

# Fractal Geometry Applied To Fracture (Part 3)

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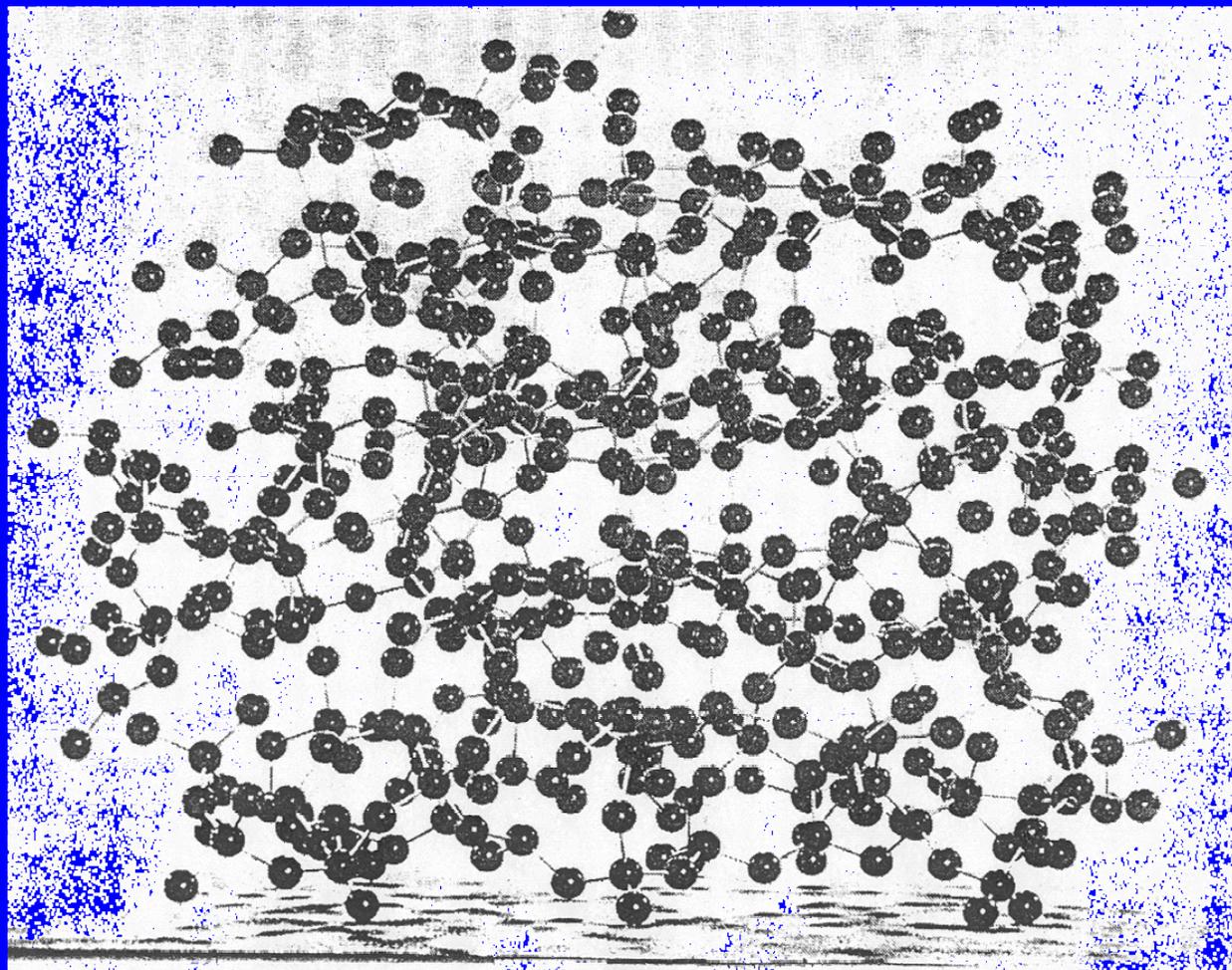
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# Technical Approach

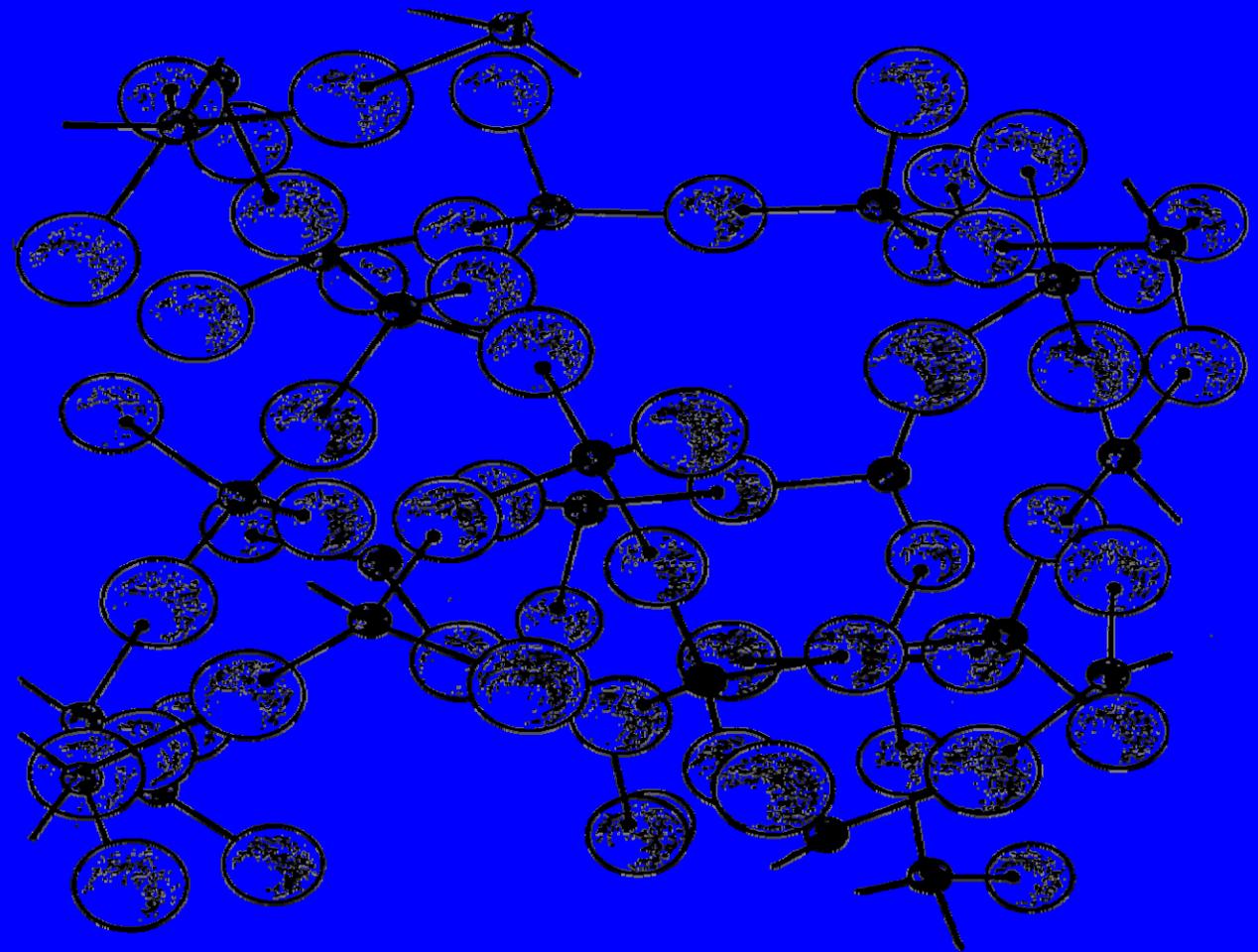
- Molecular Orbital (MO) Modeling of Fracture - determines  $a_0$ .
- Molecular Dynamics (MD) Modeling of Fracture - determines  $D^*$  (e.g. in Si)
- Experimental determination and comparison of parameters obtained in MO and MD Modeling, i.e.,  $a_0$  and  $D^*$

## Bell & Dean Model Used for MO Calculations



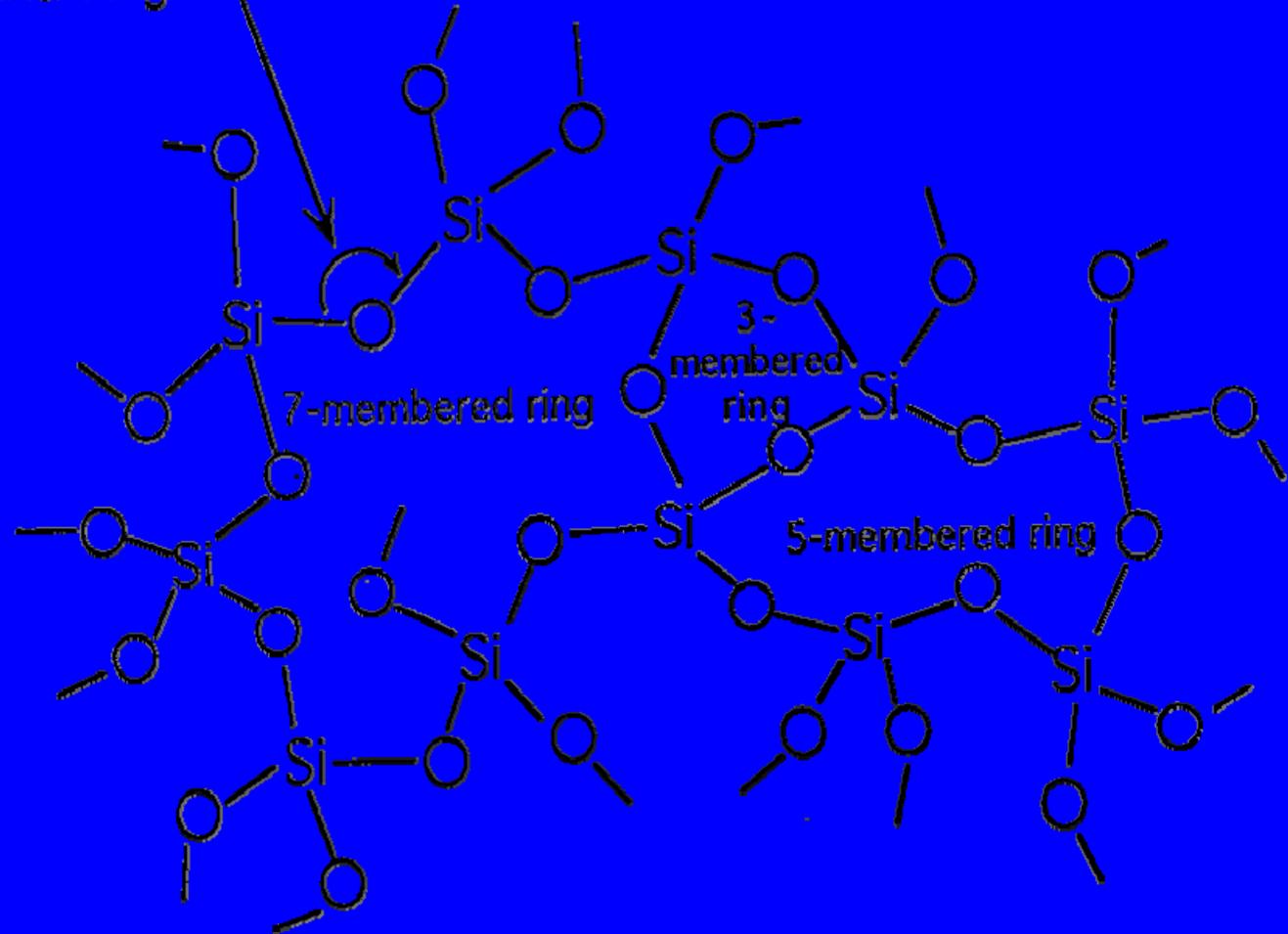
cf. Varsheneya, Fundamentals of Inorganic Glasses

(After Bell and Dean, Nature 212, 1354 [1966])



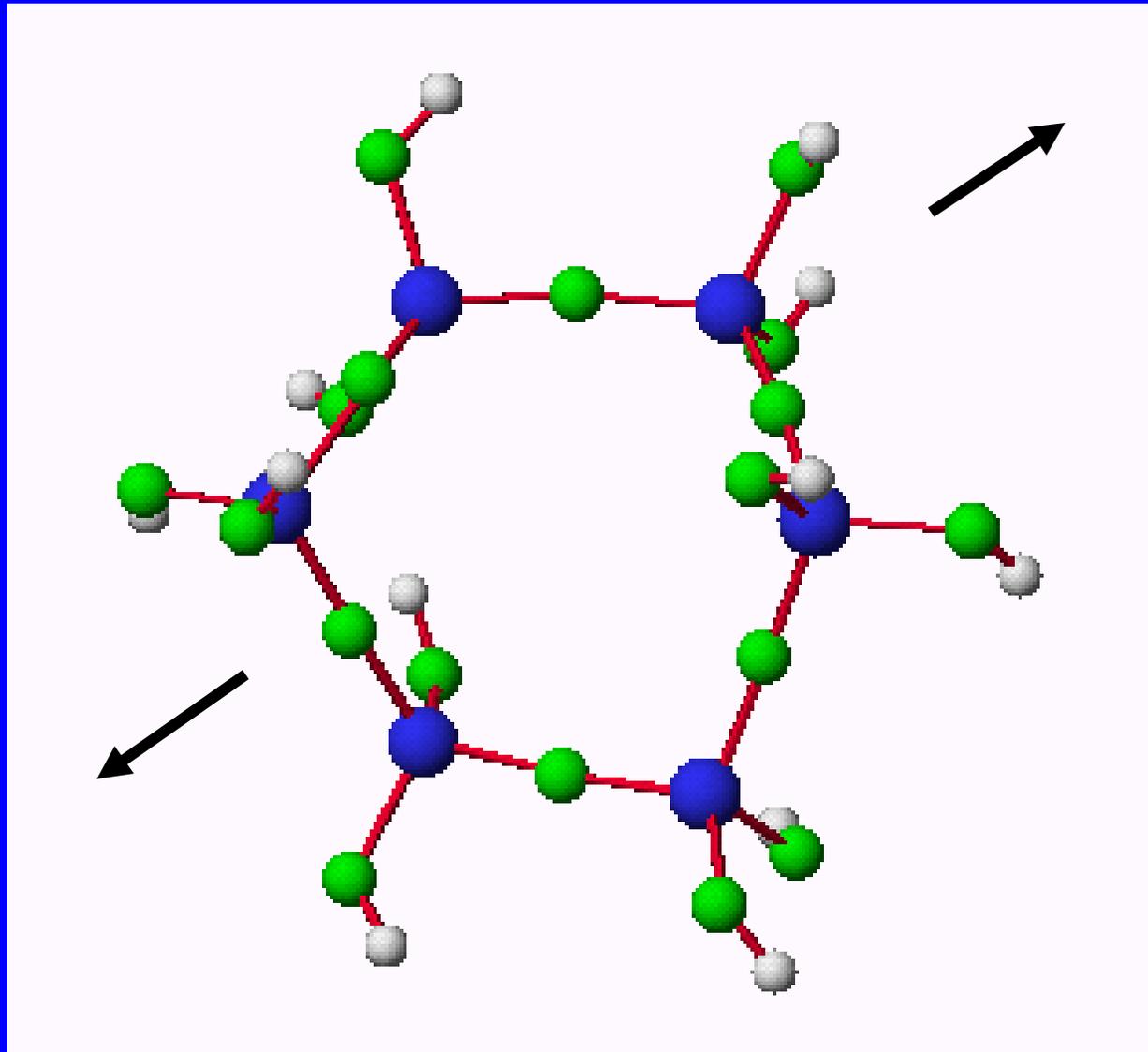
cf. Varsheneya, Fundamentals of Inorganic Glasses  
(after T. F. Soules, Glass Sci & Tech 4A, 318)

$Si_1-O_4-Si_2$   
Bond Angle



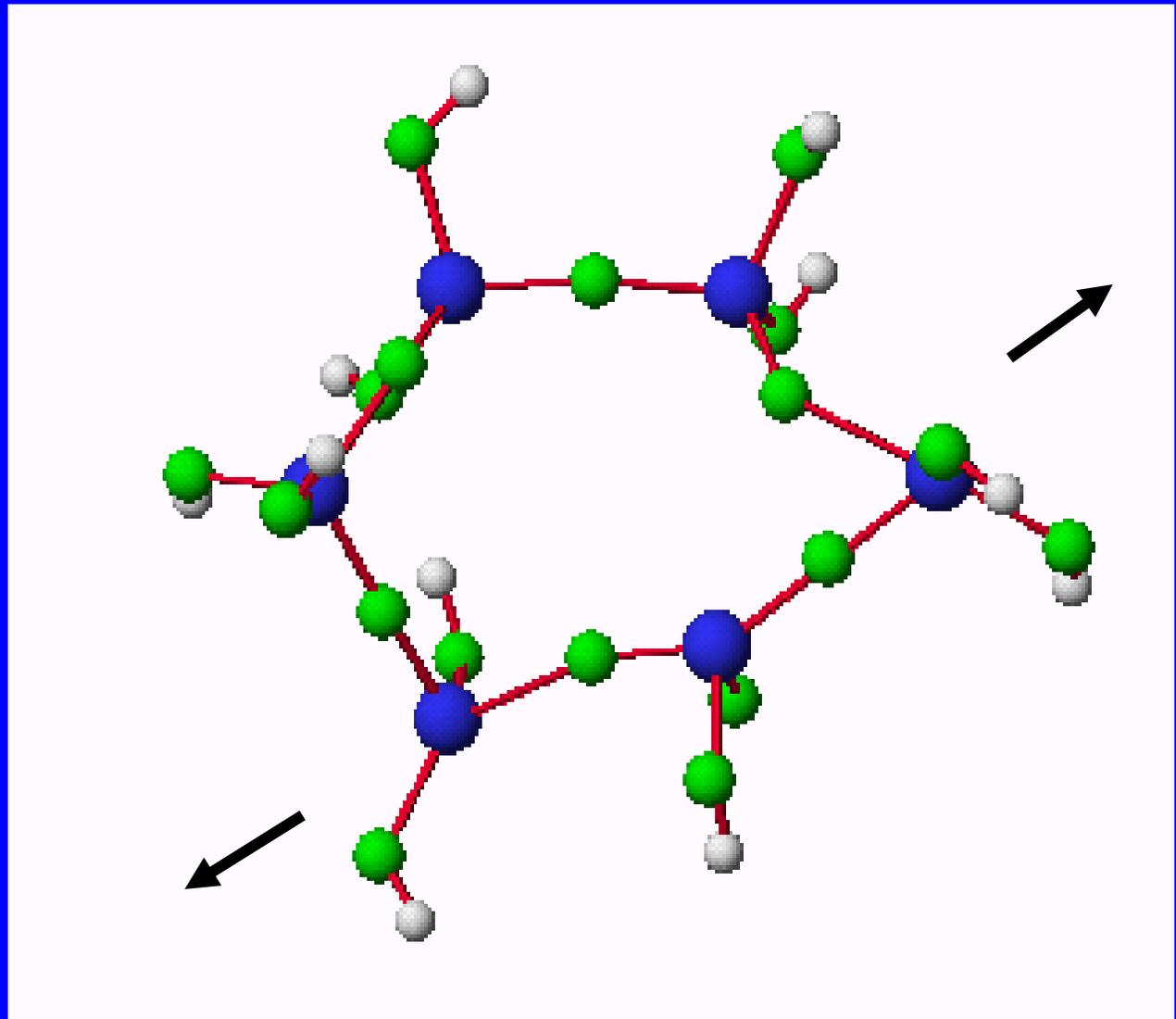
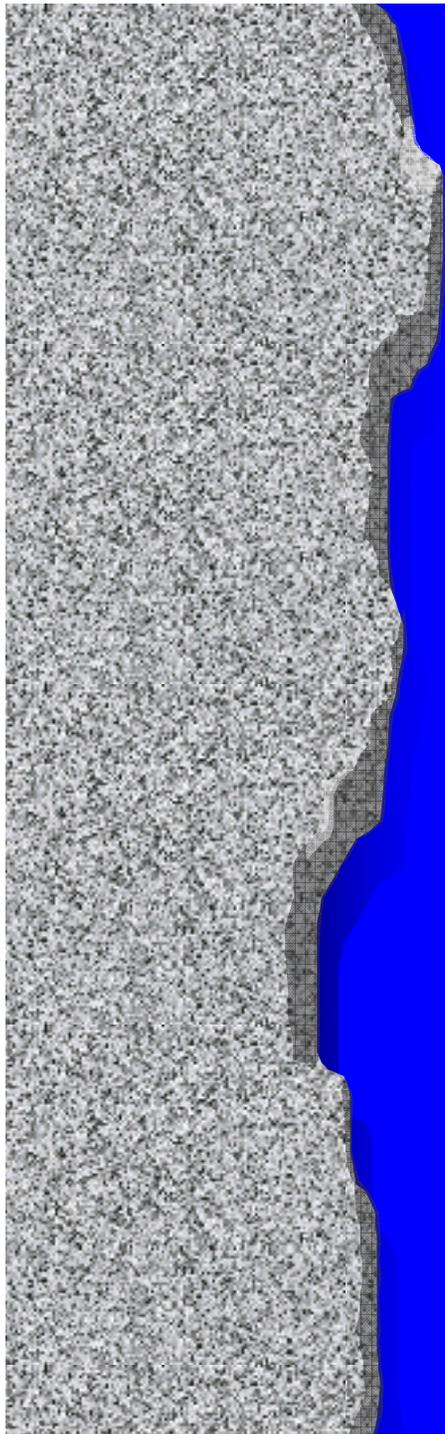
cf. Varsheneya, Fundamentals of Inorganic Glasses

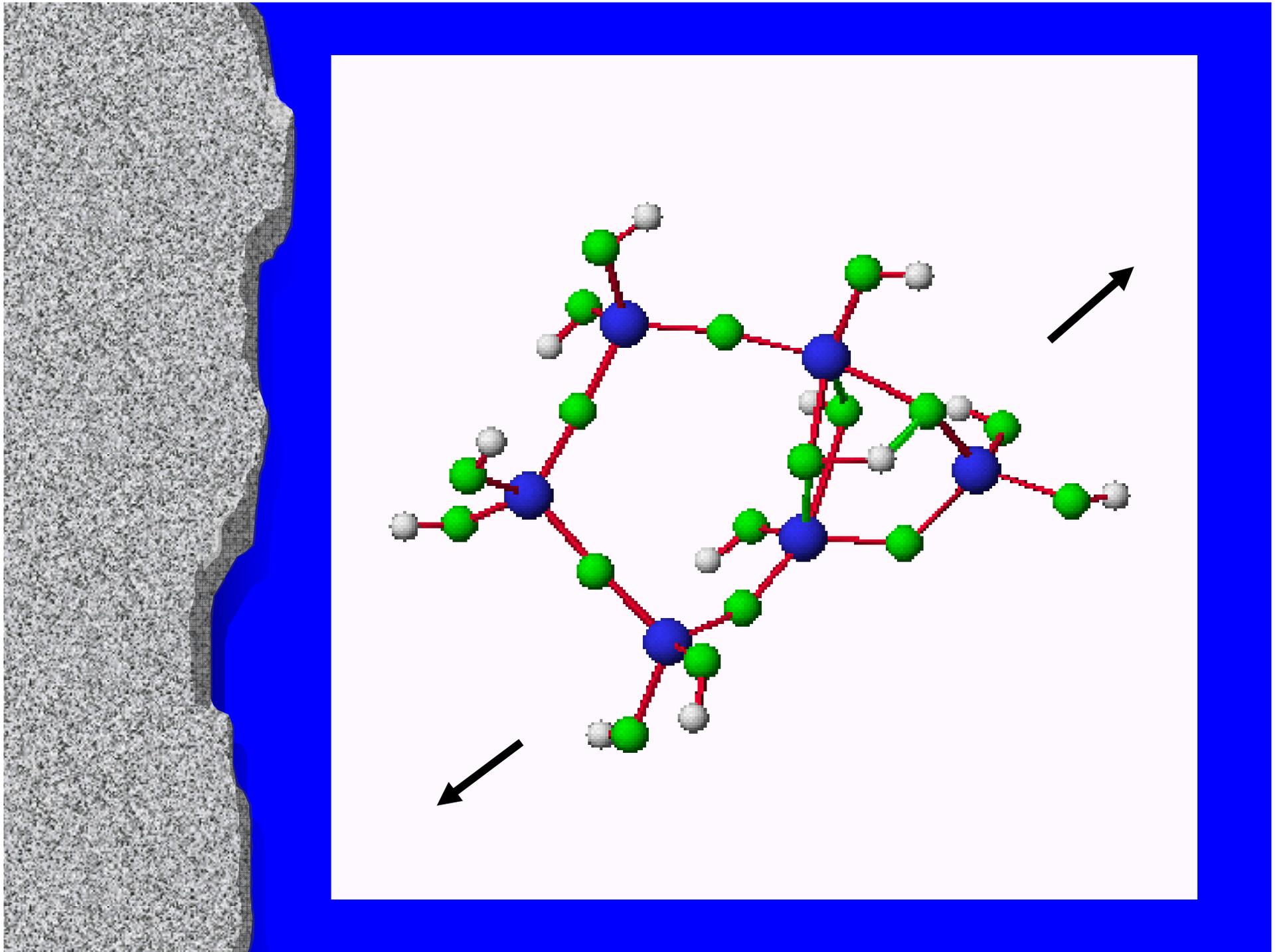
# MO Simulates Bond Breaking At The Crack Tip

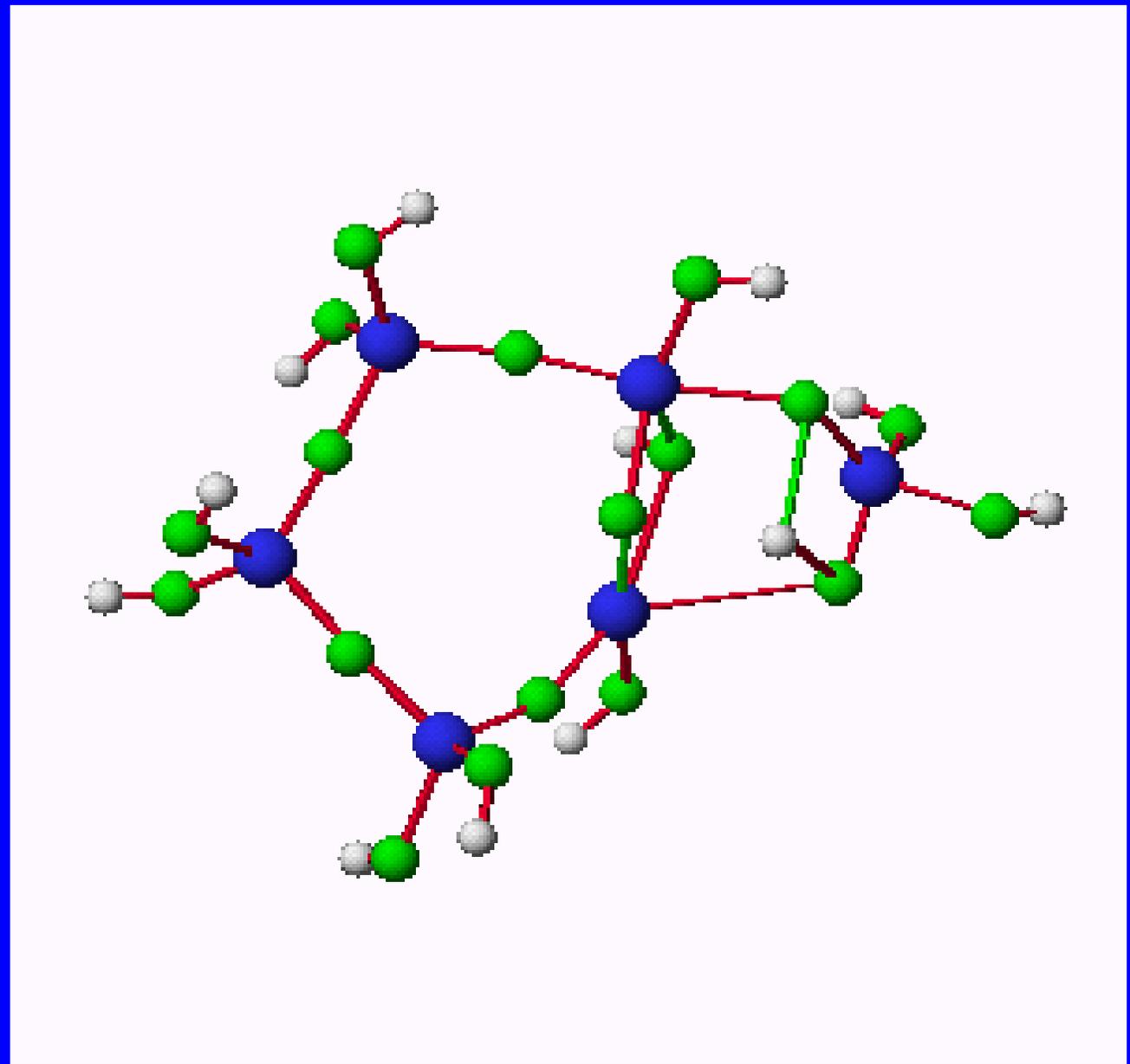
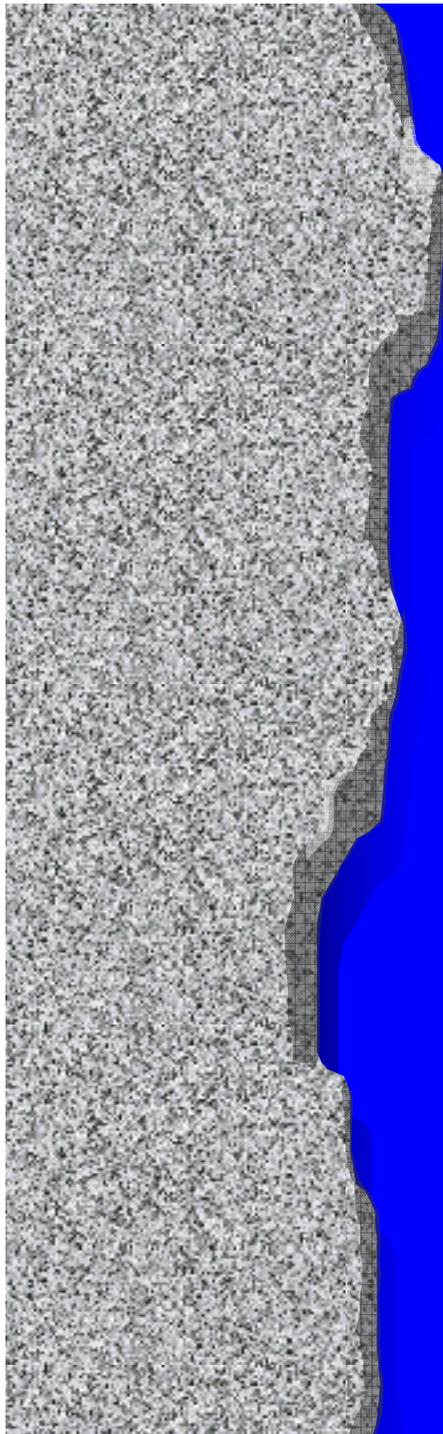


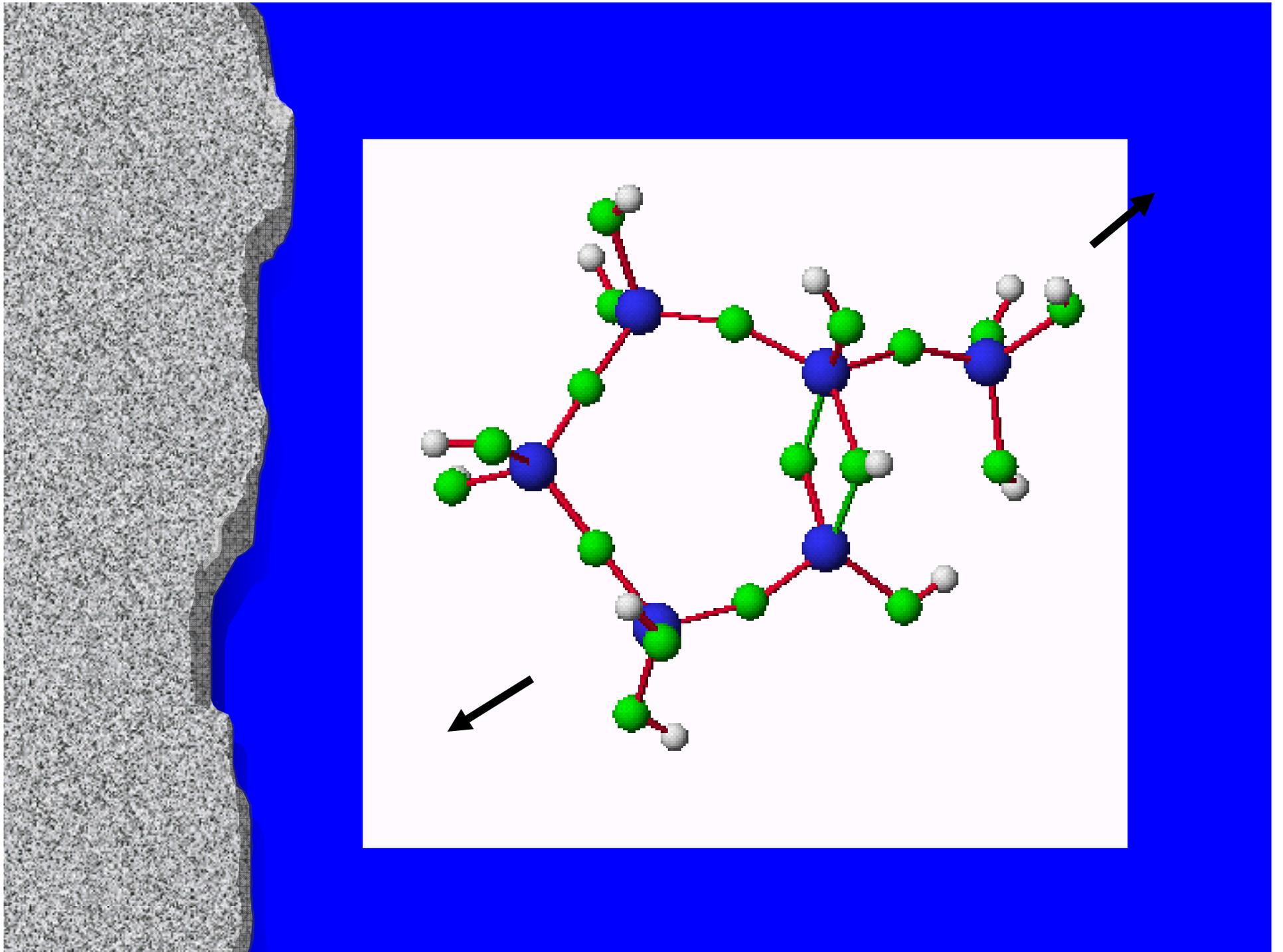
Simulated SiO<sub>2</sub>

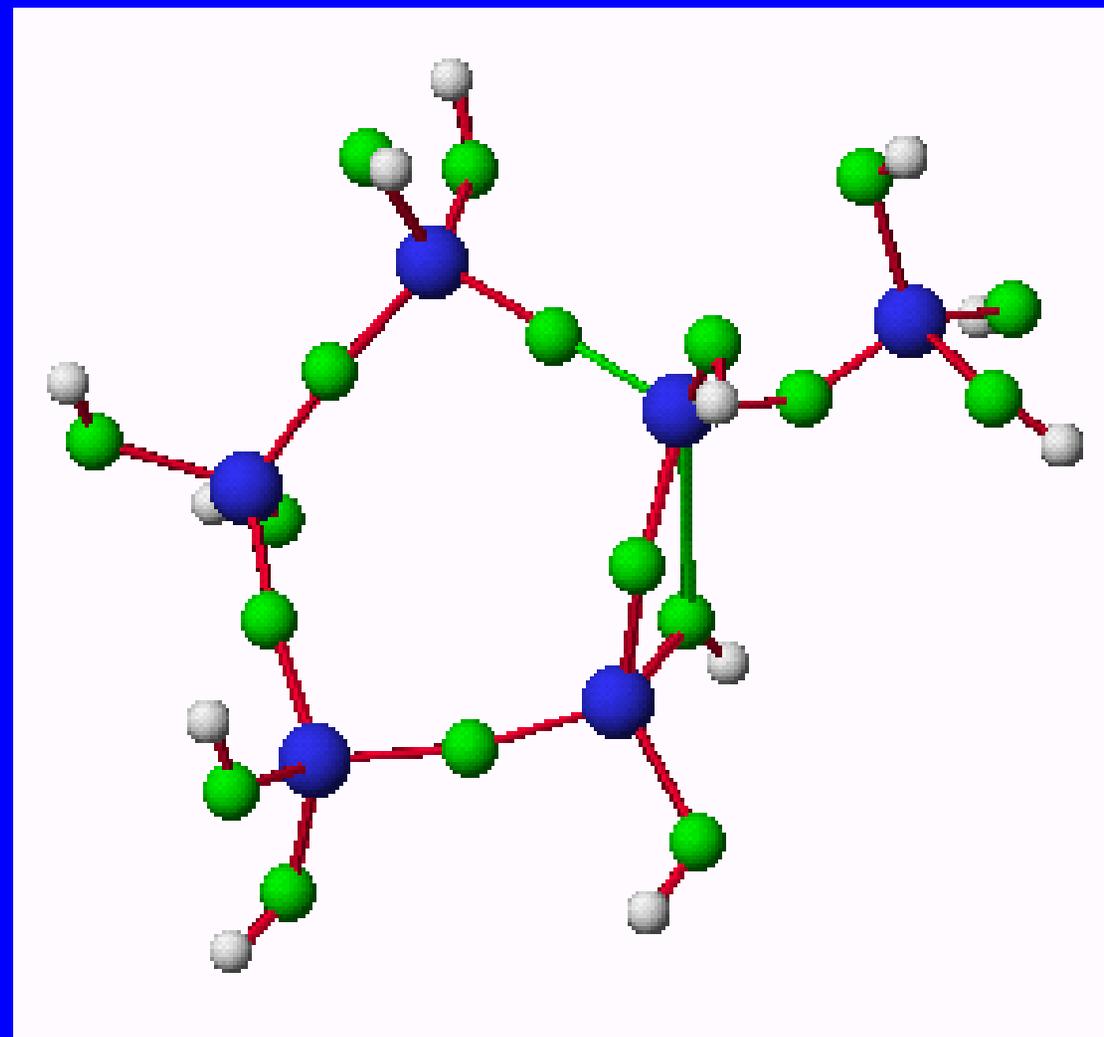
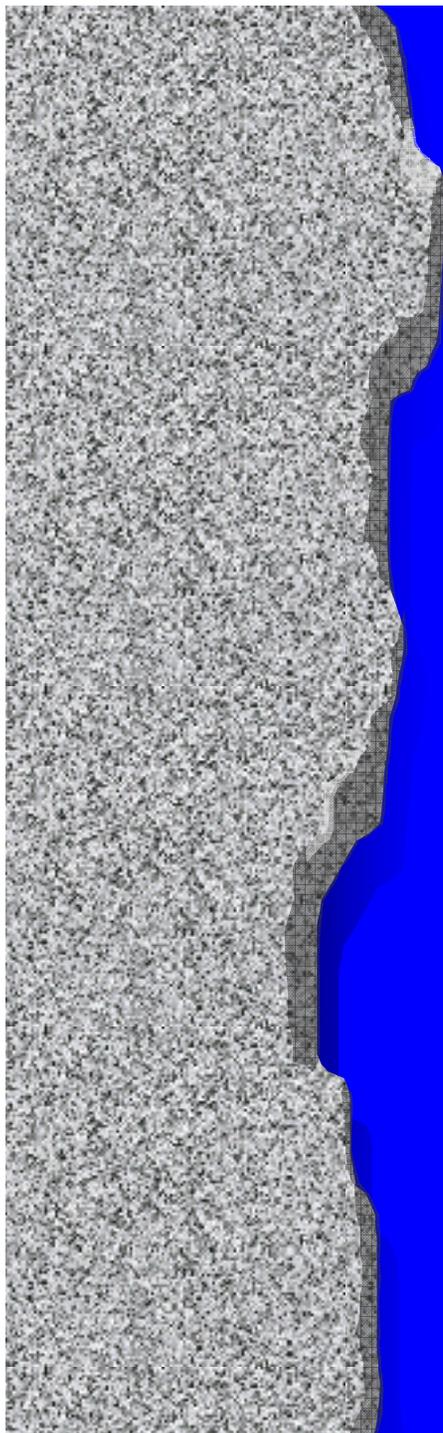
$\Delta$  displ. = 1 Å

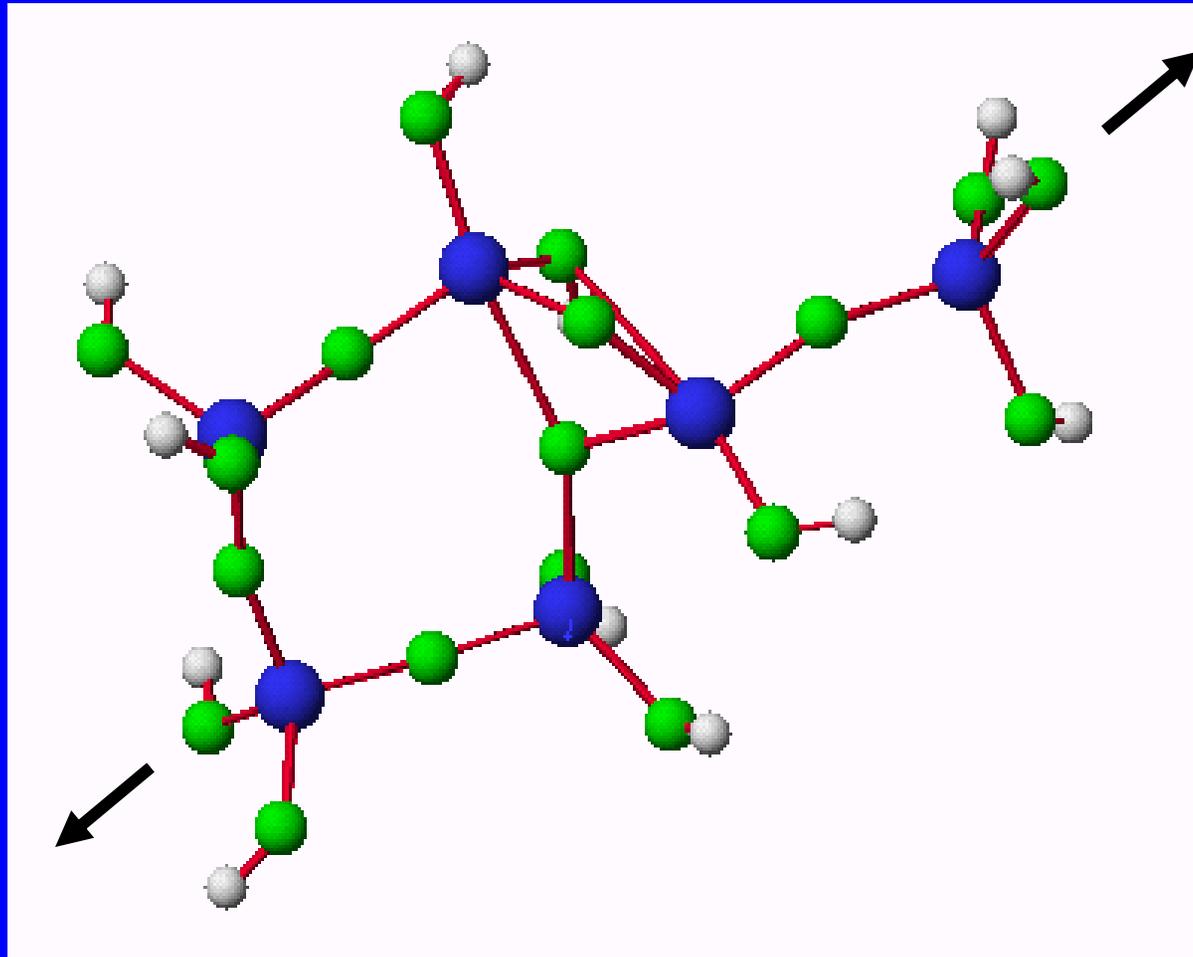
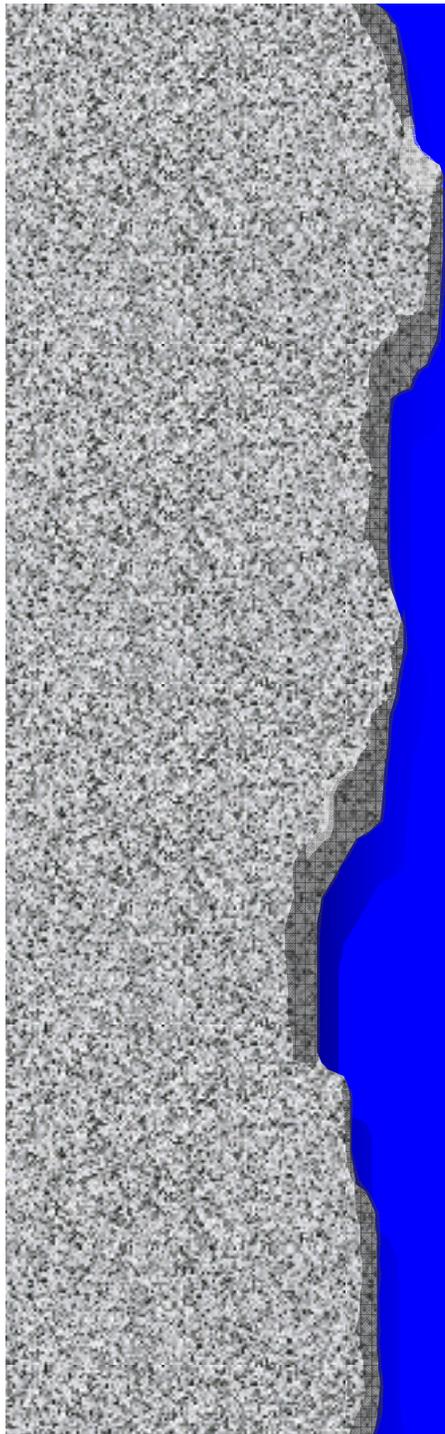


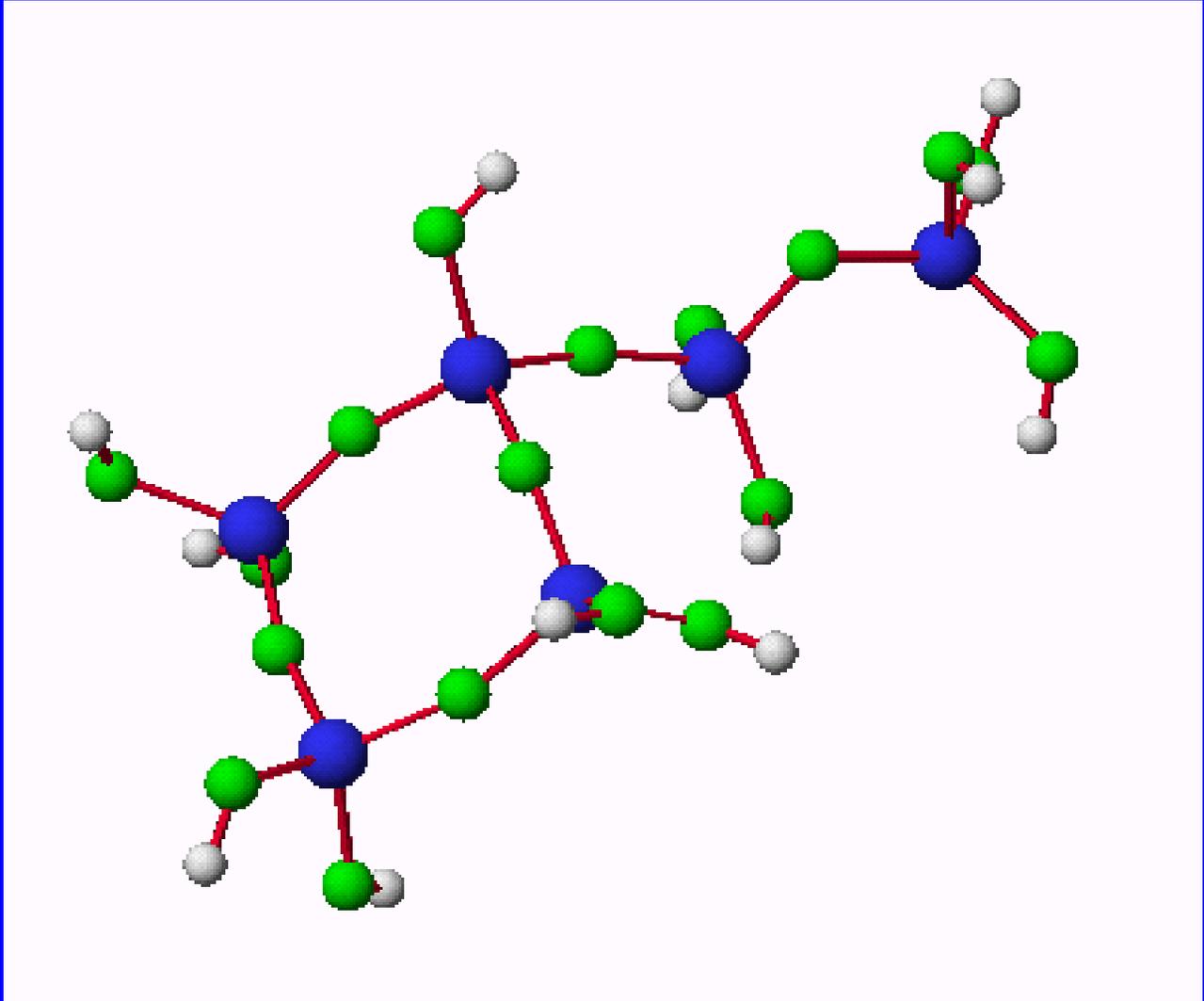
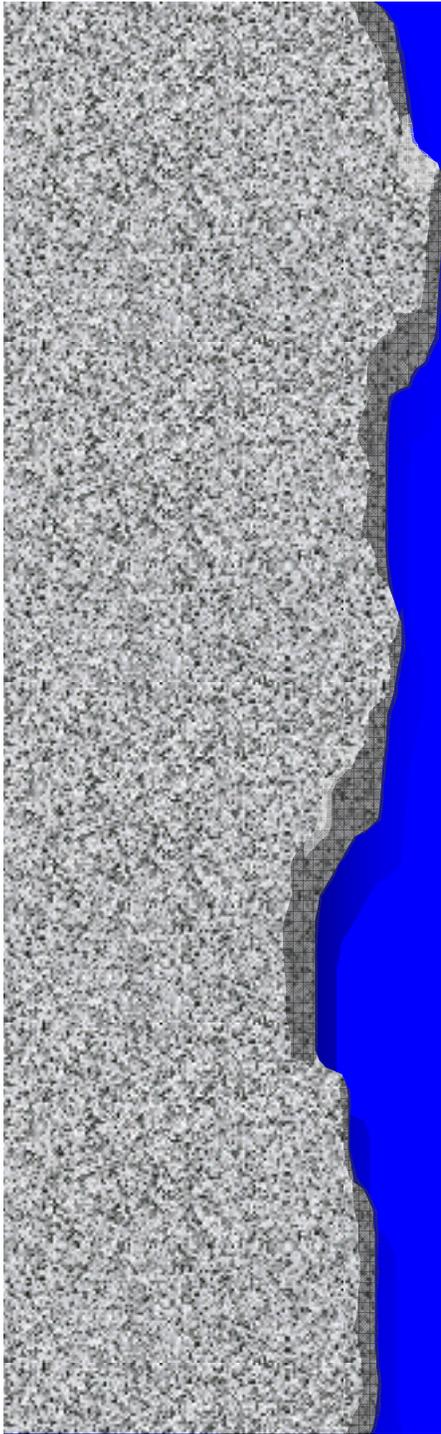


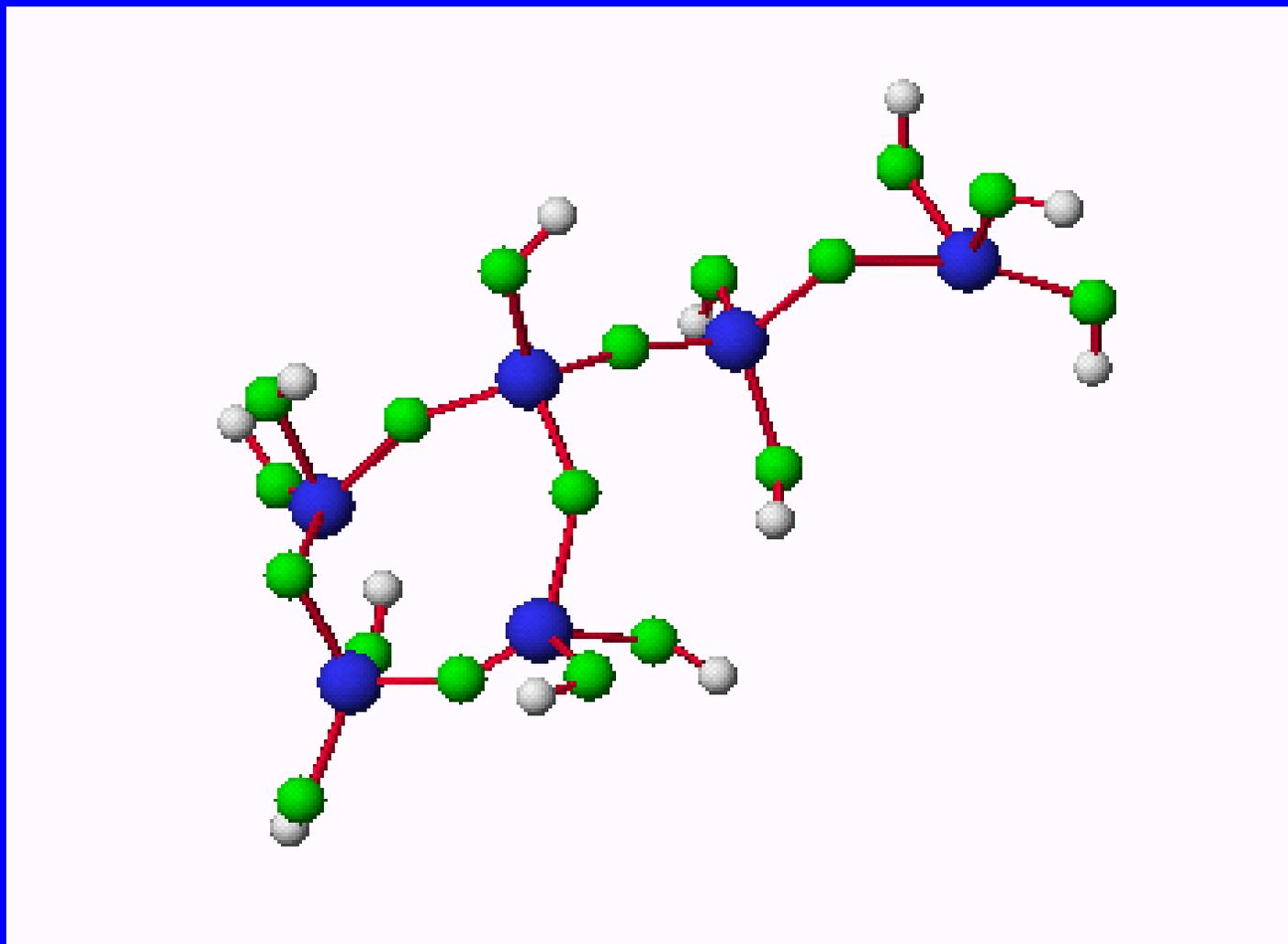
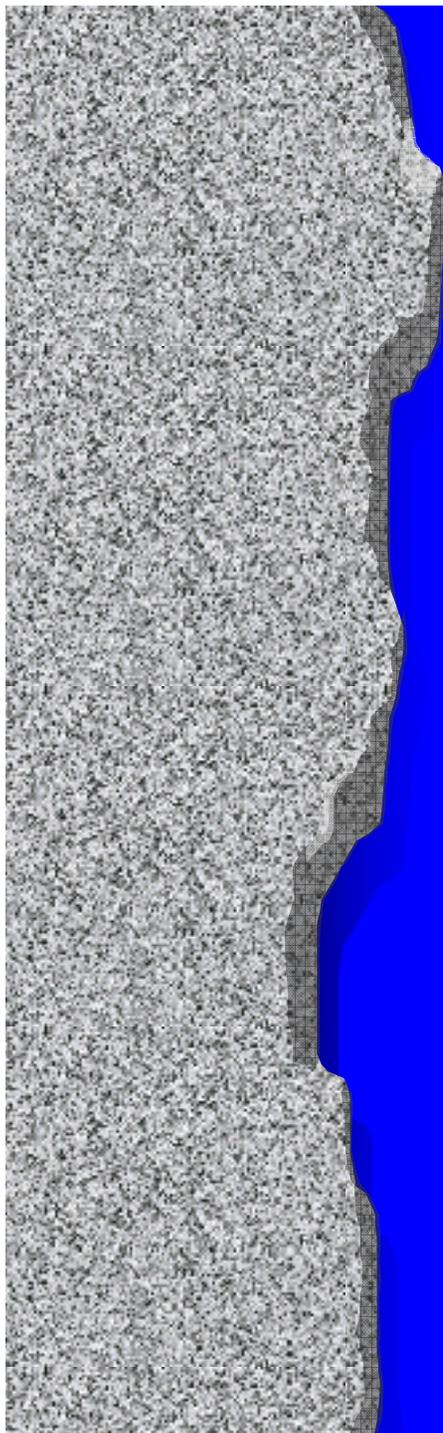


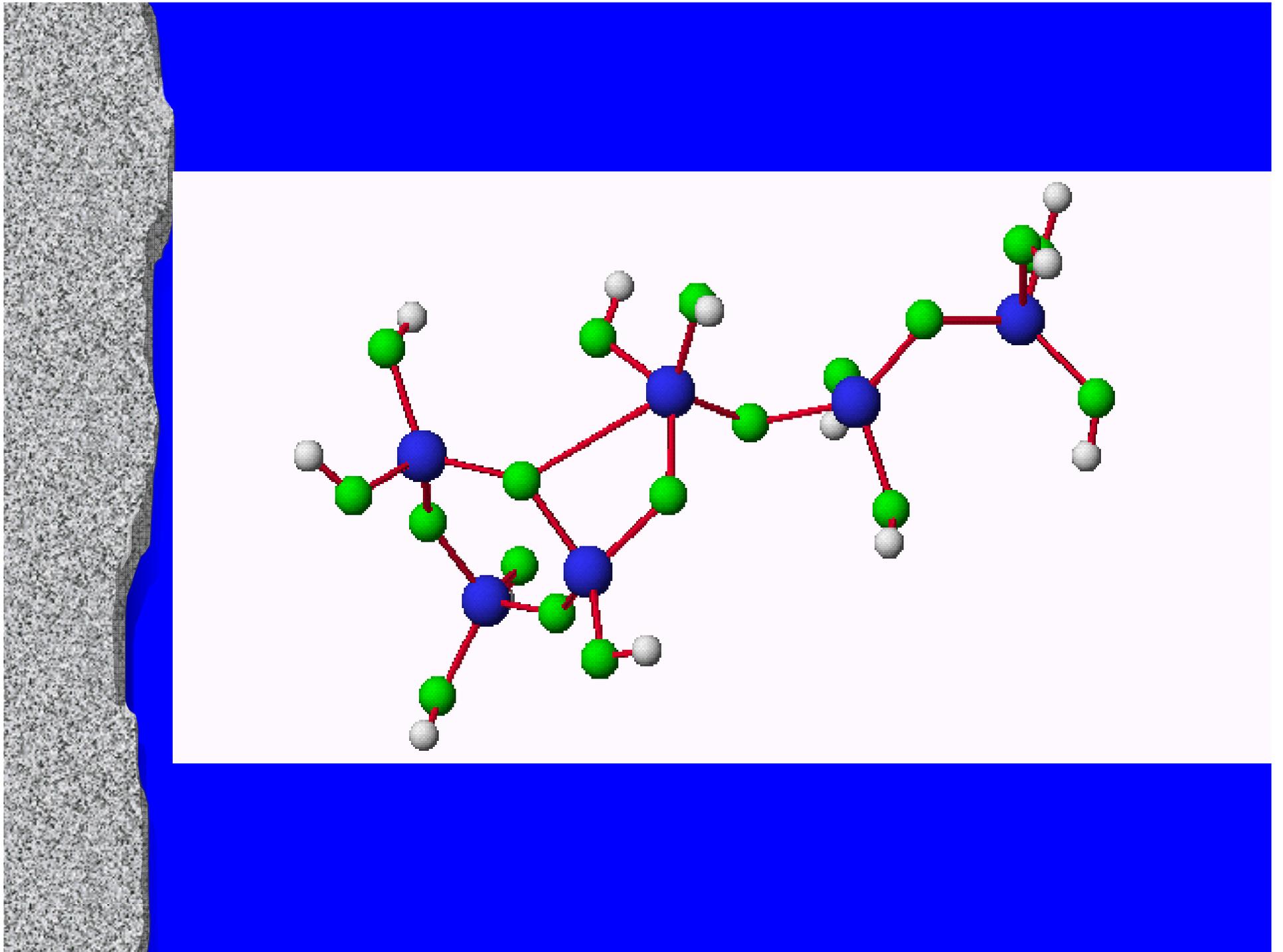


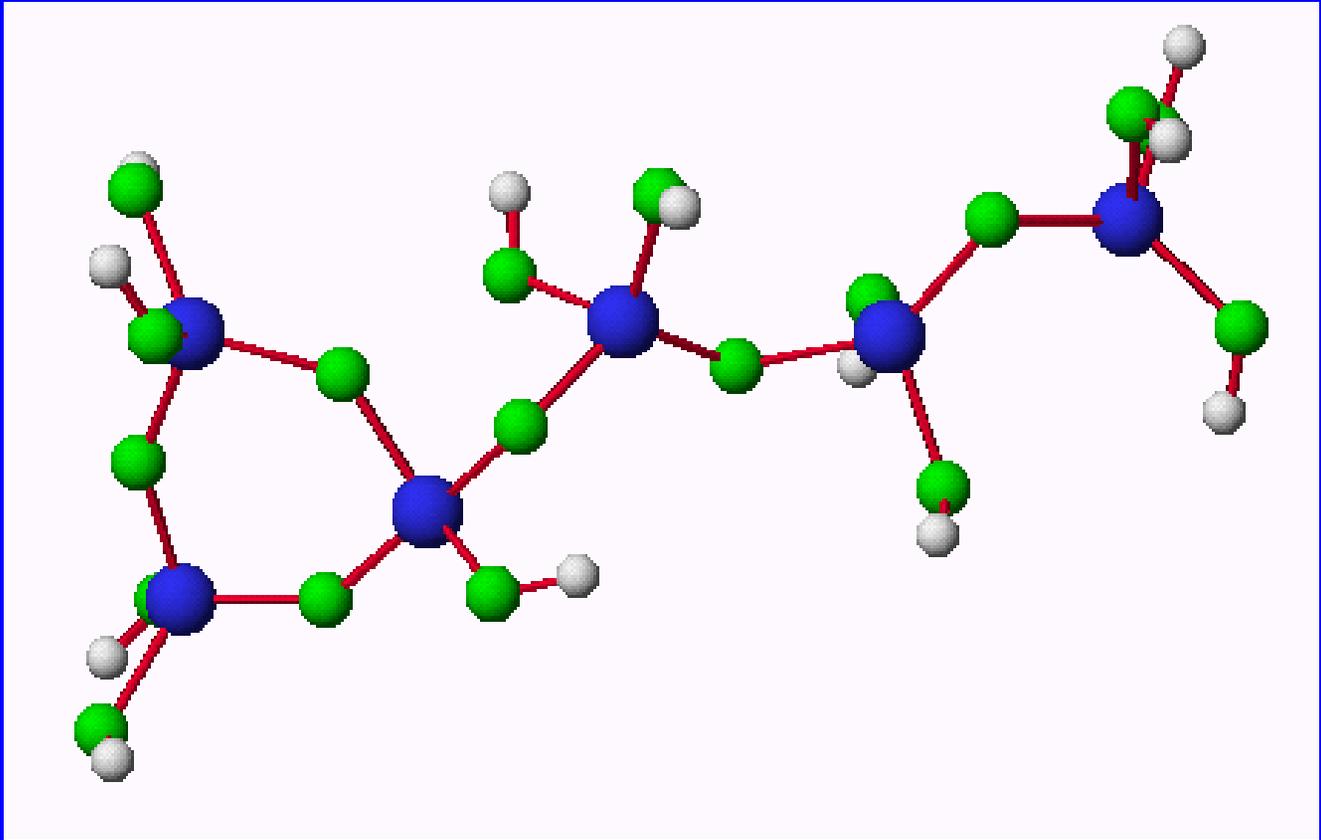
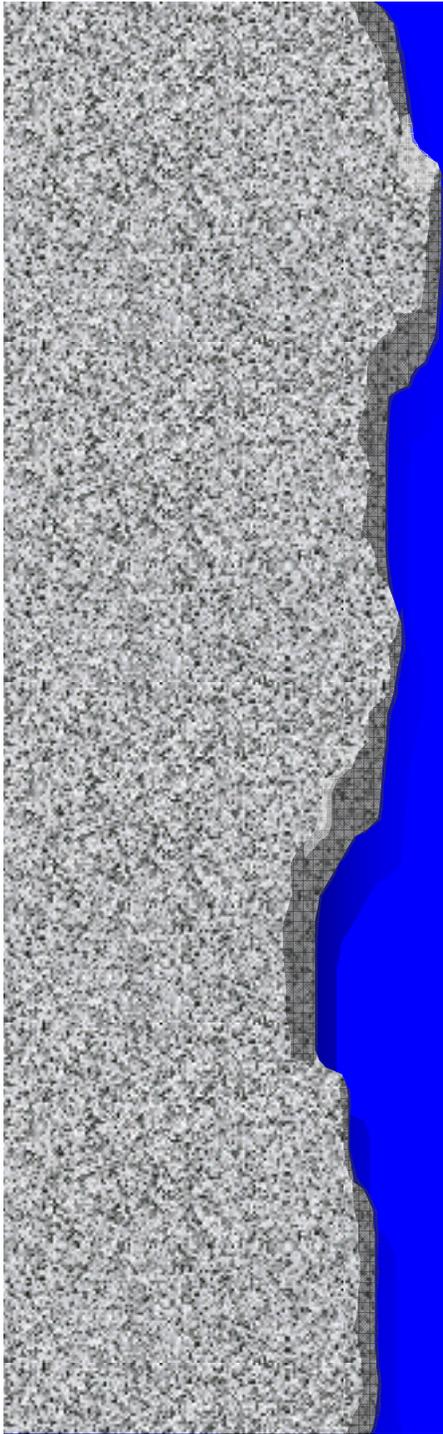




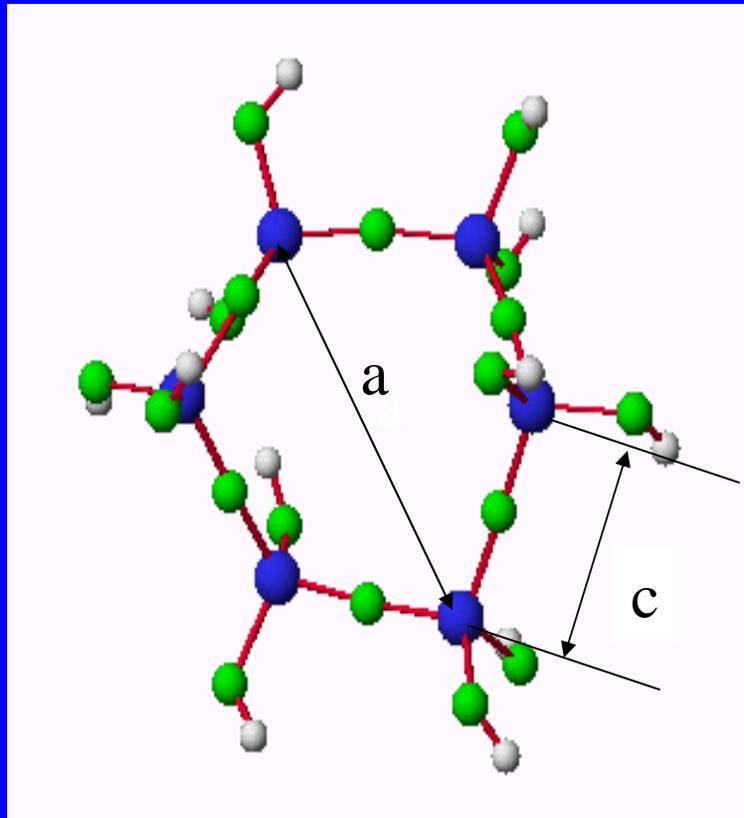




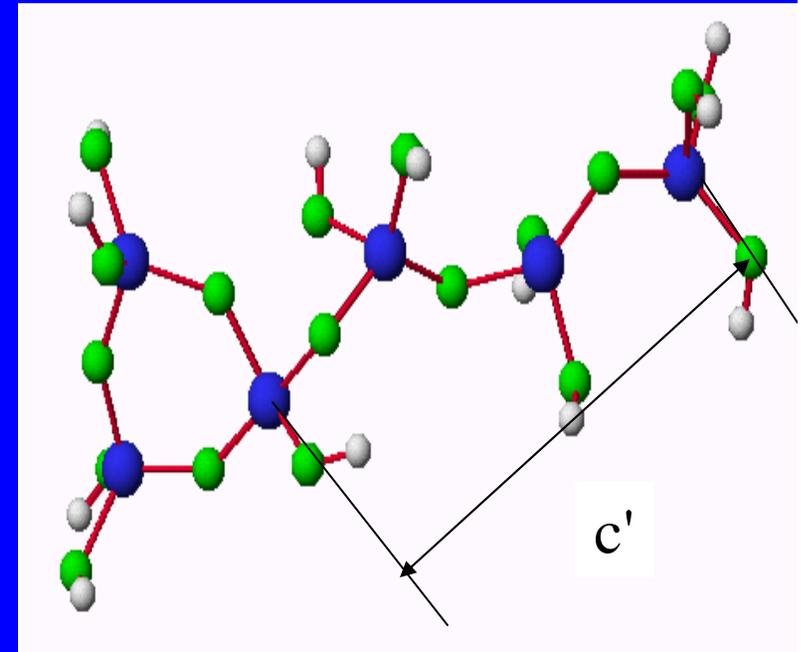




# Strain Can Be Measured In Model

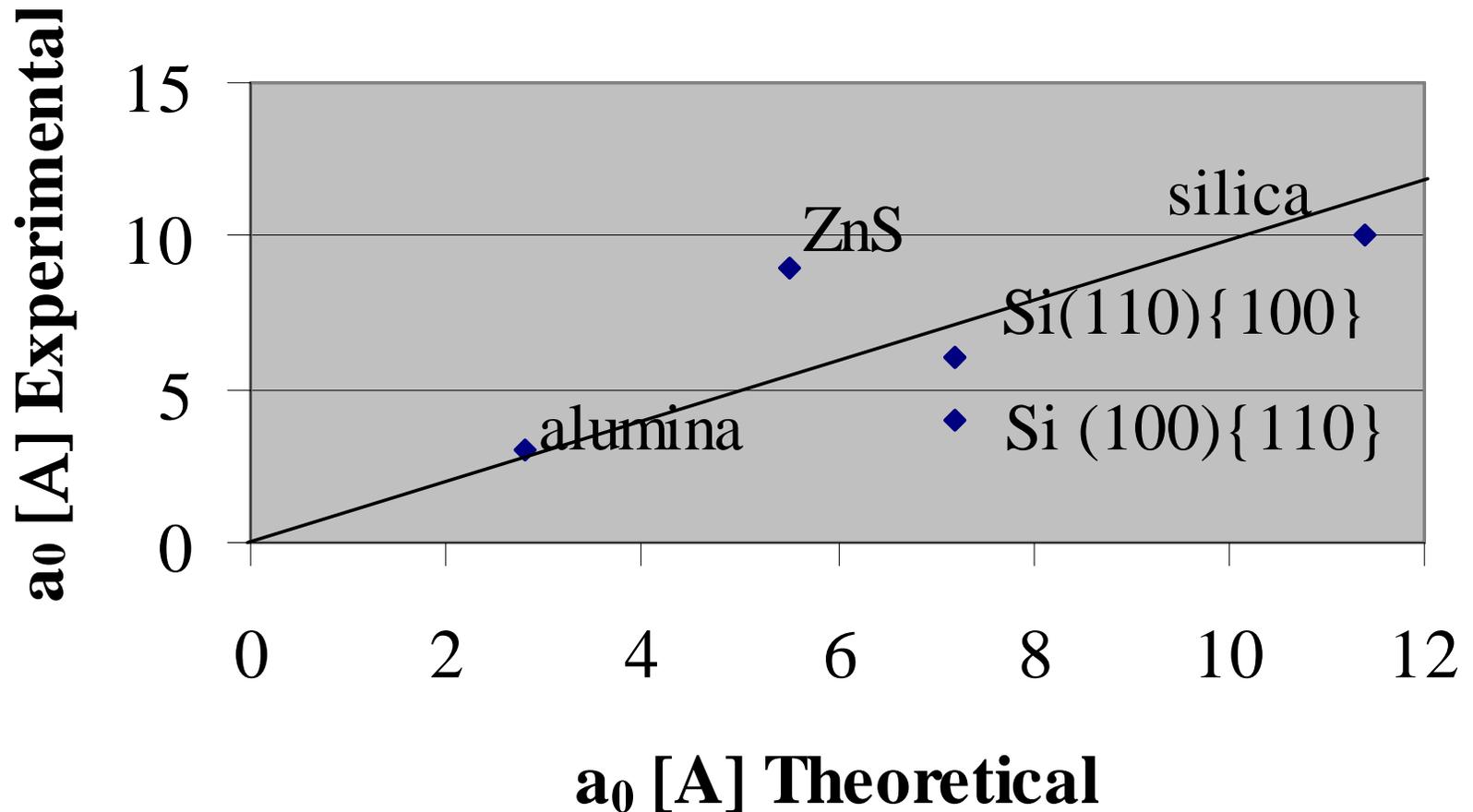


$$\begin{aligned} a_0 &= a / \varepsilon \\ &= c a / c' - c \end{aligned}$$



J. Non-Crystalline Solids  
260 (1999) 99-108.

## Modeling and Experimental Results Agree



$$a_0 = 2\gamma / (ED^*)$$

J. K. West, J. J. Mecholsky, Jr, and L. L. Hench, "The Quantum and Fractal Geometry of Brittle Fracture", J. Non-Crystalline Solids 260 (1999) 99-108.

## $a_0$ Is Related To Structure

Material Class	$a_0$ (Å)
Single Crystals	1-10
Glasses	10-20
Glass Ceramics	20-80
Polycrystalline Ceramics	3-10
Polymers	2700-14000

# Fractal Geometry Applied To Fracture (Part 4)

J. J. Mecholsky, Jr.

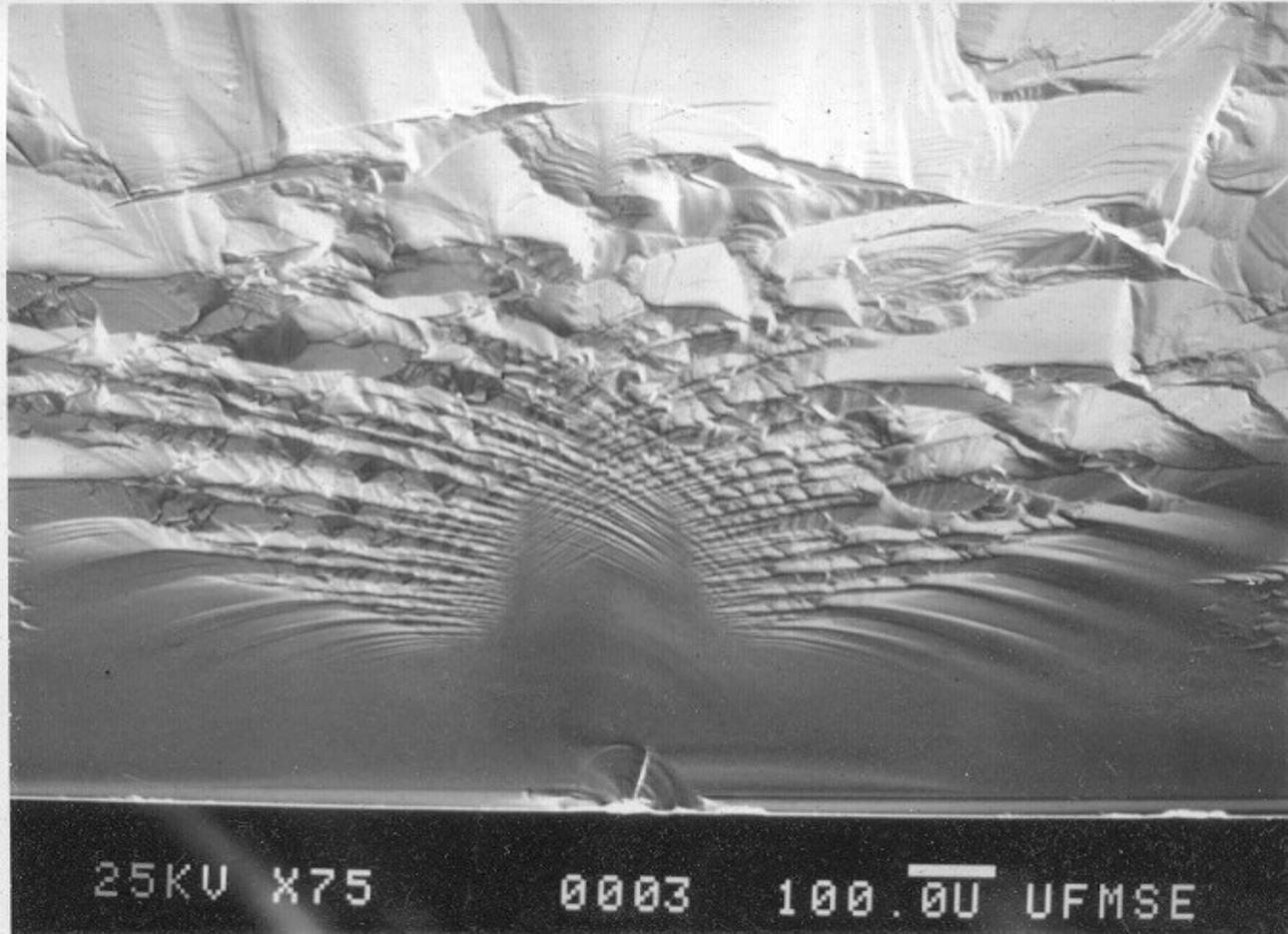
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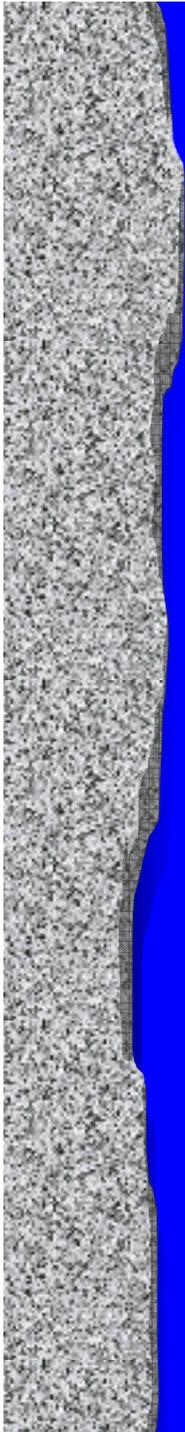
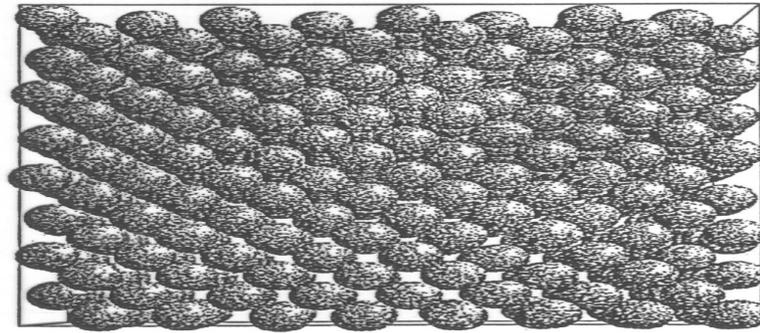
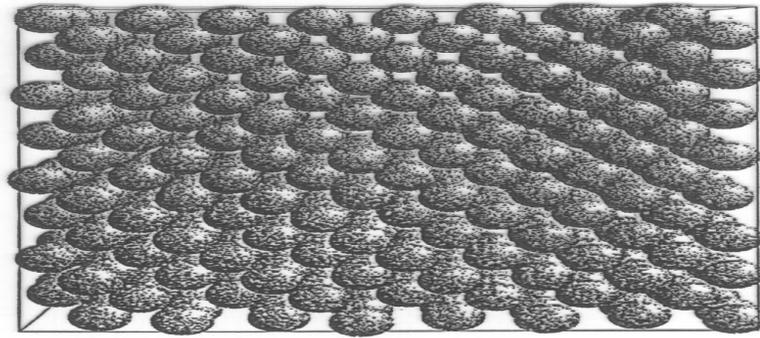


# FSA Can Be Applied To Single Crystals

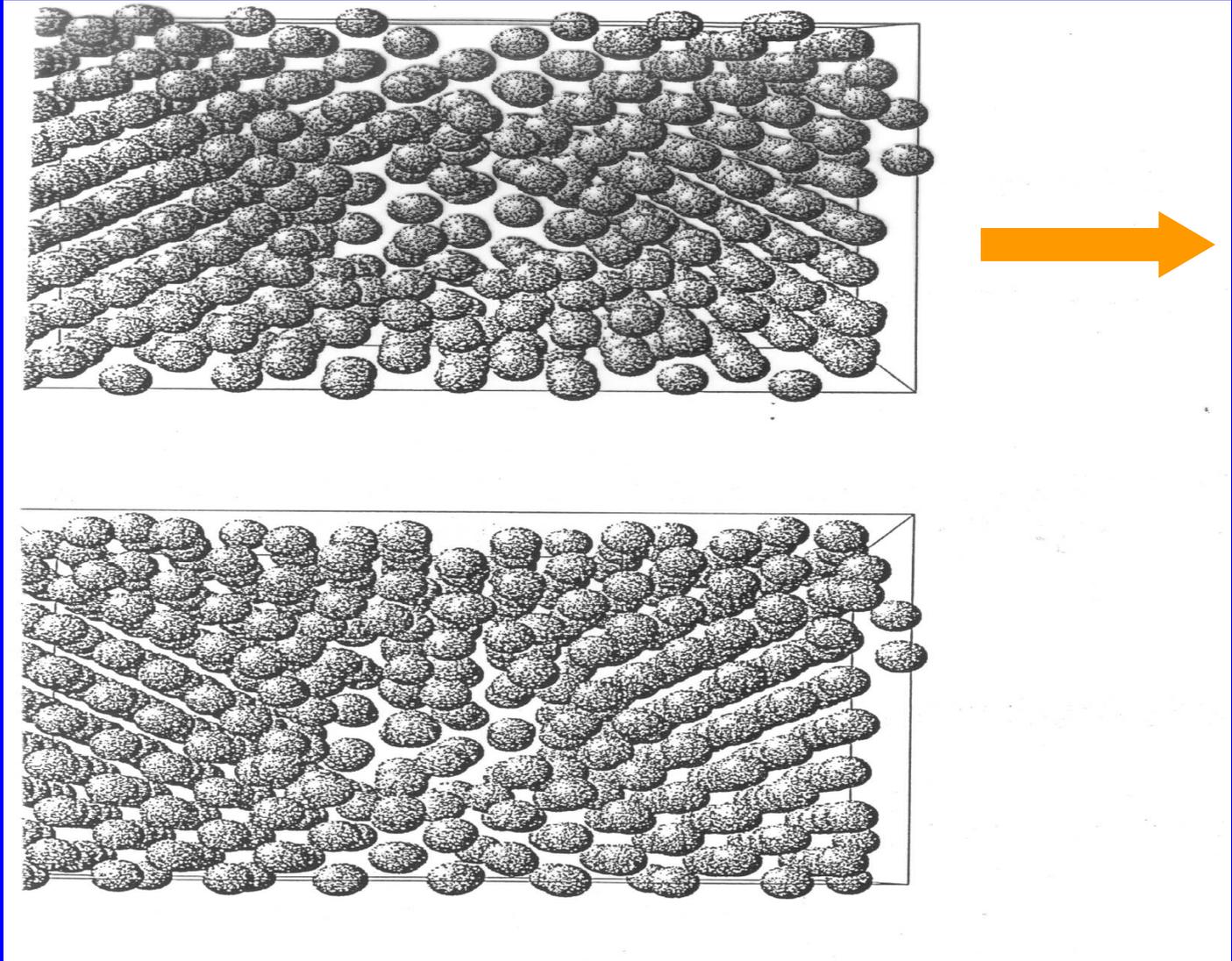
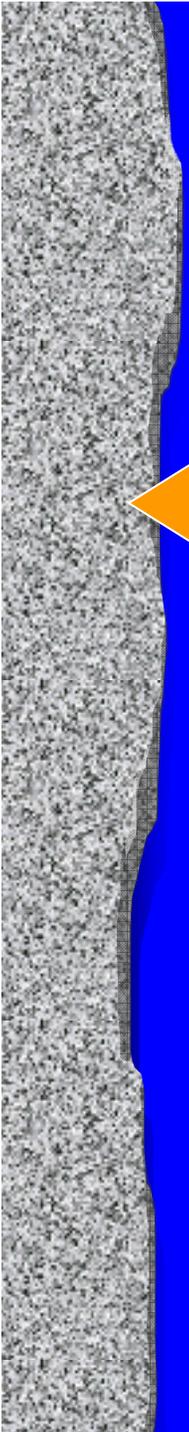


Single Crystal Silicon

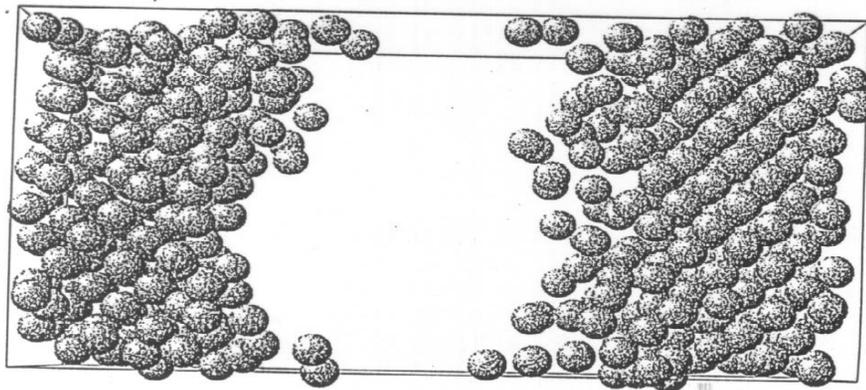
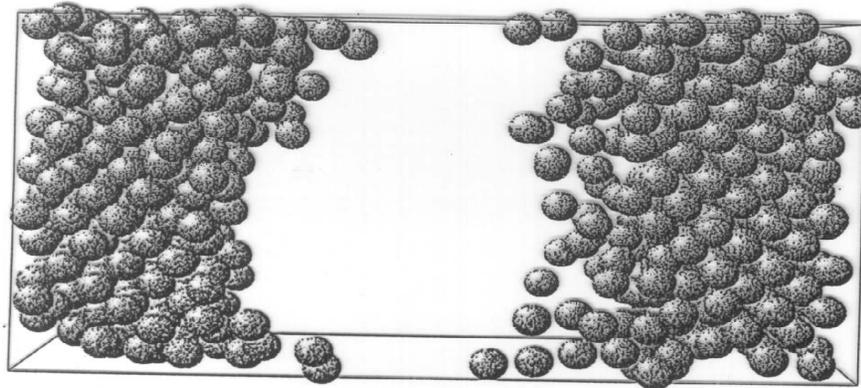
# Simulated Fracture Can Form Fracture Surface



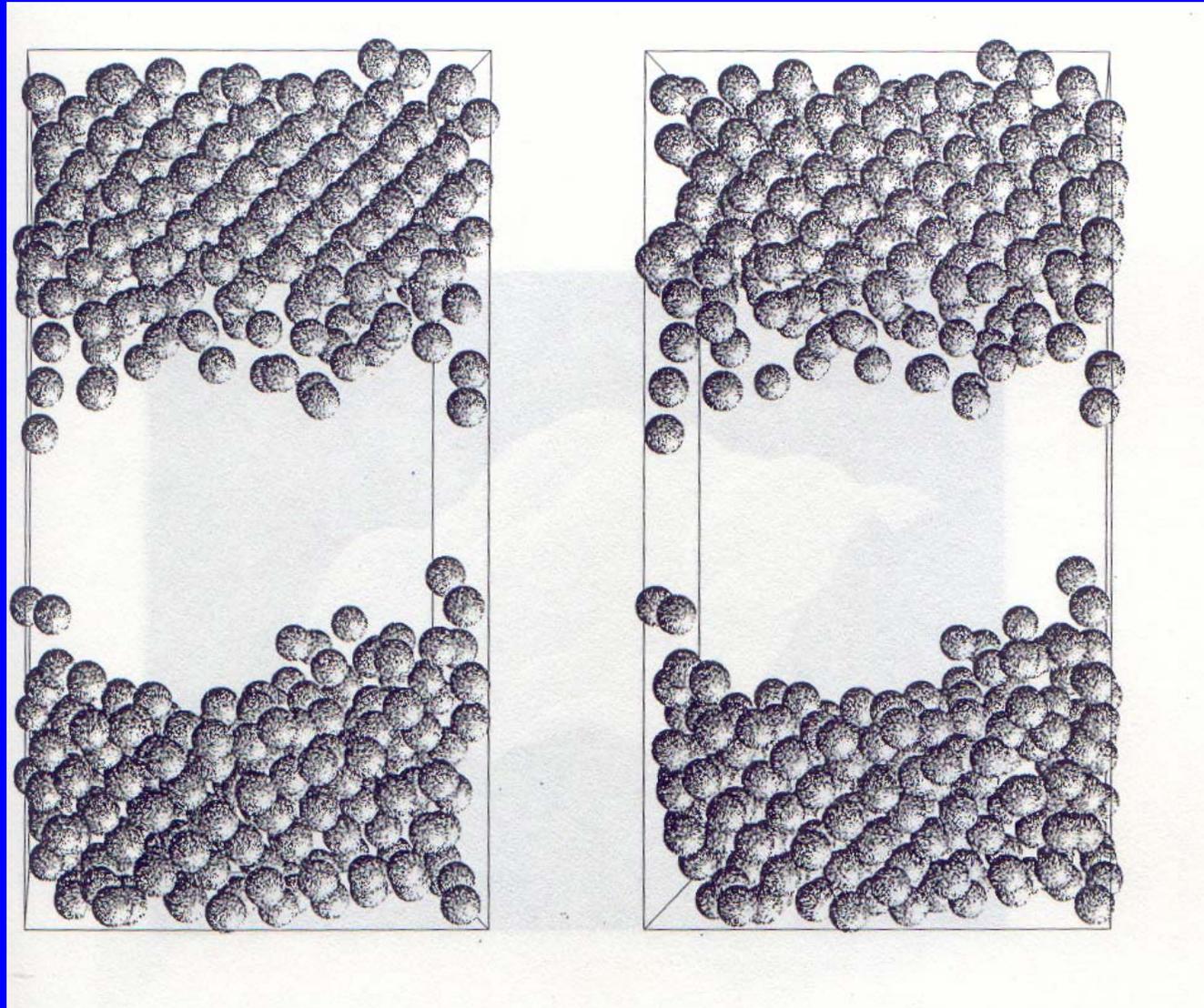
# Simulated Fracture Can Form Fracture Surface



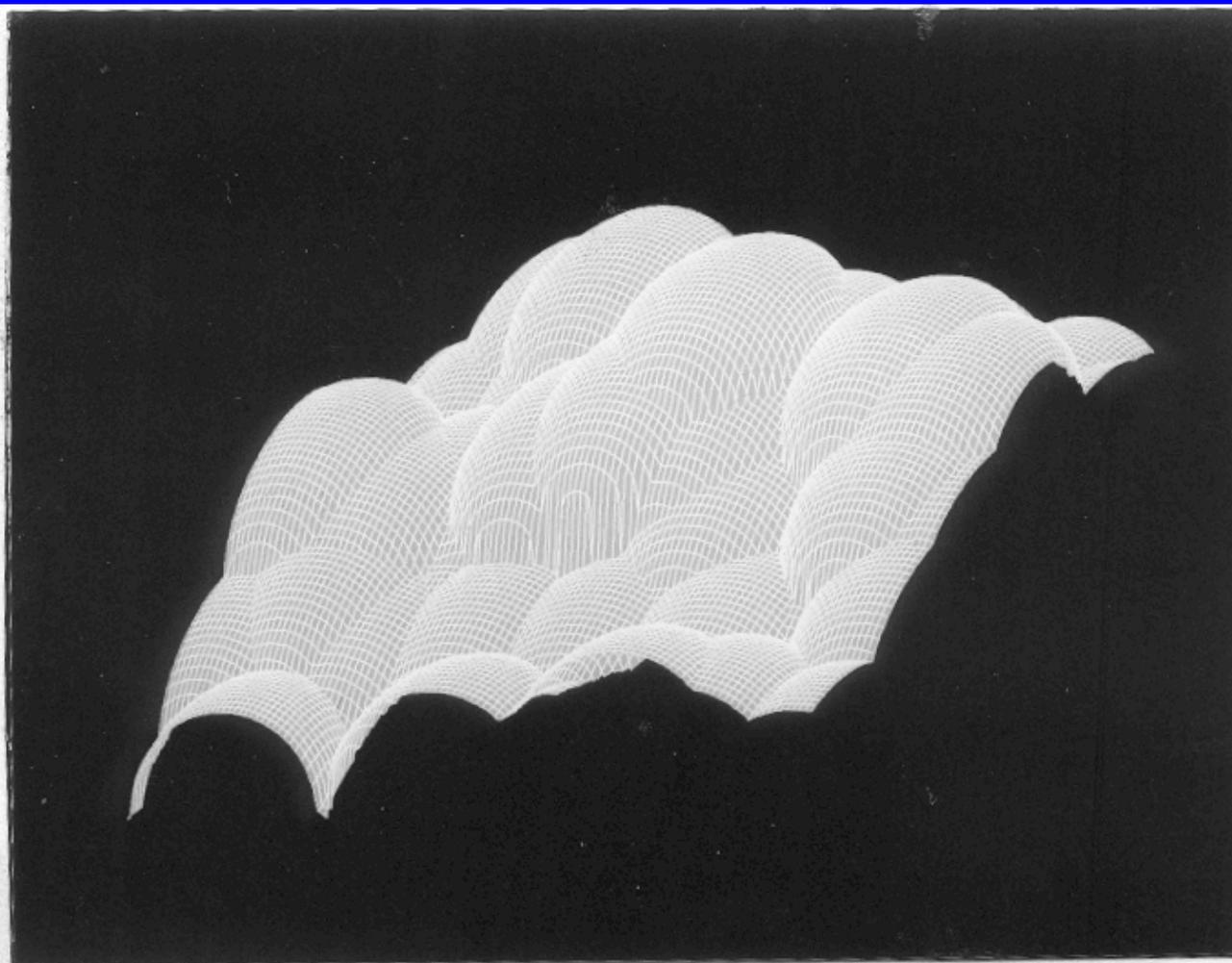
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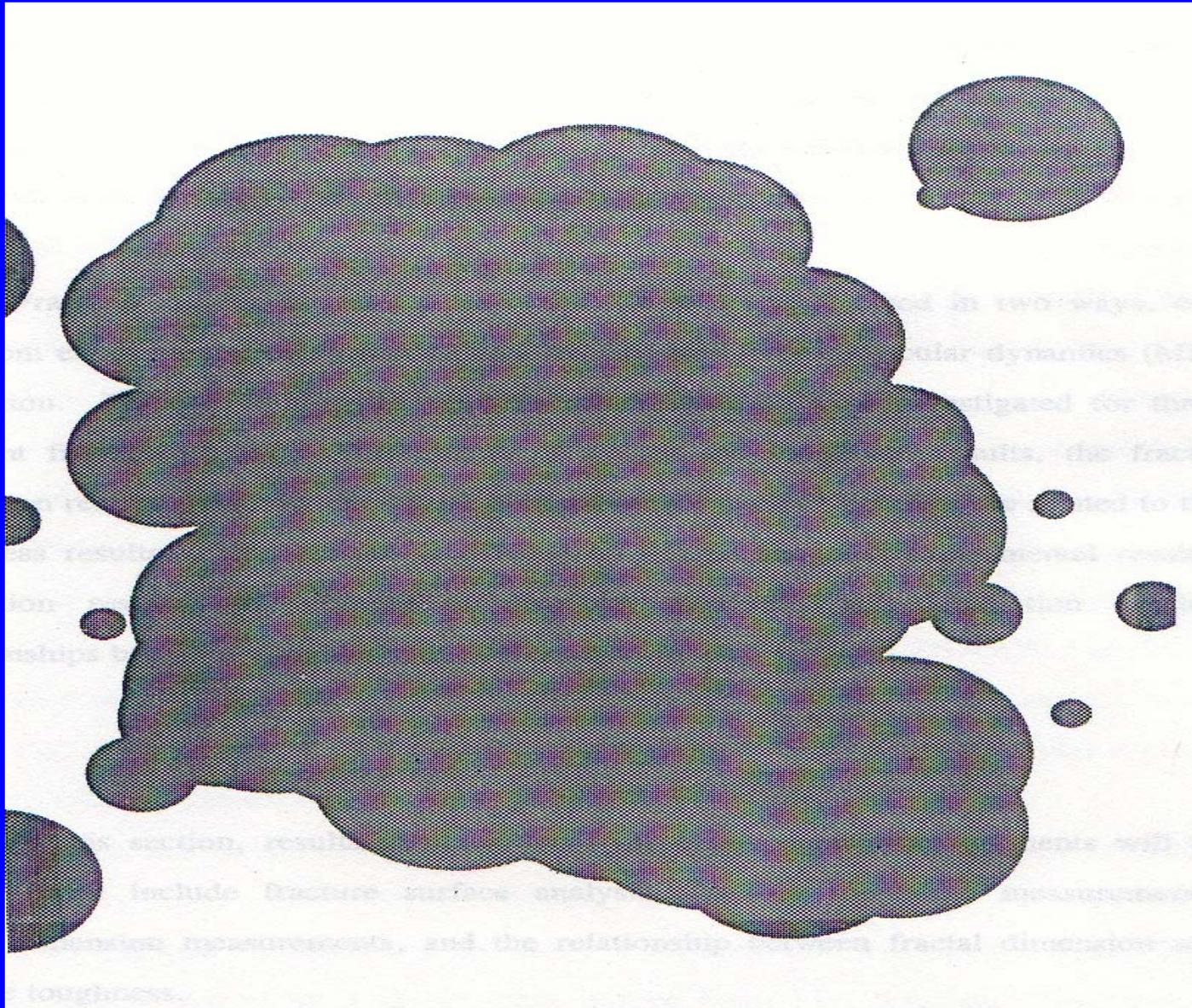
# Simulated Fracture Can Form Fracture Surface



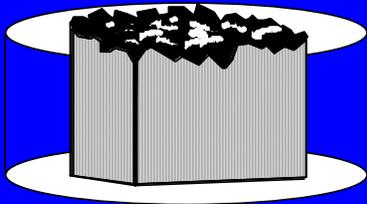
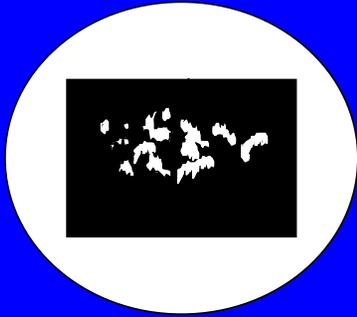
# Surface Can Be Created From MD Simulation



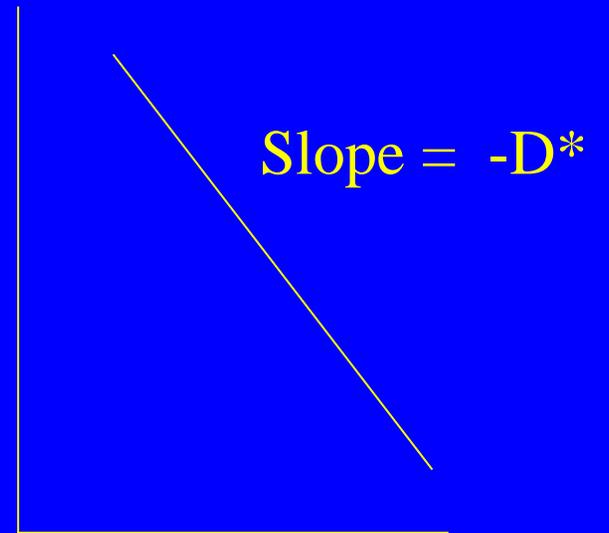
# Slit Island Contour Can Be Made From 3-D Map



# FRACTAL DIMENSION IS MEASURED ALONG CONTOUR

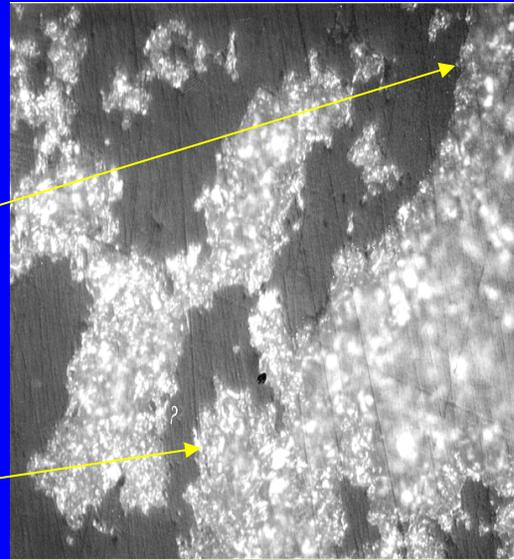


Log Length (A-B)



**A**

**B**

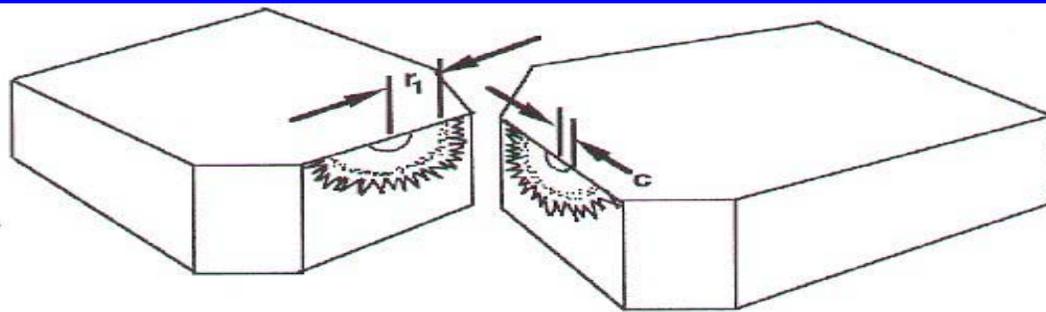


A-B = Slit Island Contour

# MD Simulations & Experimental Results Agree

Material Fracture Plane/Surface	$K_{IC}$ (MPam <sup>1/2</sup> )	Fractal Dimension (Experimental)	Fractal Dimension (MD Simulation)
<b>Si {100}/{110}</b>	<b>1.26 ± 0.06</b>	<b>2.16 ± 0.04</b>	<b>2.16 ± 0.06</b>
<b>Si {110}/{100}</b>	<b>1.23 ± 0.08</b>	<b>2.10 ± 0.04</b>	<b>2.11 ± 0.05</b>
<b>Si {111}/{110}</b>	<b>1.17 ± 0.08</b>	<b>2.06 ± 0.02</b>	<b>2.09 ± 0.04</b>
<b>Silica (amorphous)</b>	<b>0.75</b>	<b>2.11 ± 0.02</b>	<b>2.1</b>

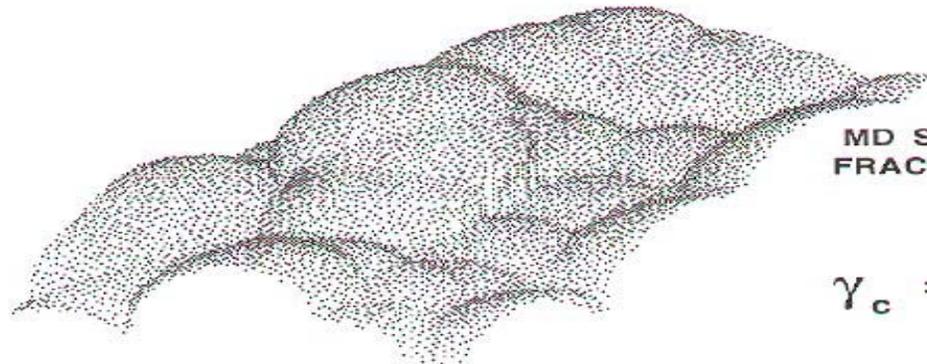
Y. L. Tsai, T. P. Swiler , J. H. Simmons and J. J. Mecholsky, Jr., in  
Computational Modelling of Materials and Processing, J. H. Simmons, et  
al. (eds) The Am. Cer. Society, Ceram. Trans. 69 (1997) 217.



CRACK SIZE (c) to MIRROR-MIST ( $r_1$ ) RATIO:  $D^* = \frac{c}{r_1}$

where  $D^*$  = FRACTAL DIMENSIONAL INCREMENT

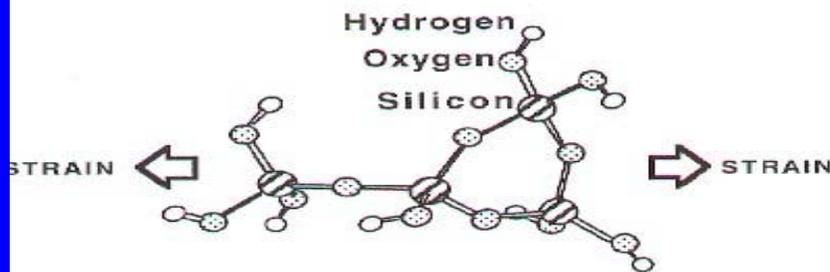
(a)



MD SIMULATION OF FRACTAL FRACTURE SURFACE

$$\gamma_c = \frac{a_o D^* E}{2}$$

(b)



AM1 MO MODEL OF FRACTURE AS ATOMIC LEVEL RING CONTRACTION

RING DIAMETER TO STRAIN FRACTION RATIO:  $a_o = a/\epsilon$

(c)

## Fracture Is A Fractal Process

- Fracture transcends many length scales; Self-similar (or self-affine), scale invariant & characterized by  $D^*$ .
- Hypothesis:  $2\gamma = [a_0 E D^*]$
- Observations seem to support the hypothesis.  
$$c/r = D^* \quad ; \quad \gamma \propto ED^*$$

( $D^*$  is a geometric & energy scaling factor.)  
( $a_0$  is a fracture surface structural element)
- MD & MO modeling provide framework for understanding macroscopic observations

# Many Tools Are Needed for Unified Fracture Theory

<u>Model</u>	<u>Scale</u>	<u>Experiment</u>	
MO	sub-atomic	fractoemission	
Ab initio	atomic	AFM	Fractal
Quantum	nano	STM	
Mechanics		Raman	
		Fluorescence	
MD	micro	crack velocity	Geometry
Monte Carlo		SEM	
Finite diff.	Meso	AFM	
FEM	Macro	fractography	
		fracture mechanics	

↓

**Fracture Process**

# Summary

- At the atomic level, quantum mechanics describes the fracture process as a ring contraction process dictated by minimum energy and availability of free volume.
- On the molecular scale, MD modeling describes creation of the fracture surface.
- On the macroscopic scale, mirror, mist & hackle form &  $c/r = D^*$
- At all length scales,  $2\gamma = [a_0 E D^*]$

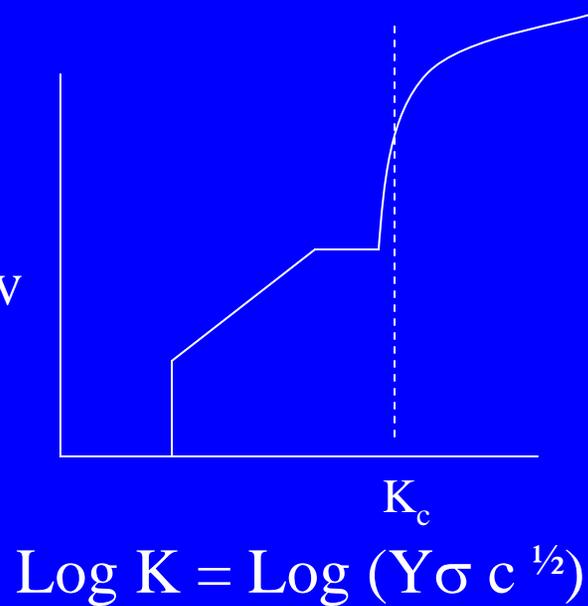
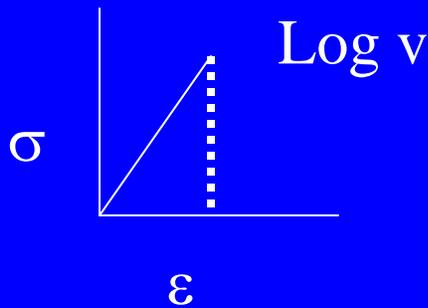
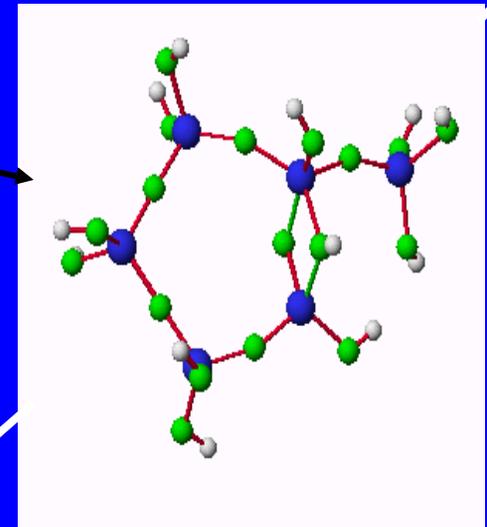
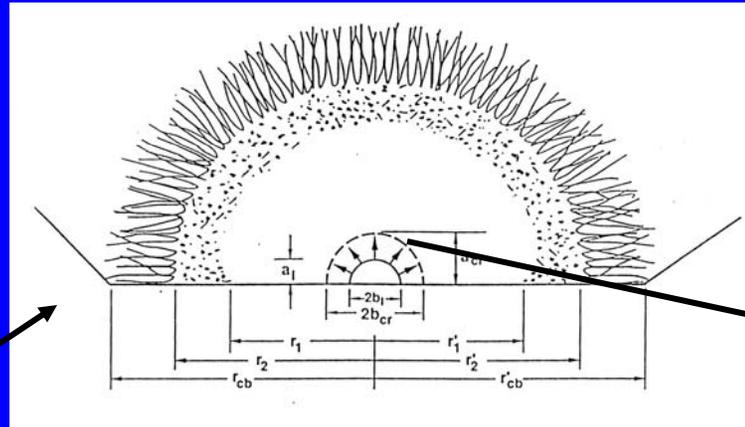
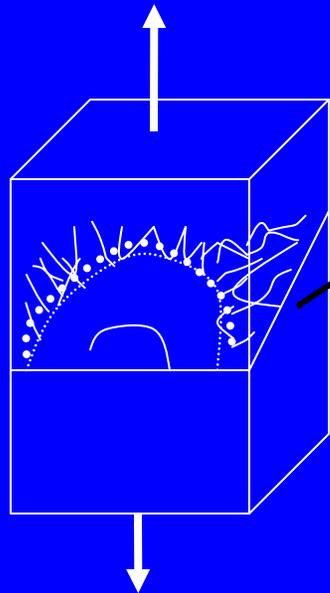
## Critical Questions Need To Be Asked

- What are the energetic & geometric steps to fracture?
- Is a flat fracture (of primary bonds) possible above absolute zero? What is bond rupture?
- Is roughness a meaningful parameter in fracture?
- How does energy scale?
- How does a crack propagate at all length scales?

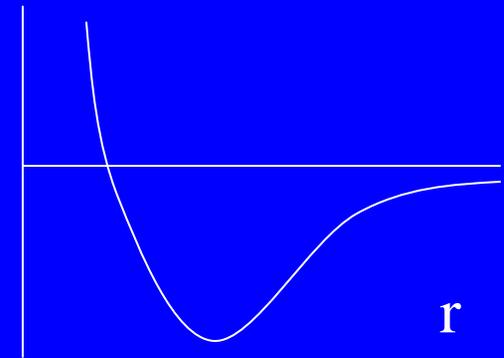
# CONCLUSIONS

- Fractal fracture implies that the same fracture process should be able to be observed at all length scales.
- Experimental data & analytical modeling have to be interactive to be successful.
- All models should be compared to (real) experimental data.
- Analytical models have to explain fractal nature of fracture, mirror, mist & hackle and crack branching.

# Bond Breaking Leads to Characteristic Features



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# University of Florida, Gainesville

