Robust Forecasting for Unit Commitment with Wind

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Abstract

Given the importance of handling high levels of uncertainty from renewables in the unit commitment problem, there has been increased attention given to the use of stochastic programming methods. Since these are computationally very demanding, there is a need for new approximations. We propose to use a point forecast of energy from wind and loads, where the point forecast is chosen and adapted by simulation to produce a robust solution. Traditionally, point forecasts represent an expectation. In our work, we suggest that we can use an appropriately chosen quantile of the forecast distribution which is optimized within a stochastic environment. The result is a policy search algorithm built around a point forecast, which is easily implementable using standard industry models and algorithms.

1. Introduction

A major challenge posed by wind penetration to grid operators is the potential source of system stress due to stochastic variations in the power output of wind farms. Imbalances between system load and generation must be compensated at all times for maintaining the AC frequency close to its standard value, but wind power is less predictable than load. With a higher share of wind turbines in the generator fleet, wind ramp-down events could be similar to the outage of one or more large generators. For instance, Wan [1] studies the large wind power drops that occurred in ERCOT (Texas) between 2004 and 2009. The severity of such events may depend less on their magnitude than on the poor predictability of their timing at the time of the unit commitment.

For operating the grid reliably, the existence of a recourse plan for dealing with uncertain wind variations, transmission constraints and credible contingencies must be guaranteed prior to dispatching the wind farms. Frequency control and regulation address imbalances in the short-term time scale. They rely on the spinning reserve required from dispatched thermal plants, and on pumped-hydro units [2]. In the future, innovative battery storage could also be untapped [3]. Higher wind penetration calls for increasing the spinning reserve [4], and maintaining reserve for being able to ramp up quickly on short notice [5]. More severe contingencies and grid congestions can be addressed by redispatching generation [6, 7], and relying on fast-start units such as combustion turbines. Demand response programs can also be implemented [8, 9]. Preemptive redispatch can be used to start up more efficient thermal plants with some advance notice, when the power system enters a state where severe contingencies become more likely. Ideally, all these provisions would be taken into account at the stage of the security-constrained unit commitment (UC) solved day-ahead and possibly intra-day for deciding which generators should be online [10, 11, 12, 13].

Due to the complexity of solving the unit commitment problem, the variability of wind is generally dealt with by imposing additional reserve requirements, and then solving the unit commitment in a deterministic fashion, using the forecast for the load and wind over the planning horizon. In this framework, one has to distinguish between the predictable and unpredictable variations of wind [14], and pay a particular attention to the choice of the forecasts [15]. Wind ramp forecasting [16] would allow, for instance, a less conservative management of the fast-ramp spinning reserve.

Another research trend is to formulate unit commitment as a stochastic program. In theory, reserve levels are then set “endogenously” as a part of the optimal solution to the program. Stochastic programming approaches had been investigated by [17] and others at the time where Lagrangian relaxation was the dominant technique for solving unit commitment [18]. With mixed-integer programming (MIP) becoming the new dominant technique for addressing unit commitment [19], stochastic mixed-integer programming approaches to unit commitment are currently under active investigation [20, 21]. The main issue with these approaches is computational. Proponents of the approach suggest that few scenarios could be enough to obtain reliable solutions, but that means that in that case the “endogenous” reserve
determination will closely reflect the exogenous scenario selection.

Fundamentally, all these research trends originate from the same need for solving an intractable multistage stochastic optimization model [22], even if that model is not written down explicitly. They differ in the type of approximations that are made.

The extent to which approximate computational schemes lend themselves to improvements through engineering and domain knowledge incorporation is important in the context of the unit commitment problem. In the present paper, we adopt the approach of using carefully engineered point forecasts. We propose a framework where point forecasts are learned from the interaction with the day-ahead optimization program and a stochastic simulator, in such a way that the robustness of the day-ahead solution to stochastic events can be improved.

The paper is organized as follows. Section 2 is dedicated to the notion of forecast in the control and optimization communities. Section 3 introduces a novel forecasting method based on quantiles, that we call X-Quantile Forecasting, where the choice of the quantile is driven by a forecast X. Section 4 argues that wind and load become coupled in the unit commitment problem and thus could be forecasted jointly. Section 5 records that the notion of reserve is dual to the notion of forecast for obtaining deterministic programs that are robust against perturbations. Section 6 describes with more details the unit commitment problem and the nature of its interaction with the subsequent operations, described in Section 7. Motivated by the idea that constraints can be modified in the day-ahead unit commitment problem once the lack of a resource has been identified through the simulation of intra-day operations, we propose in Section 8 an algorithm for learning X-Quantiles, based on the relation between quantiles and the optimality conditions relative to a simple stochastic resource procurement problem. Section 9 presents numerical results of experiments made with the proposed algorithms, and Section 10 concludes.

2. Point forecasts

A point forecast \( Y=(Y_1, \ldots, Y_T) \) for a stochastic process \( \{Y_t\}_{t=1} \) is defined as a single representative of the process up to some look-ahead time \( T \), based on information available at \( t = 0 \). The forecast \( Y \) can be a pure average path \( \mu_Y = (E[Y_1], \ldots, E[Y_T]) \), a sample path \( (Y_{1}, \ldots, Y_{T}) \) of \( \{Y_t\}_{t=1}^{T} \), or a sample path unrelated to the distribution of \( \{Y_t\} \). However, the name “forecast” conveys the idea that \( Y \) is useful to a decision maker in the context of a precise problem.

This paper will build on point forecasts where \( Y \) is obtained as a certain quantile of the distribution of the random variable \( Y_t \), written

\[
Y_t = F_t^{-1}(q) = \inf_{y \in \text{support}(Y_t)} \{ y : F(y) \geq q \},
\]

with \( q \in [0,1] \) the quantile parameter, and \( F_t \) the cumulative distribution function (cdf) of \( Y_t \). Random vectors in \( \mathbb{R}^n \) can be handled by treating each coordinate separately, with a quantile parameter \( q \) for each coordinate.

Replacing a stochastic process by a single realization is a very natural engineering practice. Control theory has identified classes of stochastic optimal control problems for which an optimal solution can be obtained by solving a deterministic program [23, 24]. In that literature, an equivalent deterministic program is constructed by taking expectations of random variables conditioned on the current information state (past and present measurements, past actions, and the prior probability distributions). [25] and others have sought to characterize the loss of optimality when random variables are replaced by a fixed value for certain classes of stochastic programs, and have concluded that the loss is very sensitive to problem data, and rather unpredictable. Deterministic formulations based on quantiles have been studied mainly in the context of chance-constrained stochastic programs. They also work almost trivially for the class of two-stage stochastic newsvendor problems: the optimal solution to \( \text{Min}_{x} E[p \min\{x, Y\} - cx] \) coincides with the optimal solution of the deterministic equivalent \( \text{Min}_{x} \exp [\{x, \mathcal{Y}\} - cx] \) with \( \mathcal{Y} = F^{-1}(q) \), \( q = (p-c)/p \).

An attractive property of quantile-based forecasts is that when the distribution of \( X_t \) is fixed but unknown, quantiles can still be estimated from data by using modified supervised learning algorithms [26, 27]. For instance, loss-function-based regression on the samples \( y_i \) minimizes the empirical expectation of the pinball loss function \( l_q(y, y_i) = q |y - y_i| \) if \( y_i > y \), \( (1-q) |y - y_i| \) if \( y_i < y \).

Quantile estimation methods applied to wind probability density forecasting have been compared in [28].

3. X-Quantile forecasts

In this paper, we introduce forecasting methods where the quantile \( q \) in (1) becomes time-dependent, as a function of another forecast \( X_t \). We call such methods X-Quantile forecasting. By doing so, a large family of forecasting methods can be obtained, that we restrict by choosing a class of functions for \( q \). For instance,
point forecast $\mathbf{X} = (X_1, \ldots, X_T)$ of a process $\{X_t\}$ can be used to construct

$$Y_t = F_t^{-1}(\rho(X_t)),$$

$$\rho(X_t) = [1 + \exp\{-(\alpha + \beta^T X_t)\}]^{-1},$$

for some $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}^d$ ($d$: dimension of $X_t$).

To illustrate how $X$-quantile methods can be useful in the context of unit commitment in the presence of wind, consider the problem of mitigating the phase-shift error of wind predictions, which is critical in the context of ramp events prediction. Consider the random function with $\delta \sim N(0,1)$,

$$f_\delta(t) = 2 + \cos(t - 0.3\delta) + 2 \cos(2(t - 0.3\delta))$$

that models a time-varying wind power affected by an uncertain phase shift. Figure 1 shows sample paths and different forecasts (evaluated by Monte Carlo over $10^4$ samples):

- The mean function $f(t) = \mathbb{E}_{\delta}\{f_\delta(t)\}$;
- The quantile functions for $q = 1/1000, 1/8, 2/8, \ldots, 7/8, 999/1000$;
- $X$-quantile (1'-2) with $X_t = f(t)$, $\alpha = 1$, $\beta = -1$;
- $X$-quantile (1'-2) with $X_t = f(t)$, $\alpha = 0.5$, $\beta = -1.5$.

In contrast to the mean or fixed quantiles, the $X$-Quartiles can attenuate the slope of potential ramp events, by modulating the profile of the zero-cost wind energy that is seen and dispatched by the day-ahead model. Using such forecasts can help reduce the fast-reserve requirements in the unit commitment (at the cost of scheduling less wind).

4. Joint forecasts of load and wind

Irrespective of the assumption that load and wind follow independent processes or are coupled through a hidden temperature process [29], there can be a benefit in considering joint forecasts of load and wind, given their interaction in the unit commitment. Indeed, during periods in the day where the load builds up, less ramp-up capacity is available for hedging against stochastic wind ramp-downs. As a result, it can be beneficial to lower wind forecasts, so as to hide some wind power from the UC solver. Figure 2 depicts the situation in the spirit of a graphical model [30], where $X \rightarrow Y \leftarrow Z$ indicates that the random variables $X$ and $Z$ are not necessarily independent conditionally to $Y$.

![Figure 2. The load forecasts (LF) and wind forecasts (WF) are used in the day-ahead unit commitment (UC); they are based on the Load and Wind stochastic processes that also drive the real-time adjustments (RT).](image-url)
5. Reserves

Point forecasting can be put in perspective with perturbation theory for multistage stochastic programs. Such a theory allows one to optimally reduce a set of scenarios for solving an approximate stochastic program as close as possible to the original one [31].

We add the notion that scenario reduction, applied to the extreme case of producing a single realization (forecast), has to be accompanied by a modification of the objective or constraints of the original stochastic program, so as to reflect the stochastic variability discarded by the scenario reduction process. In the context of unit commitment, a natural answer is to add reserve requirements, possibly with different grades of ramping-up capabilities.

If a multistage stochastic formulation of the unit commitment problem with the true probability distributions could be solved exactly, there would be no need to specify reserve requirements in advance. By virtue of the optimization over an astronomical number of branching scenarios, units would set aside capacity to be ready to react to the stochastic events.

6. Unit Commitment

A unit commitment problem (UC) can be viewed as a look-ahead mixed-integer program (MIP) of the form

$$\text{Min } \sum_{t=1}^{T} c_t^\top x_t + d_t^\top y_t$$

s.t. $B_t x_t + B'_t y_t = h_t$,

$A_{t-1} x_{t-1} + A'_t y_{t-1} + B_t x_t + B'_t y_t = h_t$,

$L_t \leq x_t \leq U_t, \quad L'_t \leq y_t \leq U'_t$,

$x_t \in \mathbb{R}^n, \quad y_t \in \mathbb{Z}^m$,

where $x_t$ and $y_t$ are further decomposed into a number of generator-indexed vectors, bus-indexed vectors, branch-indexed vectors, and slack variables. Generator-indexed subvectors of $y_t$ determine the on-off state and startup and shutdown indicators of the generators at time $t$. A generator-indexed subvector of $x_t$ determines the power productions at time $t$. Constraints on generators typically include warm-up time, max ramp-up and ramp-down rates, min and max production, min up- and down- times, and must-run status. The nonzero elements of $c_t$ and $d_t$ are relative to the costs of the generators and to the slack variables. A large value can be chosen for the Value Of Lost Load (VOLL) [32] so as to prevent load shedding at the time of the unit commitment.

If the program takes grid constraints into account, $x_t$ would also contain the voltage angles, bus injections and branch flows at time $t$. The relation between angles and flows is determined by the DC power flow linear equations. The flows are subject to the ratings of the branches.

Commercial solvers can be used to solve (4) over a one-day period ($T=24$). It is not unusual for system operators to run a unit commitment model over a one-week period ($T=168$), or to consider 15-minute increments.

7. Hourly Adjustment

The hourly adjustment model (RT) is a set of redispatch and start-up rules (possibly implemented by solving an optimization program) for correcting the imbalances between the net hourly production and consumption, and possibly restoring the spinning reserves. Imbalances result from the discrepancy between the day-ahead forecasts used in the UC model and the actual production and consumption levels, and from events affecting the grid. Corrective actions are generally taken according to the following order:

1. Adjust the output of online generators according to their operating constraints and economic merit (economic dispatch).
2. In case of production shortage, fire up fast-start combustion turbines. This is possible in the hourly time frame. Generators participating to the non-spinning reserve are usually those who can respond in 30 minutes or less.
3. In case of production overage, implement wind spillage. In last resort, shut down units.
4. Implement load shedding to prevent a system-wide black-out.

In simulations, one can keep track of the cost of hourly adjustments, based on the cost characteristics of the units. In reality, the adjustment costs depend on many factors, including on how markets are implemented, and on the interactions with the neighboring systems, which play the dual role of being a tertiary reserve provider and a source of imbalances.

8. Learning X-Quantiles

We propose an algorithm for optimizing X-Quantiles in the context of the UC problem. It is (i) based on the optimality relation between quantiles and two-stage stochastic newsvendor problems, and (ii) specific to the regression model (2). We state the algorithm in the case of learning the forecast of a scalar-valued wind power process $\{Y_t\}$, given an auxiliary input forecast $X$ that would typically include the forecasted load levels.
1. Set $k=0$, $\alpha^{(k)}=0$, $\beta^{(k)}=0$, thus $q_t^{(k)}=\rho(X_t)=0.5$.

2. Solve the day-ahead unit commitment (UC) with the current forecasting parameters.

3. Simulate the hourly adjustments (RT) of the schedule on random scenarios. For each scenario $j$, collect the pairs of forecast errors $Y_t^{1} - Y_t^{j}$ and the instantaneous adjustment costs $C_t^{j}$ from the RT model, $Z_t = (Y_t - Y_t^{j}, C_t^{j})$. (5)

To reflect that a forecasting error at time $t$ has a non-negligible impact on adjustment costs that may also affect subsequent periods, construct $C_{t,\lambda}^{j} = \sum_{t=1}^{T} \lambda^{t-1} C_t^{j},$ (6)

where $\lambda \in [0,1]$ is a fixed parameter of the algorithm.

4. For each $t$, define the index sets

$$J_t^+ = \{ j : Y_t > Y_t^{j} + \varepsilon T/2 \},$$

$$J_t^- = \{ j : Y_t < Y_t^{j} - \varepsilon T/2 \},$$

where $\varepsilon \geq 0$ is a fixed dead-band parameter for discarding forecast errors below some threshold. Estimate the empirical expected marginal overage cost $c_t^+$ and underage cost $c_t^-$:

$$c_t^+ = |J_t^+|^{-1} \sum_{j \in J_t^+} C_{t,\lambda}^{j}/(Y_t - Y_t^{j}),$$

$$c_t^- = |J_t^-|^{-1} \sum_{j \in J_t^-} C_{t,\lambda}^{j}/(Y_t^{j} - Y_t).$$

5. Define estimated first-order optimal quantiles

$$q_t^{(k)} = c_t^- h(c_t^+ + c_t^-),$$

and solve the following regression problem, where $\omega \geq 0$ is a fixed regularization parameter of the regression:

$$\min_{\alpha, \beta} \left( (T^{-1}/2) \sum_{t=1}^{T} \rho(\rho(Y_t) - q_t^{(k)})^2 + (\omega/2) \| \beta \|^2 \right),$$

$$\rho(z) = [1 + \exp\{-(\alpha + \beta^T z)\}]^{-1}. \tag{9}$$

Let $(\alpha^*, \beta^*) \in \mathbb{R} \times \mathbb{R}^d$ be the optimal solution to (9).

6. Update $\alpha$ and $\beta$, using a stepsize $\mu^{(k)} \to 0$:

$$\alpha^{(k+1)} = (1 - \mu^{(k)}) \cdot \alpha^{(k)} + \mu^{(k)} \cdot \alpha^*$$

$$\beta^{(k+1)} = (1 - \mu^{(k)}) \cdot \beta^{(k)} + \mu^{(k)} \cdot \beta^*.$$ (10)

7. Go back to Step 2 until a stopping criterion is met. For instance, stop when $\|\alpha^{(k+1)} - \alpha^{(k)}\|_{c}$ and $\|\beta^{(k+1)} - \beta^{(k)}\|_{c}$ are below a certain threshold.

We sketch the justification of the algorithm as follows. Steps 2 to 4 extract information on the regret of having the forecast too high or too low, by simulation of the black-box models. The information is aggregated in two scalar values $c_t^+$ and $c_t^-$ and plugged into a newsvendor problem, whose optimal solution will depend on the quantile parameter $q_t^{(k)}$. Step 5 performs a regression to express the sequence of quantiles $q_t^{(k)}$ in terms of the input forecast $X_t$. The new coefficients $\alpha^*$, $\beta^*$ for the model (2) are then used to update the forecasting parameters. We select a sequence of stepsizes $\mu^{(k)}$ such that

$$\mu^{(k)} > 0, \sum_{k=1}^{\infty} \mu^{(k)} = \infty, \sum_{k=1}^{\infty} [\mu^{(k)}]^2 < \infty.$$ Note that $1/k$ satisfy those conditions.

From the regression model, a wind forecast is available not only at the discrete time steps of the unit commitment, but at any period of time where wind distributions are available.

The regression problem is mathematically equivalent to minimizing the expected squared error of a neural network having one sigmoidal neuron, where the expectation is replaced by the mean squared error of the training set plus a regularization term for penalizing model complexity (by weight decay) [33]. The regularization term $(\omega/2) \| \beta \|^2$ attenuates the temporal variations of the predicted quantiles and helps mitigating the effect of the noise from the imperfect estimation of the quantiles $q_t^{(k)}$.

The look-ahead time $T$ may not be sufficient to generate enough samples for learning a model (9) that generalizes well, especially if $X_t$ is a vector. In that case, one could repeat Steps 2 to 4 with different initial conditions, before proceeding to Step 5.

9. Experiments

In this section, we evaluate the X-Quantile learning algorithm in a simplified setting. To do that, we build a simplified UC-RT simulator. We model the generator’s day-ahead offers by a continuous stack of bids, with no associated ramping constraint (Figure 3). The curve approximates a large, diversified, generator fleet by taking the limit to an infinite number of generators.

Given discretized hourly load and wind levels, a dynamic programming algorithm determines the percentage of the wind that should be used at each hour, and the generator capacity that should be online.

![Figure 3. Generator bid stack used in the tests](image-url)
The online capacity must at least meet the net load and the reserve requirement. The quantity used for the net load, and the spinning capacity, are priced separately. A linear startup-cost penalty is incurred when the online capacity is increased, providing an incentive to keep plants online when the net load drops. In the hourly adjustment, we have wind spillage for the excess wind, and a penalization for generation beyond the online capacity, at first by using the residual capacity of fast-start turbines, and finally by shedding load (VOLL 5000$/MWh).

The load and wind model are based on random functions evaluated at random times for accentuating phase-shift uncertainties. Namely:
1. Let $f(t)$ be a continuous basis profile.
2. Sample $t_1, \ldots, t_n$ uniformly in $[0, T]$ and sort them to obtain $t(1) < \ldots < t(n)$, with $n = kT$, $k$ integer.
3. Subsample the times: $\tau_j = t(kj)$, $j = 1, \ldots, T$.
4. Let $p_j = f(\tau_j) + \epsilon_j$ be the sample at time $j$, where $\epsilon_j$ is an independent noise.

Sample paths obtained with this stochastic model are shown in Figure 4. For the load we used $f(t) = 5 + \cos(\pi(t/12 + 0.7)) + 0.8 \cos(\pi(t/6 + 0.9))$.

For the wind we used $f(t) = 1.25 + 0.5 \cos(\pi t/16) + 0.5 \cos(\pi t/8)$.

More randomness could be obtained by randomizing the coefficients of the basis function $f(t)$.

Transmission constraints and losses are not modeled in this experiment, so the net load is the load minus the amount of wind used.

The stochastic models are used to build the quantile estimators, and then to generate an independent "simulation set" for evaluating the cost of hourly adjustments.

In our experiments, we compare a wind forecast strategy based on a fixed optimized quantile, to a wind forecast strategy based on time-varying quantiles, where both strategies are optimized using the same stochastic simulator.

To do that, first, we estimate by Monte Carlo simulation the expected cost of a schedule combined with the subsequent cost of adjustments over 24 hours, when the schedule is induced by the wind forecast made by evaluating a fixed quantile of the wind distribution. When wind power is scheduled, there can be substantial cost savings in the schedule, by the horizontal translation effect of a negative amount of needed capacity on the curve of Figure 3. A delicate tradeoff is that, the higher the percentage of wind used, the more costly the hourly adjustments become if the wind ramps are imprecisely forecasted. On the simulation results of Figure 5, the optimal fixed quantile is around $q = 0.4$. Note that this result strongly depends on the problem parameters, and in particular on the level of uncertainty for the phase errors of the wind. Scaling up or down the magnitude of the wind would also affect the optimal fixed quantile.

Second, we implement the proposed learning approach for selecting time-varying quantiles. The $X$ process is two-dimensional: we use the load forecast for the first dimension, and the mean process of the wind itself for the second dimension. On our test problem, we found that choosing $\varepsilon = 0.01-0.1$ and $\lambda = 0.01-0.2$ were satisfying settings. Figure 6 illustrates the progress of the algorithm along the iterations, where initially $q^0(t) = 0.8$ for each $t$. Figure 7 (Top) shows the solution found by the algorithm. A way to see that the wind quantile changes as a function of the hour of the day is to observe that the solution crosses the iso-quantile lines that have been drawn on the picture. The thin lines represent the curves $F^{-1}_t(q)$ for $q = 1/100, 1/828, \ldots, 7/899/100$. The values of $q_t$ are comprised between 0.28 and 0.60. The value of the solution is close to the value with the fixed quantile solution. The advantage of using the learning approach...
is that the optimization algorithm avoids the direct enumeration strategy previously used for identifying a best fixed quantile.

With $\varepsilon = 1$, we found that most of the forecast errors were discarded, and that the algorithm was quickly stuck in a solution that uses wind energy more aggressively. Interestingly, the expected cost of that solution actually corresponds to an improvement.

Our numerical results are reported in Table 1, where we also report the standard error of the estimator for the expected cost, computed over 50000 wind scenarios.

For our upcoming experiments with a MIP model, we expect to be able to transfer some knowledge gained from our experiments on the simplified model.

Finally, we stress that setting the wind power seen in the unit commitment on the basis of cost simulations makes the cost models even more critical. For instance, underestimating startup costs will underestimate the cost of excessively high wind forecasts, and could thus overestimate the optimal quantile of wind to be used to compute the point forecasts from the distributions.

### 10. Conclusion

In this paper, we have introduced a family of forecasting methods based on the broad idea of using time-varying quantiles of the day-ahead wind power distribution, motivated by specific issues in wind prediction and stochastic unit commitment. We have assumed that the unit commitment is solved through a look-ahead optimization based on a single scenario, that can be distorted by the choice of the wind forecasting method. An iterative procedure has been proposed for adjusting the parameters of the forecasting method, based on the expected cost of the schedule and of the subsequent adjustments, estimated by Monte Carlo simulation. Some preliminary numerical results were presented with a simplified model of the unit commitment and of the real-time adjustment process.

There are several potential benefits from scheduling wind energy using time-varying quantiles driven by external forecasts, such as load forecasts. The consequences of having overestimated the available wind power can be quite different in terms of adjustment costs, depending on the current load level and load ramping behavior. Time-varying quantiles provide a convenient way to relax the reliability of scheduled wind power during periods where it appears advantageous to do so in terms of simulated costs.

### Table 1. Comparison of different solutions

<table>
<thead>
<tr>
<th>Wind forecast</th>
<th>Expected cost</th>
<th>Std. error</th>
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<td>Fixed $q_t$:</td>
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<tr>
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<td>Time varying $q_t$:</td>
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<td>Fig. 8 (Bottom)</td>
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References


