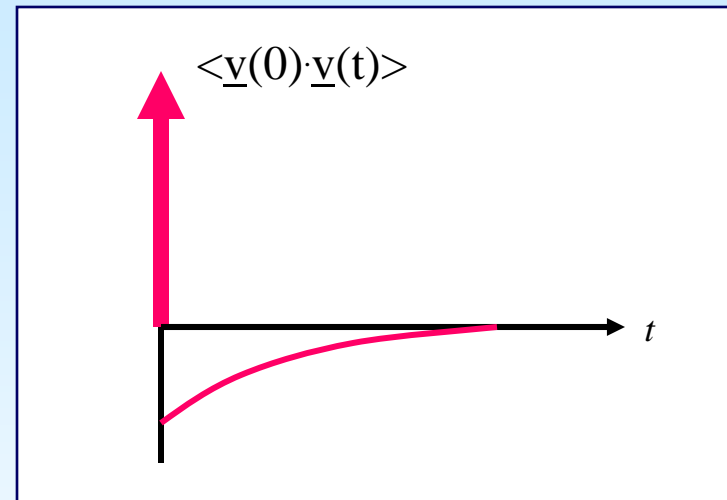
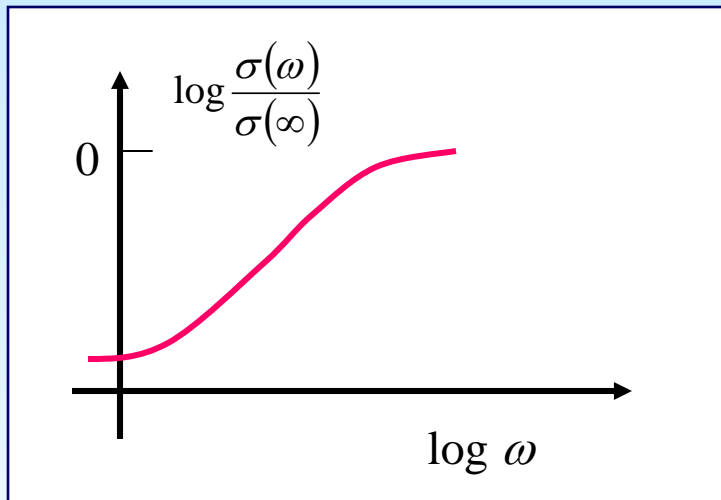
Delivered 5/5/06 at Lehigh University



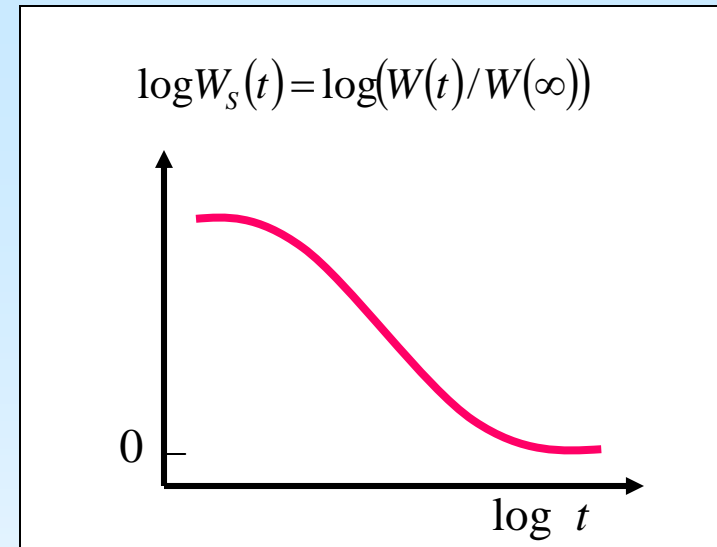
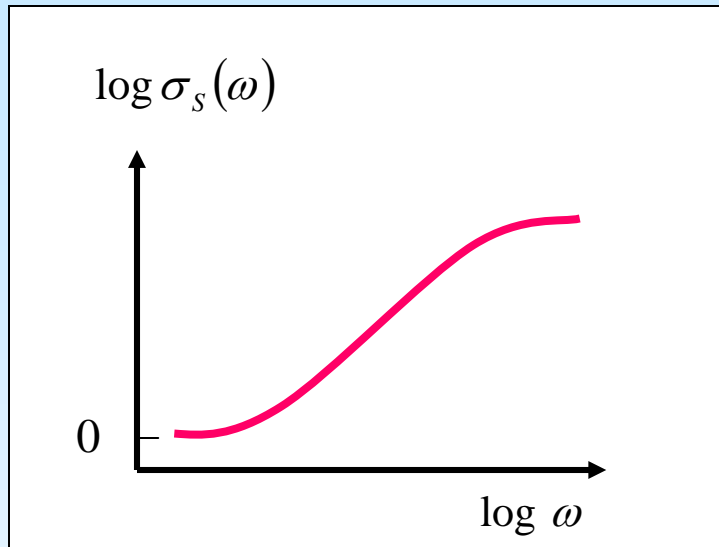
## Video Module 2 : Dynamics of Ions (includes Part 3)

1. Materials and phenomena : the rôle of disorder
2. Microscopy in time :  $\sigma(\omega)$ ,  $\varepsilon(\omega)$  and scaling properties
3. Dynamics of ions in motion : searching for simple rules

## Introducing the time-dependent correlation factor, $W(t)$



$$\frac{\sigma(\omega)}{\sigma(\infty)} = 1 + \int_0^{\infty} \dot{W}(t) \cdot \cos(\omega \cdot t) \cdot dt$$
$$\frac{\hat{\sigma}_{HOP}(\omega)}{\sigma(\infty)} = 1 + \int_0^{\infty} \dot{W}(t) \cdot \exp(-i\omega \cdot t) \cdot dt$$

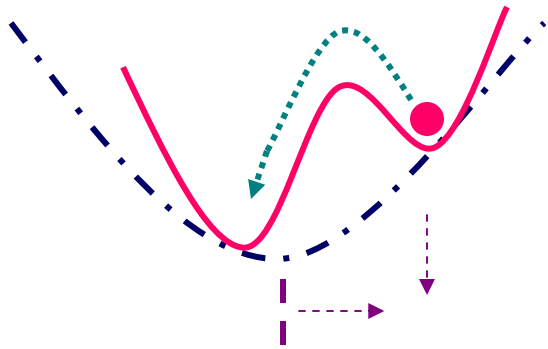


$$\hat{\sigma}_s(\omega) = \frac{\hat{\sigma}_{HOP}(\omega)}{\sigma(0)} = 1 + i\omega \int_0^{\infty} (W_s(t) - 1) \cdot \exp(-i\omega \cdot t) \cdot dt$$

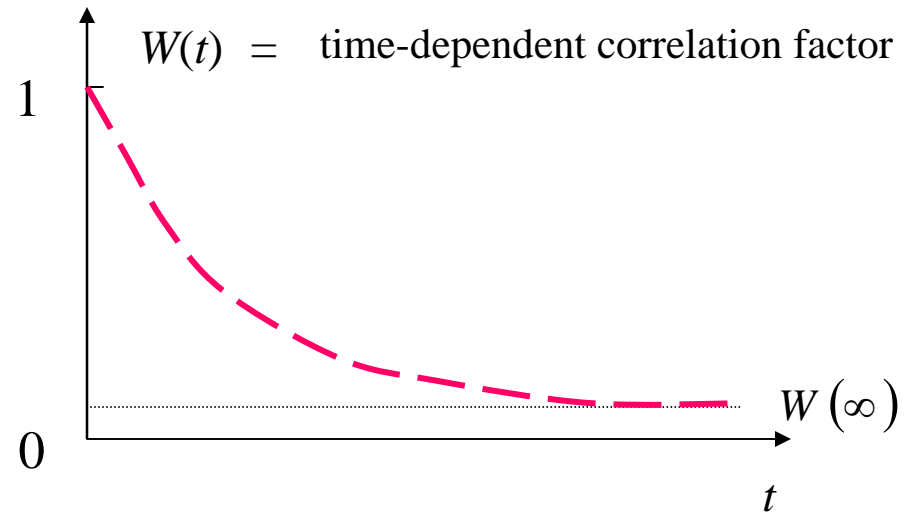
As soon as  $W_s(t)$  is known, the scaled conductivity is also known. Very realistic results are obtained by the **MIGRATION** concept, the acronym standing for **MI**smatch **G**enerated **R**elaxation for the **A**ccommodation and **T**ransport of **ION**s

In the model,  $\omega_0$  marks the onset of the dispersion.  
 Scaled time:  $t_s = t\omega_0$ , scaled frequency:  $\omega_s = \omega/\omega_0$

single-particle route



many-particle route



$$-\frac{\dot{W}(t)}{W(t)} = -B \cdot \dot{g}(t) [+ \Gamma_0]$$

$$-\frac{\dot{g}(t)}{g(t)} = \Gamma_0 \cdot W(t) \cdot N(t)$$

$$N(t) - N(\infty) = (B \cdot g(t))^2$$

← if motion is completely localised

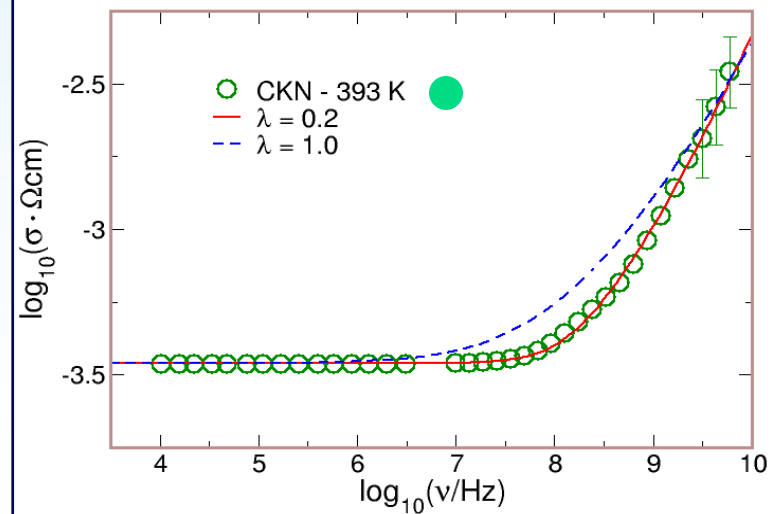
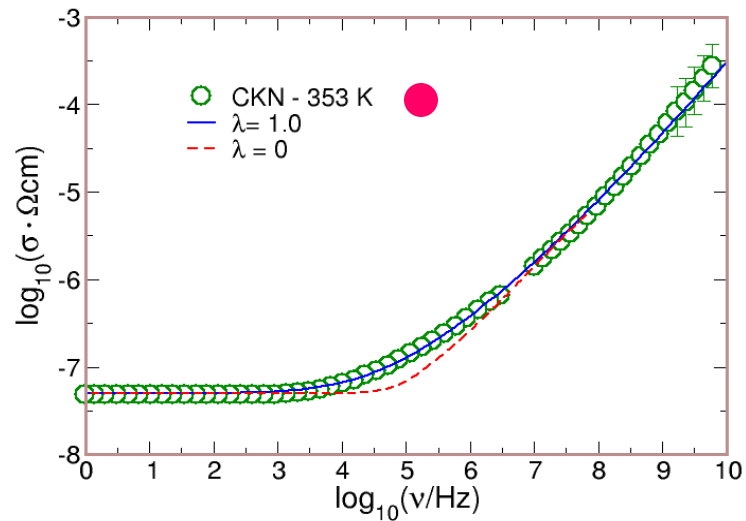
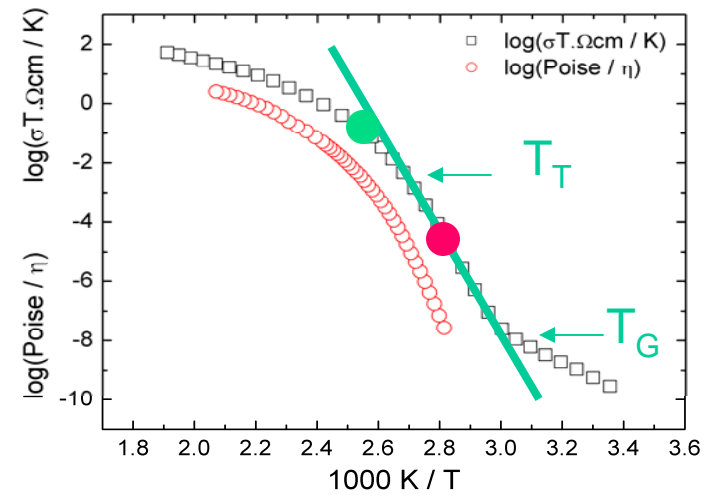
$W(t) =$ correlation factor	}	$W(t)$ and $W_S(t_S)$
$g(t) =$ mismatch function		$\sigma(\omega)$ and $\sigma_S(\omega_S)$
$N(t) =$ number function		$\varepsilon(\omega)$ and $\varepsilon_S(\omega_S)$

In  $0.4 \text{ Ca(NO}_3)_2 \cdot 0.6 \text{ KNO}_3$  (CKN)

above  $T_G$

$K = \lambda + 1$  changes

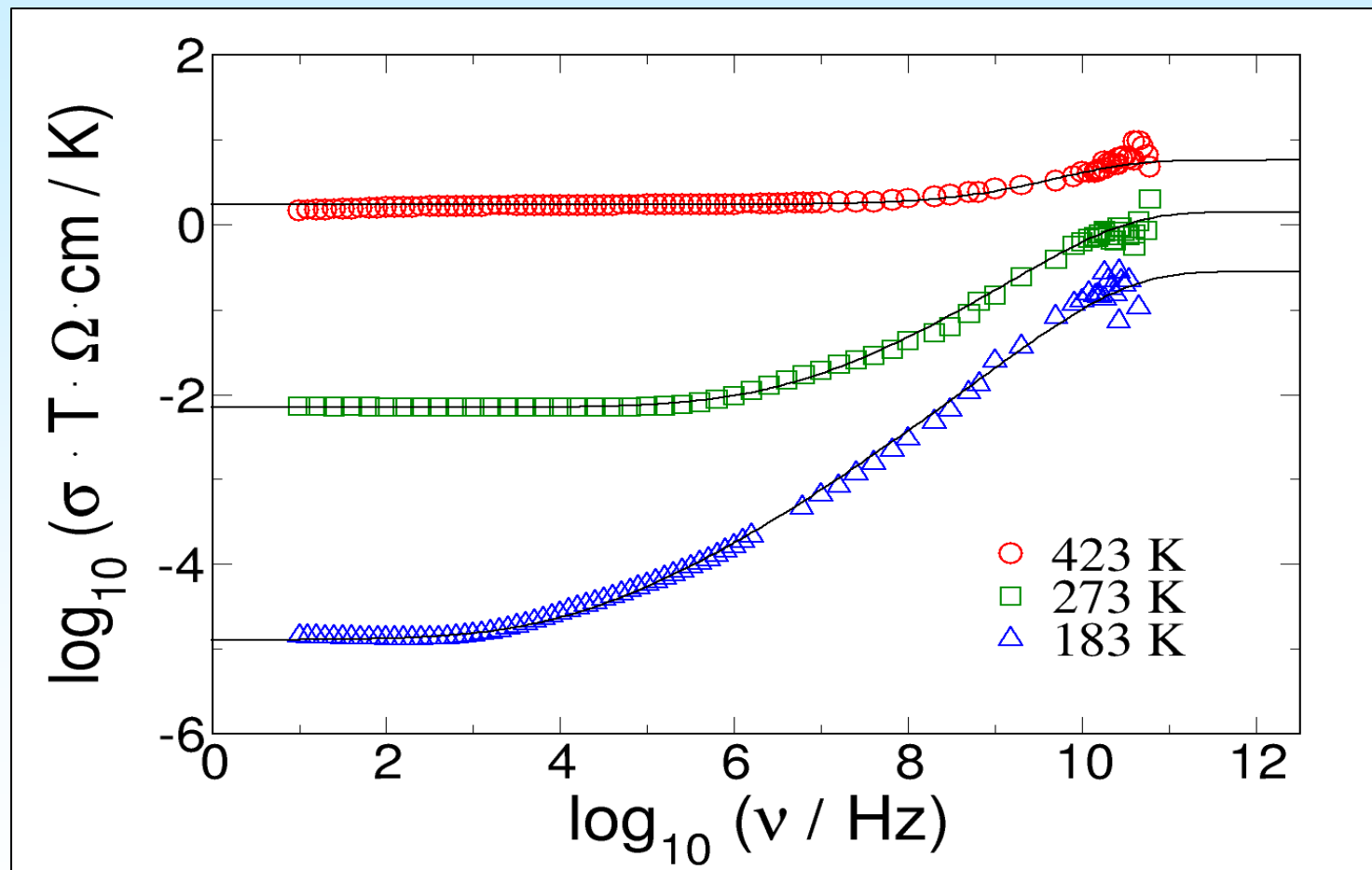
from 2.0 to 1.2





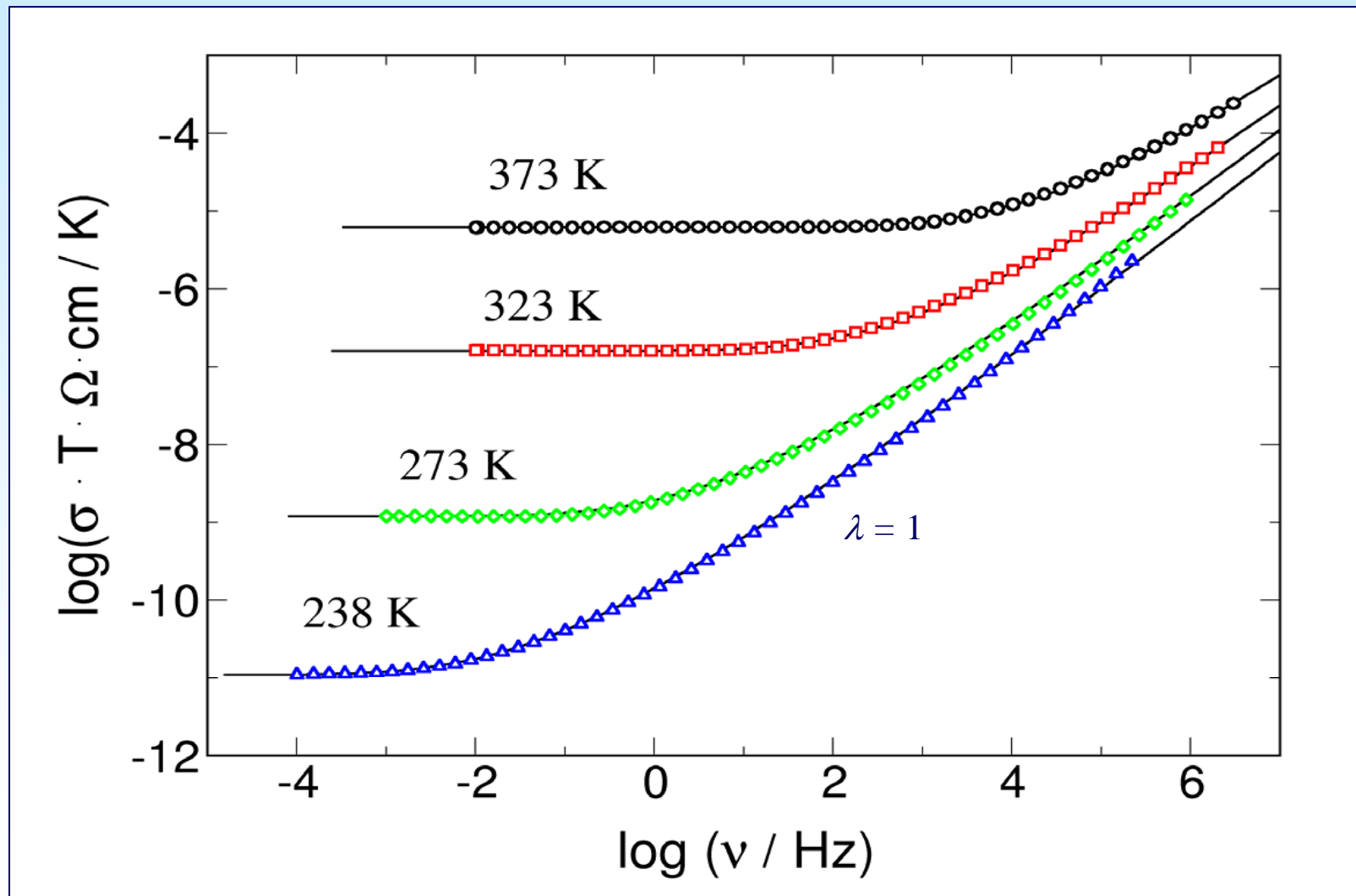
# $\text{Ag}_2\text{S} \cdot \text{GeS}_2$ glassy electrolyte:

vibrational contribution removed, set of model spectra included

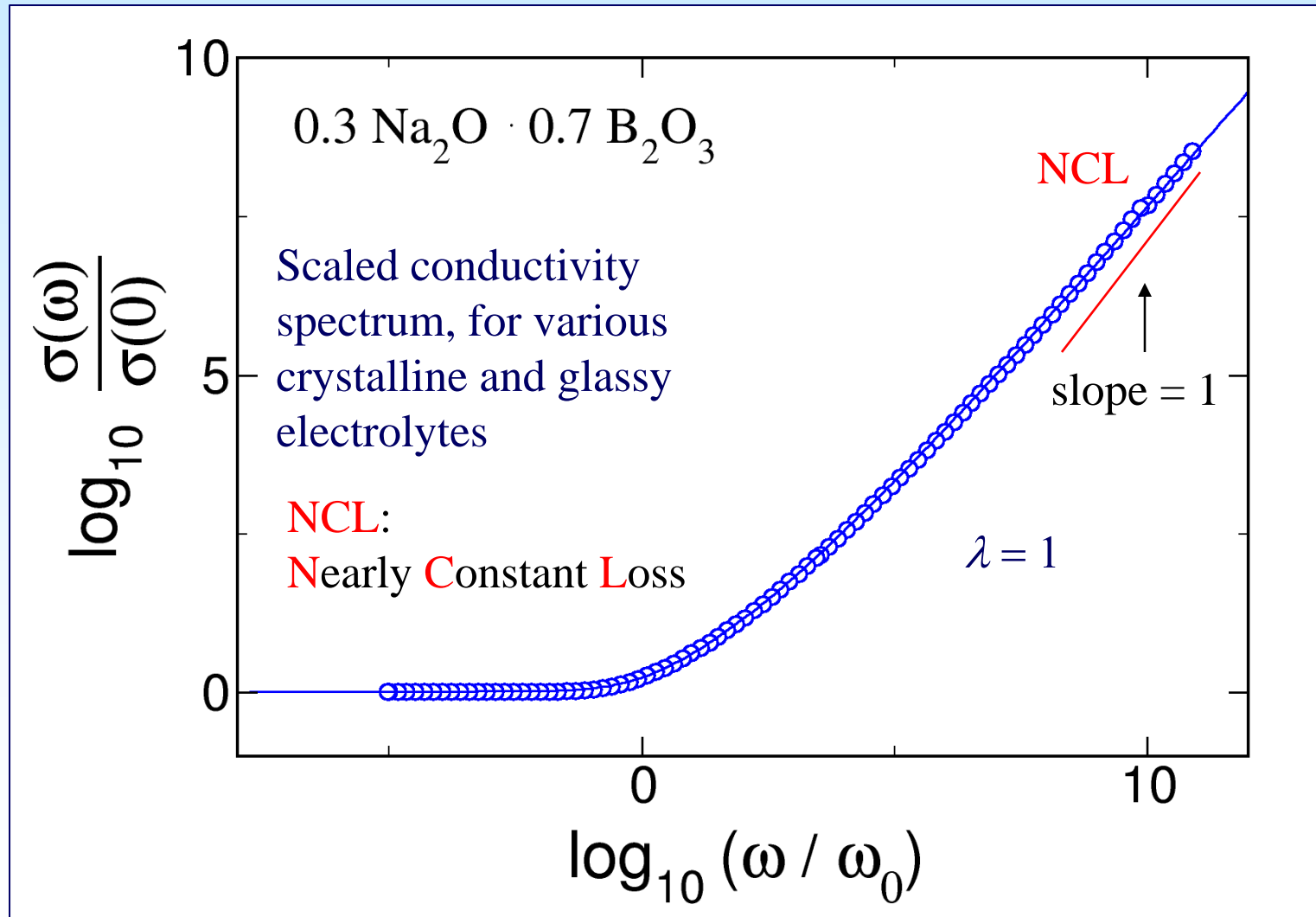


$\sigma(0) \cdot T$  and  $\sigma(\infty) \cdot T$  both Arrhenius activated

# Conductivity isotherms for glassy $0.3 \text{ Li}_2\text{O} \cdot 0.7 \text{ B}_2\text{O}_3$

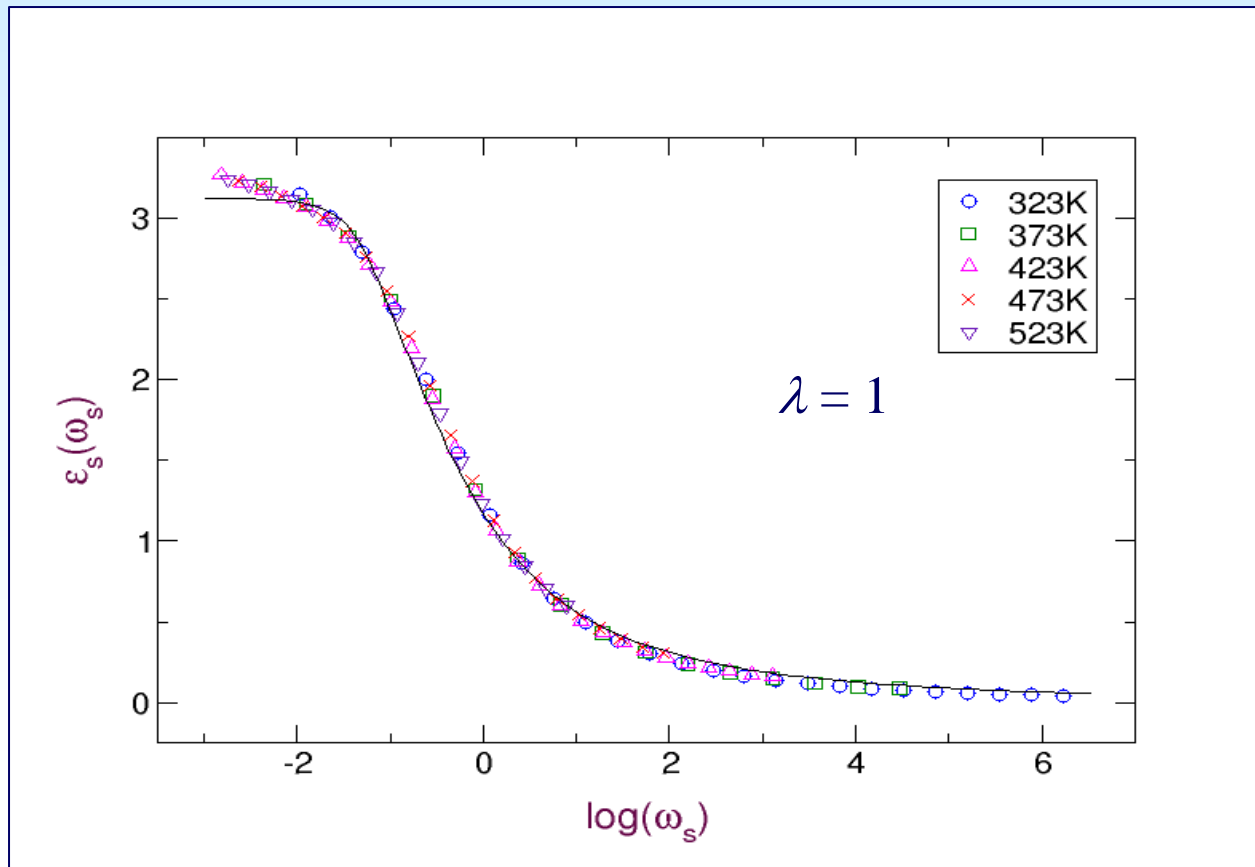


## Gradual transition into NCL behavior :



## Scaled permittivity for 0.2 Na<sub>2</sub>O · 0.8 GeO<sub>2</sub> glass

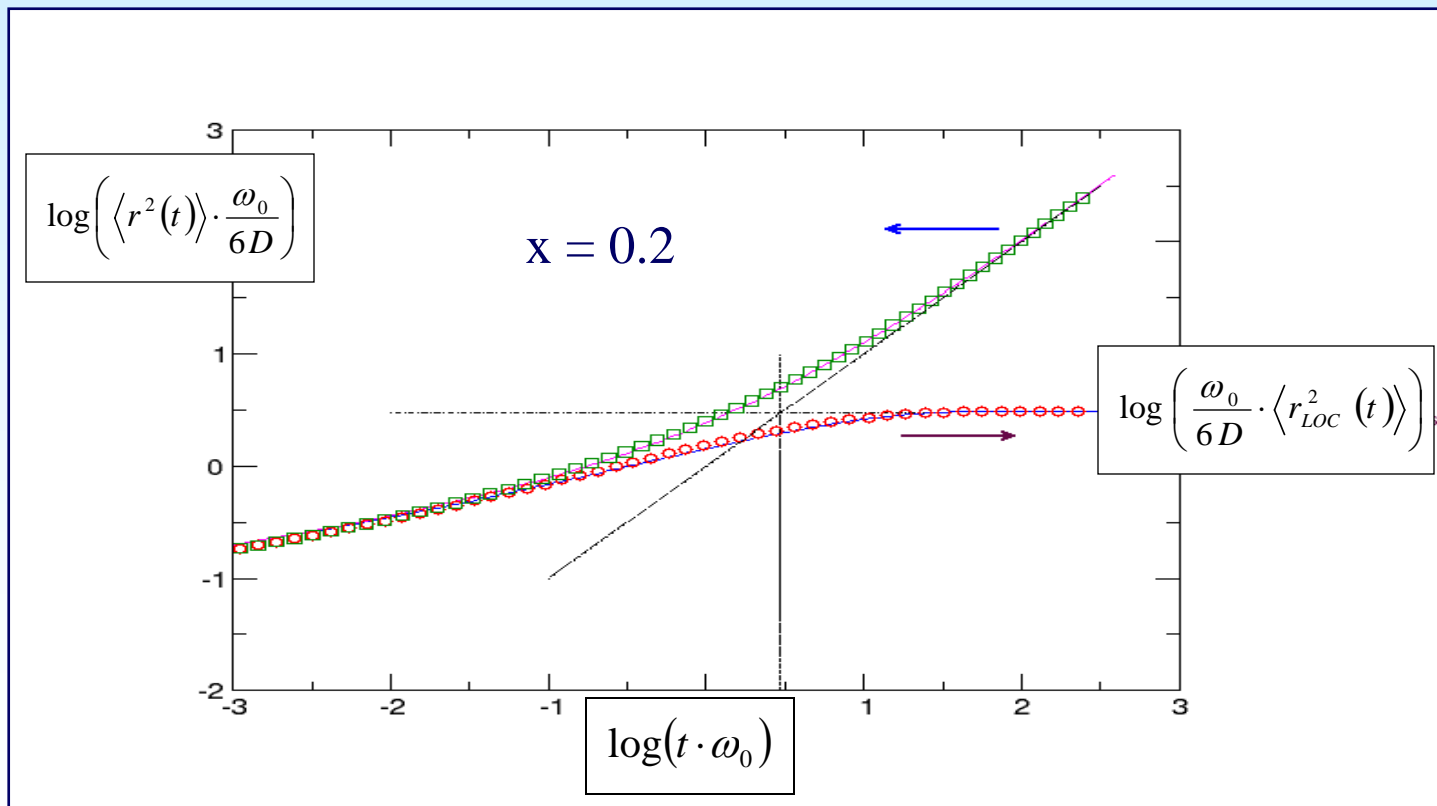
$$\varepsilon_s(\omega_s) = \frac{\omega_0 \cdot \varepsilon_0 \cdot (\varepsilon(\omega_s) - \varepsilon(\infty))}{\sigma(0)} = \frac{1}{\omega_s} \cdot \text{Im} \hat{\sigma}_s(\omega_s)$$



Localized mean square displacement for glasses  $x \text{ Na}_2\text{O} (1-x) \text{ GeO}_2$

$$\langle r_{LOC}^2(t) \rangle = \langle r^2(t) \rangle - 6Dt = 6D \cdot \int_0^t (W_s(t') - 1) dt'$$

Therefore:  $\varepsilon_s(\omega) \approx \frac{\omega_0}{6D} \cdot FT \left[ \frac{d}{dt} \langle r_{LOC}^2(t) \rangle \right]$  and  $\varepsilon_s(0) = \frac{\omega_0}{6D} \langle r_{LOC}^2(\infty) \rangle$



$\langle r_{LOC}^2(\infty) \rangle^{0.5} \propto x^{-1/3}$  and about 65 % of average Na - Na distance

Treatment of localised motion:

$$-\frac{\dot{W}_{NCL}(t)}{W_{NCL}(t)} = -B_{NCL} \cdot \dot{g}(t) + \Gamma_0$$

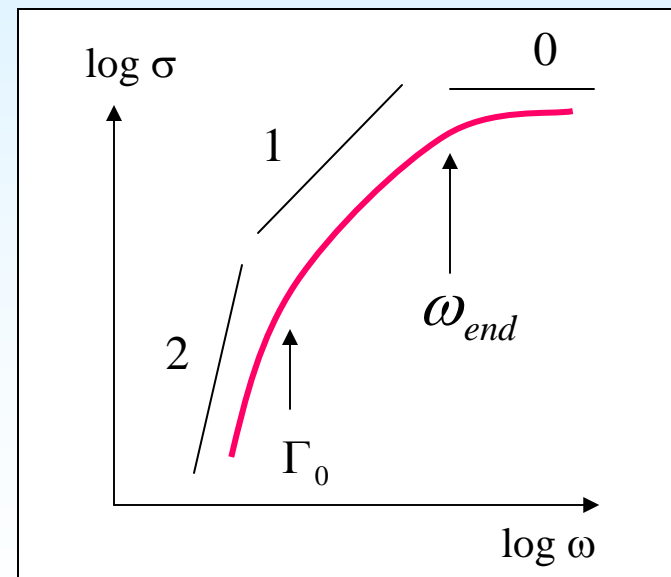
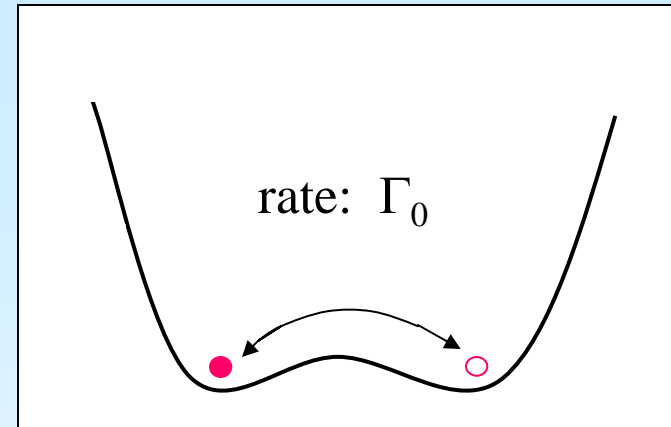
$$-\frac{\dot{g}(t)}{g(t)} = \dots \text{ as before, with } g(t) \text{ close to } 1$$



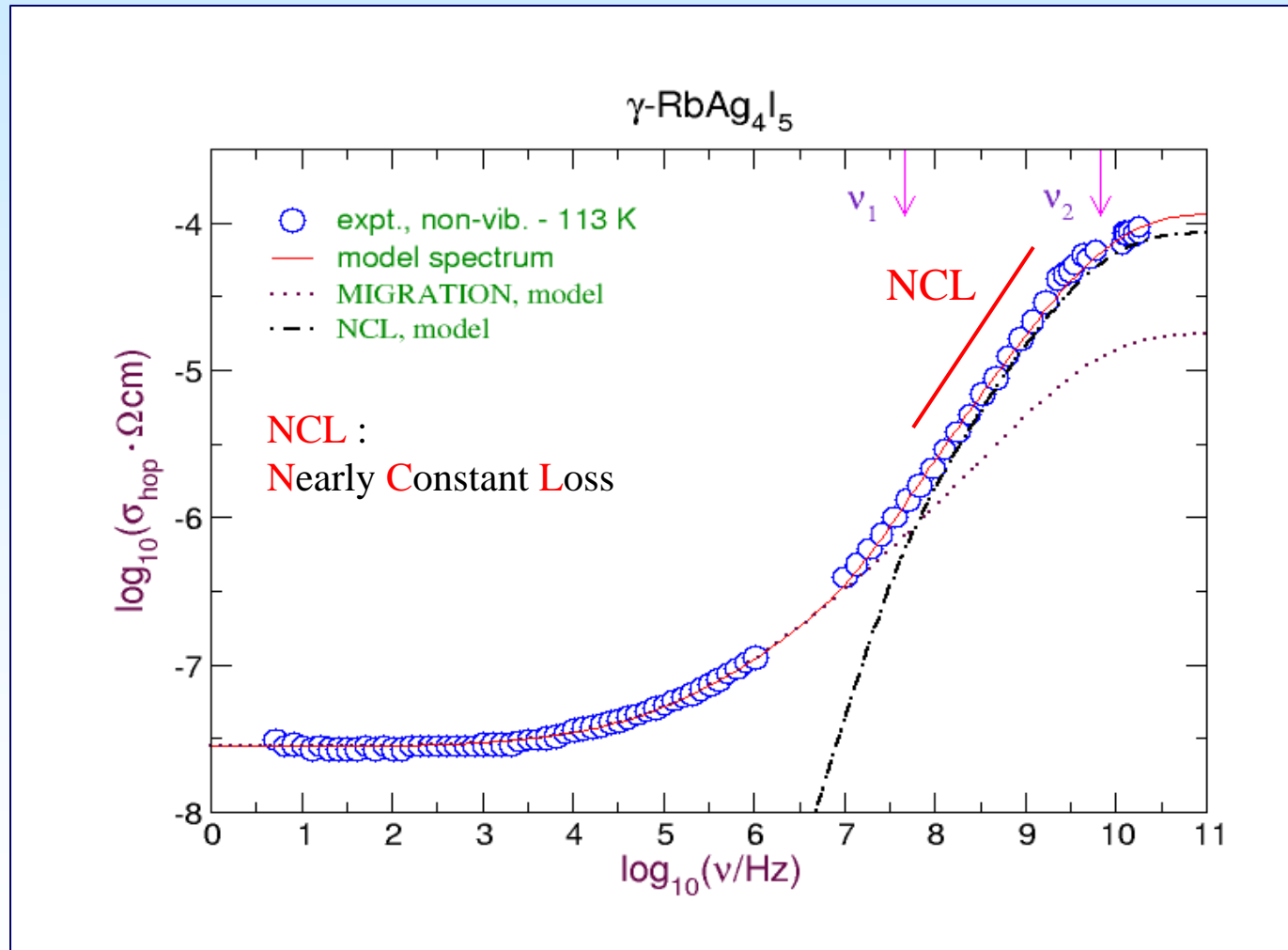
$$W_{NCL}(t) = \frac{\Gamma_0}{(\omega_{end} + \Gamma_0) \exp(\Gamma_0 t) - \omega_{end}}$$

$$\omega_{end} = \Gamma_0 \cdot B_{NCL}^2$$

i.e., Debye for  $B_{NCL} = 0$



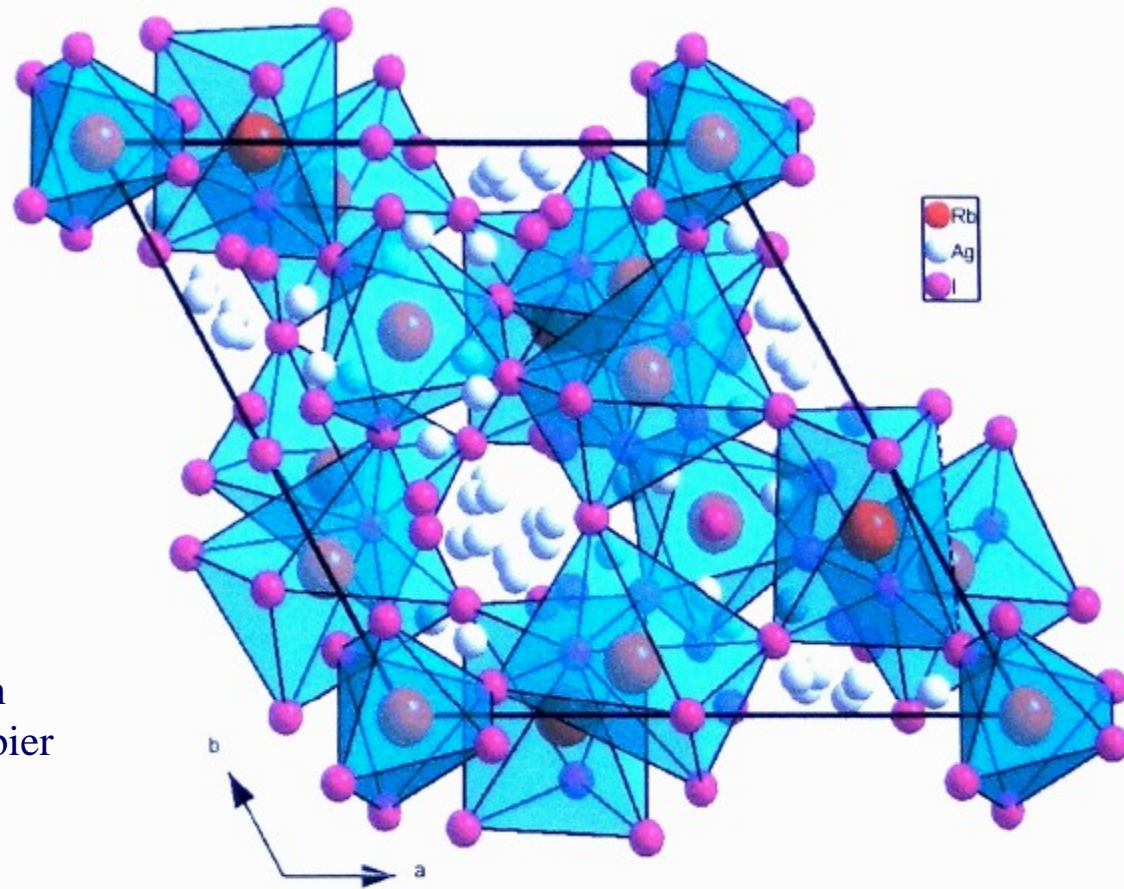
Conductivity spectrum of  $\text{RbAg}_4\text{I}_5$ , *below* the 121.8 K first order  $\beta$ - $\gamma$  phase transition, after removal of the vibrational component



$$\nu_1 = \Gamma_0/2\pi$$

$$\nu_2 = \omega_{\text{end}}/2\pi$$

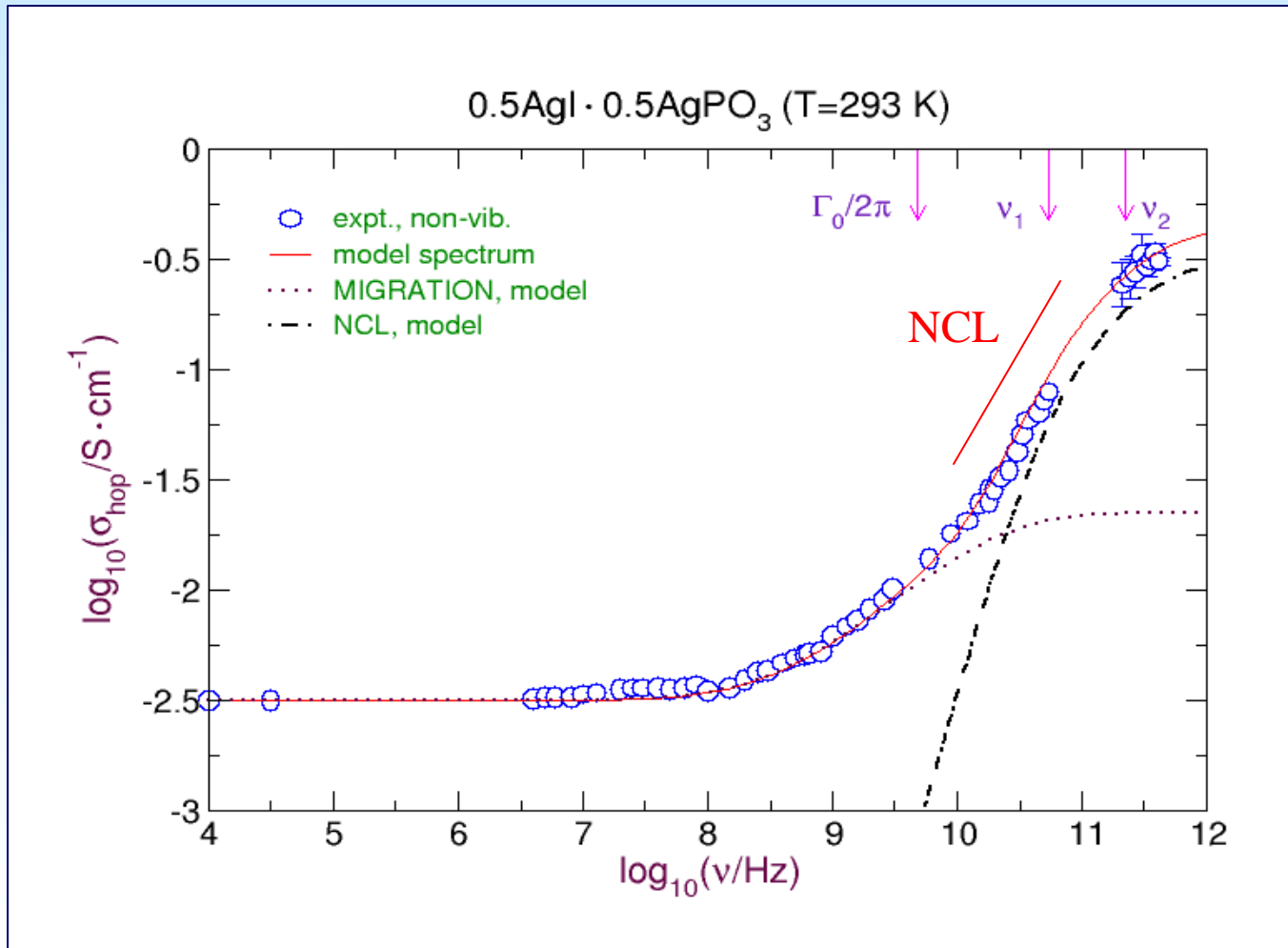
# Structure of $\gamma - \text{Rb Ag}_4 \text{I}_5$



M. Jansen  
R. Dinnebier  
A. Fitch

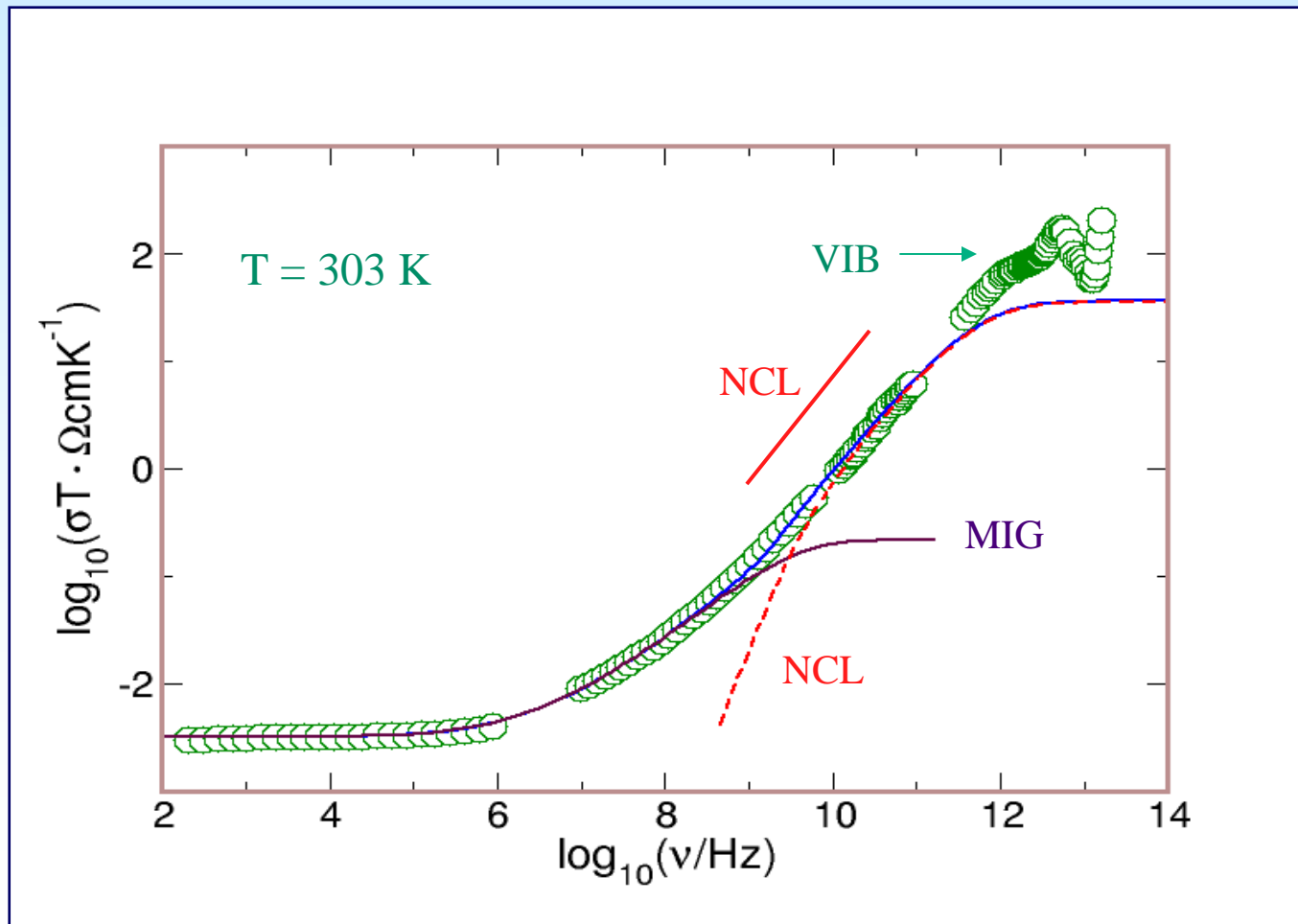


Conductivity spectrum of a silver iodide - silver metaphosphate glass,  
after removal of frequency-squared vibrational component

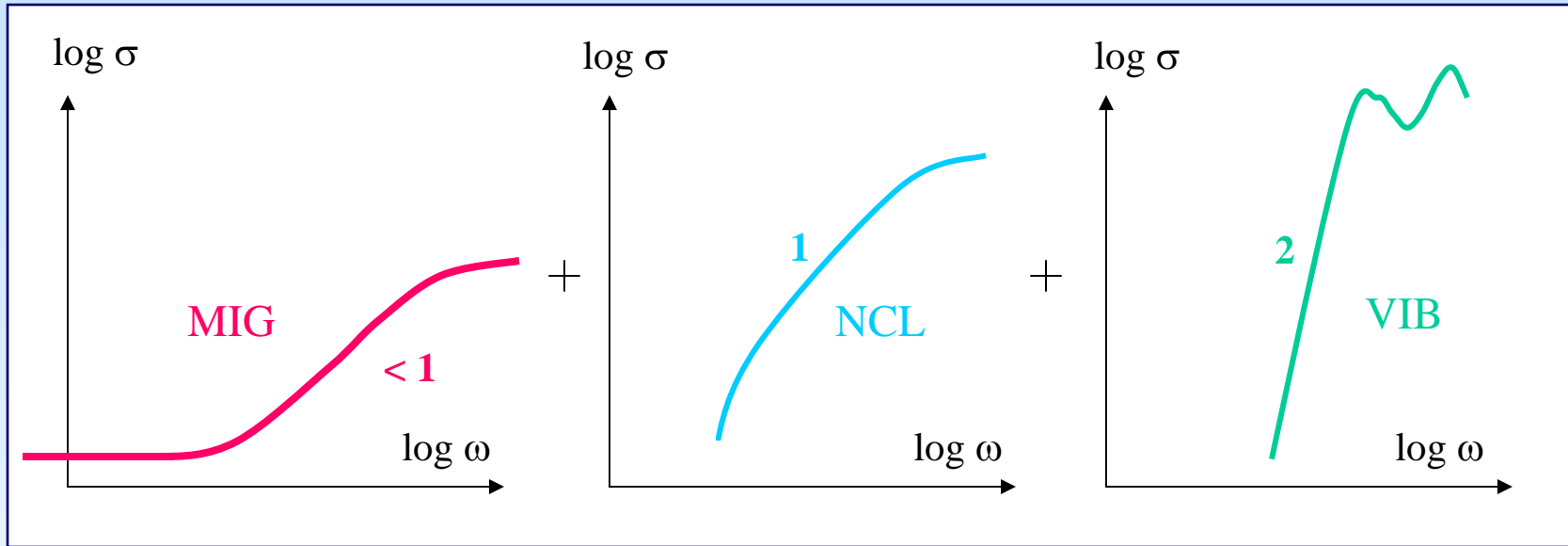


We now find :  $\Gamma_0/2\pi \ll (\Gamma_0/2\pi)_{\text{loc}} = \nu_1$  ,  $\nu_1$  being activated with only 0.05 eV.  
Localised motion is ubiquitous, e.g., Ag<sup>+</sup> - nBO<sup>-</sup> dipoles

Conductivity spectrum of  $\text{NaPF}_6$  (1 molal) in a polyurethane,  
prepared by crosslinking a tri-functional random PEO : PPO copolymer,  
4 : 1 by mol,  $M_w \sim 3600$  g / mol



# Conclusion



Ionic materials with disordered structures  
(crystalline, glassy, polymeric)  
always show this kind of superposition  
caused by

**potentially translational  
hopping motion**

**strictly localised  
non-vib. motion**

**vibrational  
motion**

all three of them being collective and cooperative in character.

## Thanks to :

Deutsche Forschungsgemeinschaft for SFB 458

M.D. Ingram, Aberdeen

M. Jansen, Stuttgart

R. Dinnebier, Stuttgart

A. Fitch, Grenoble

A.S. Nowick, Columbia

H. Jain, Lehigh

A. Heuer, Münster

R.D. Banhatti, Münster

C. Cramer, Münster

D. Wilmer, Münster

P. Singh, Münster

S.J. Pas, Münster

R. Belin, Montpellier / Münster



THANK YOU FOR YOUR ATTENTION