Advanced Vitreous State – The Physical Properties of Glass

Dielectric Properties of Glass

Lecture 2: Dielectric in an AC Field

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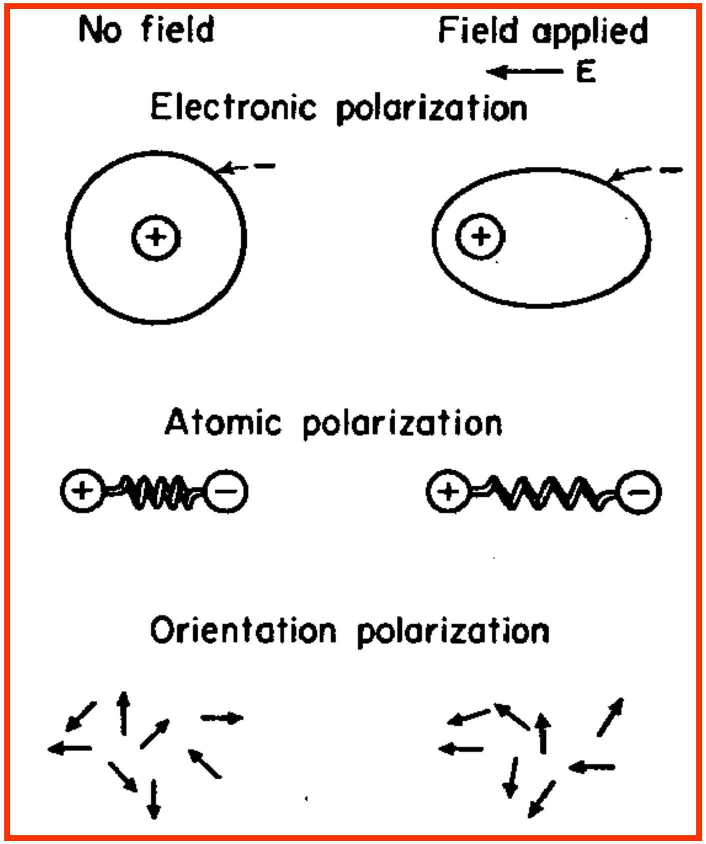
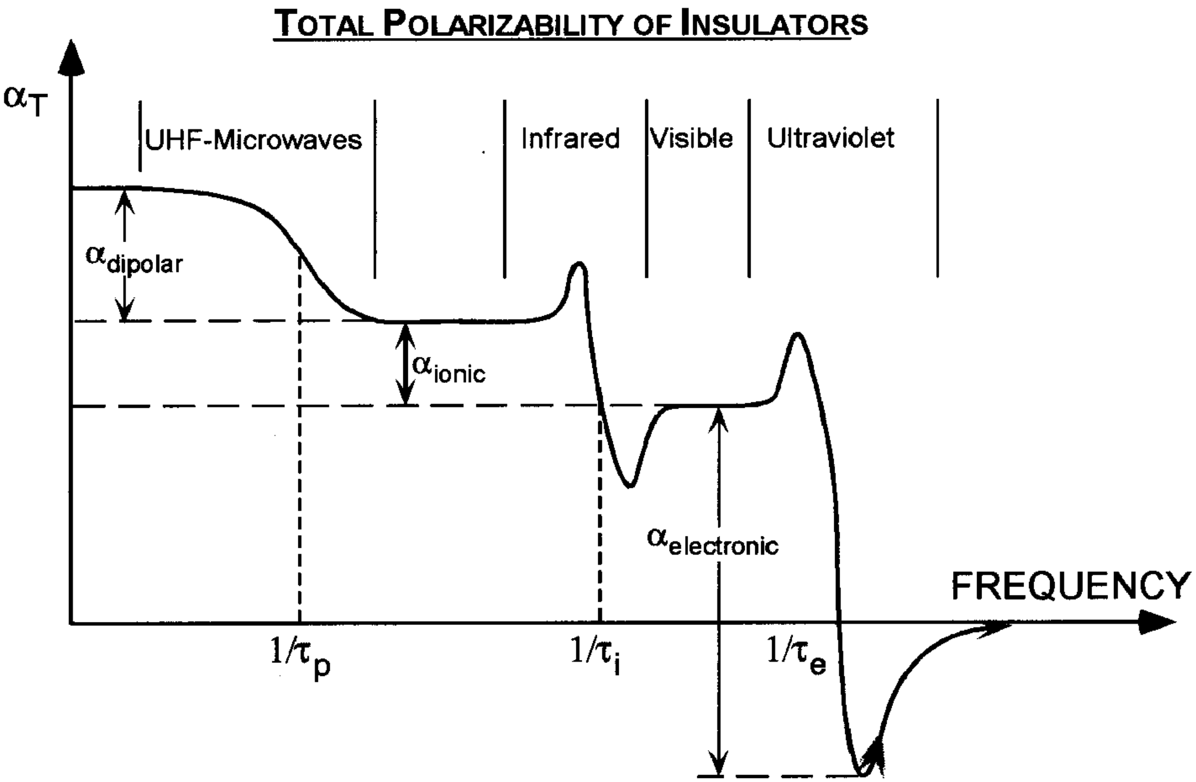
Classic sources of polarizability in glass vs. frequency



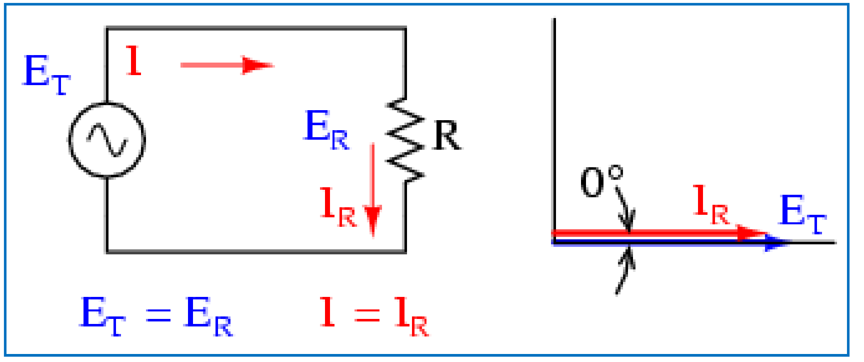


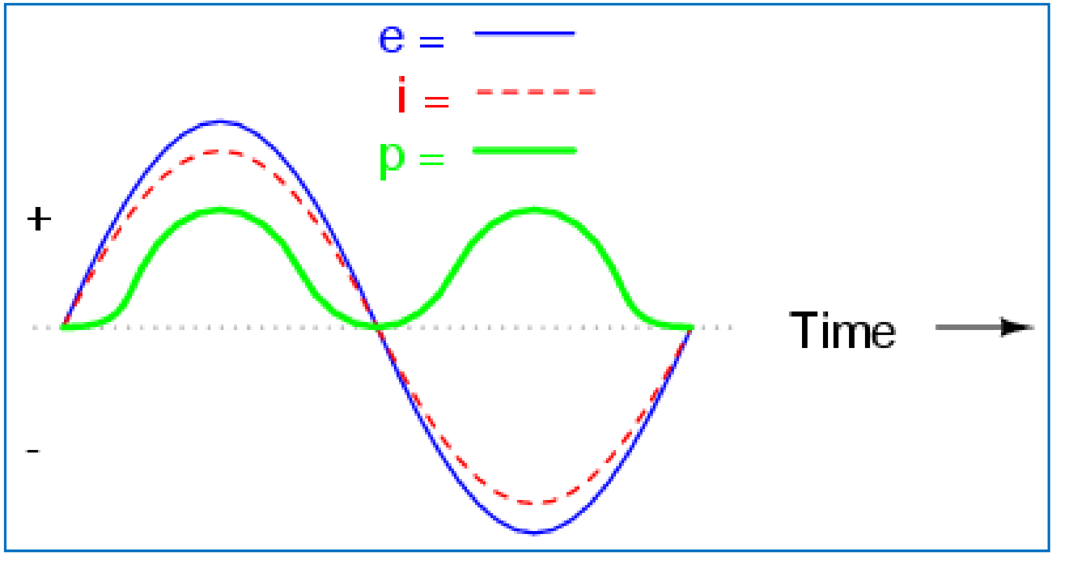
$$\alpha = \alpha_e + \alpha_a + \alpha_d$$

At optical frequencies $(\varepsilon_r - 1)/(\varepsilon_r + 2)$ $= \dfrac{n^2 - 1}{n^2 + 2} = \dfrac{N q_e^{\,2}}{3\epsilon_0 m} \sum_j \dfrac{f_j}{\omega_{oj}^2 - \omega^2 + i\Gamma_j \omega}$

Dielectric in AC Field: **Macroview i.e. a bit of EE**

$E = E_0 \sin \omega t$
$i = i_0 \sin \omega t$





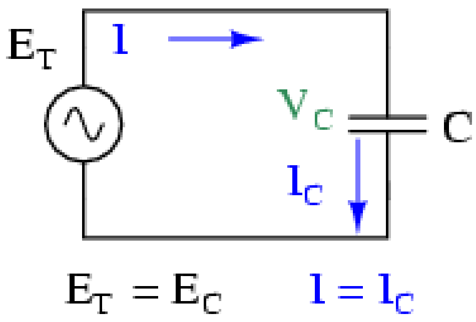
Voltage and current are "in phase" for resistive circuit. Power or energy loss, p, $\propto Ri^2$

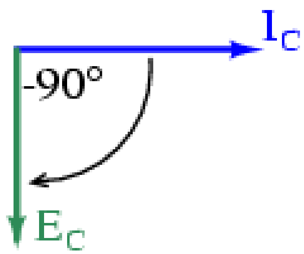
http://www.ibiblio.org/kuphaldt/electricCircuits/AC/AC_6.html

Ideal vs. Real Dielectric

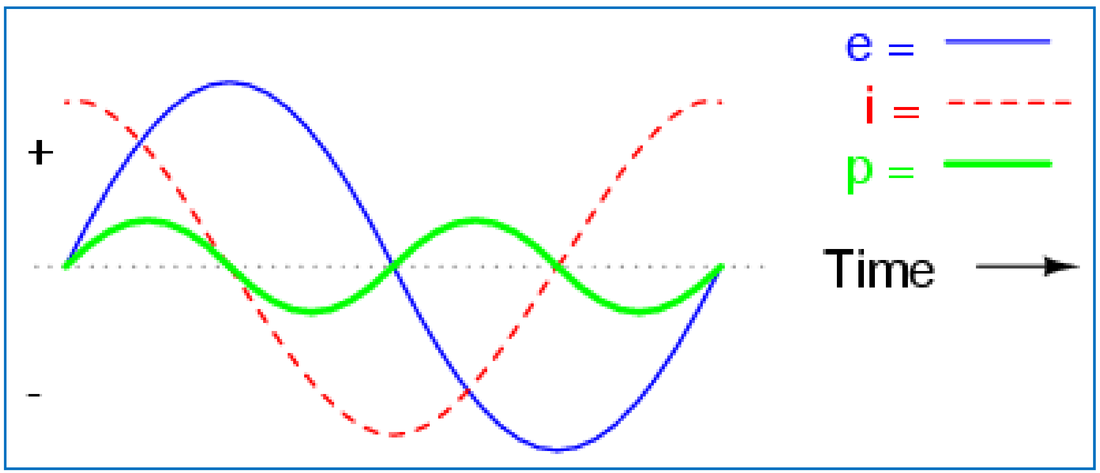
$E=E_0 \sin \omega t$

$i=i_0 \cos \omega t$





C is frequency independent

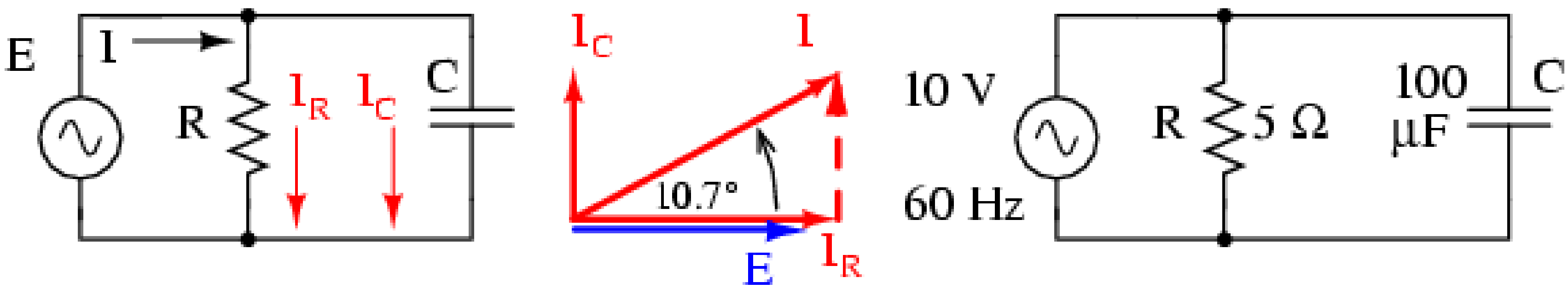


In an ideal dielectric, current is ahead of voltage (or voltage lags behind the current) by 90°.

The power is positive or negative, average being zero i.e. there is no energy loss in a perfect dielectric.

http://www.web-books.com/eLibrary/Engineering/Circuits/AC/AC_4P2.htm

Real dielectric: A parallel circuit of R and C



$I = I_R + I_L$

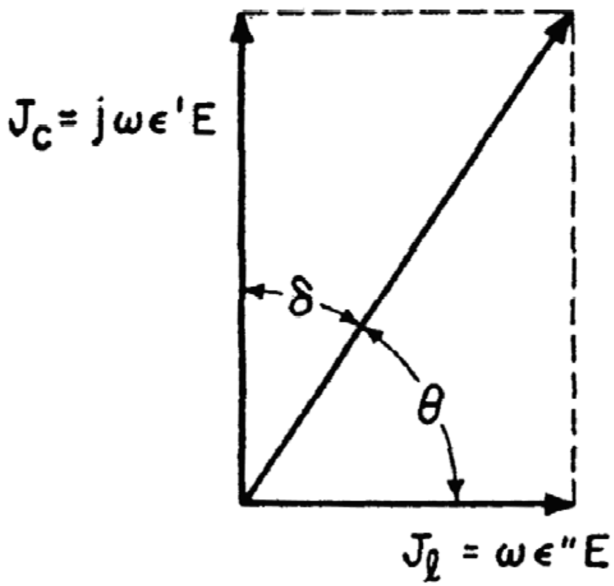
$E = E_R = E_C$

The total current can be considered as made of a lossy resistive component, I_L (or I_R) that is in-phase with voltage, and a capacitive current, I_C, that is 90° out-of-phase.

Unlike ideal dielectric, real dielectric has finite conductivity that causes loss of energy per cycle.
In this case, the current is ahead of voltage by <90°.

Complex Relative Permittivity

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$$



Charging and loss current density.

ε_r = dielectric constant

ε'_r = real part of the complex dielectric constant

ε''_r = imaginary part of the complex dielectric constant

j = imaginary constant $\sqrt{(-1)}$

There are many parameters to represent the dielectric response (permittivity (ε*), susceptibility (χ*), conductivity (σ*),modulus (M*), impedance (Z*), admittance (Y*), etc.) emphasizing different aspects of the response. However, they are all interrelated mathematically. One needs to know only the real and imaginary parts of any one parameter.

$\varepsilon^*(\omega,T) = \varepsilon' - j[\sigma(\omega,T)/\omega]$

$\varepsilon_0\varepsilon_r''(\omega,T) = \sigma'(\omega,T)/\omega$

Energy loss in a dielectric

Loss tangent or loss factor

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

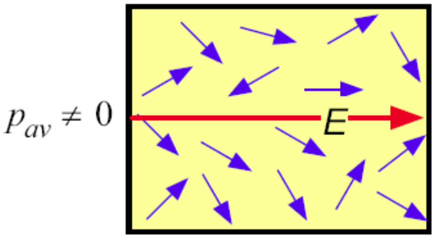
Describes the losses in relation to dielectric's ability to store charge.

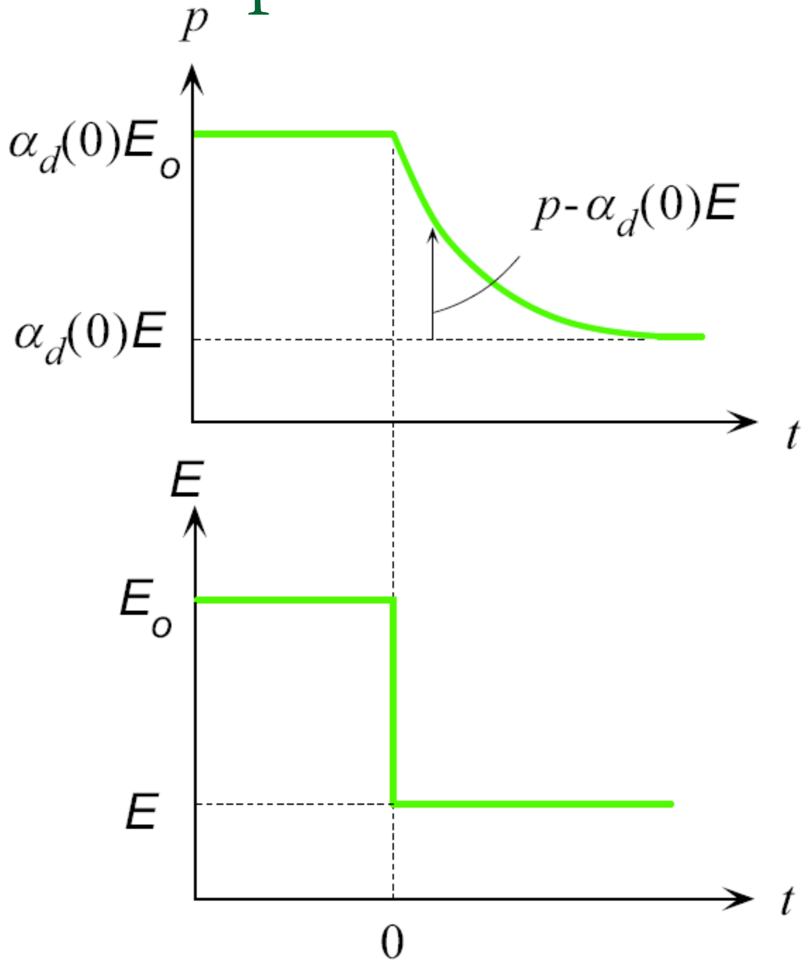
Energy absorbed or loss/volume-sec

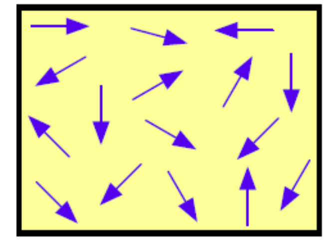
$$W_{\mathrm{vol}} = \omega E^2 \varepsilon_o \varepsilon''_r = \omega E^2 \varepsilon_o \varepsilon_r' \tan\delta$$

Loss tangent of silica is $1\mathrm{x}10^{-4}$ at 1 GHz, but can be orders of magnitude higher for silicate glass (Corning 7059) = 0.0036 @ 10 GHz.
Depends on ω and T.

Depolarization of dipolar dielectric







The dc field is suddenly changed from $\mathcal{E}_o$ to $\mathcal{E}$ at time t = 0. The induced dipole moment p has to decrease from $\alpha_d(0)\mathcal{E}_o$ to a final value of $\alpha_d(0)\mathcal{E}$. The decrease is achieved by random collisions of molecules in the gas.

Dipolar Relaxation Equation

$$\frac{dp}{dt} = -\frac{p - \alpha_d(0)\mathcal{E}}{\tau}$$

p = instantaneous dipole moment = $\alpha_d \boldsymbol{E}$,

dp/dt = rate at which p changes, α_d = dipolar orientational polarizability,

$\mathcal{E}$ = electric field, τ = relaxation time

When AC field $\boldsymbol{E}=\boldsymbol{E_0}$ exp (jωt), the solution for p or α_d vs. ω:

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau}$$

ω = angular frequency of the applied field, j is $\sqrt{(-1)}$.

Debye Equations

$$\varepsilon_r' = 1 + \frac{[\varepsilon_r(0) - 1]}{1 + (\omega\tau)^2} \qquad \varepsilon_r'' = \frac{[\varepsilon_r(0) - 1]\omega\tau}{1 + (\omega\tau)^2}$$

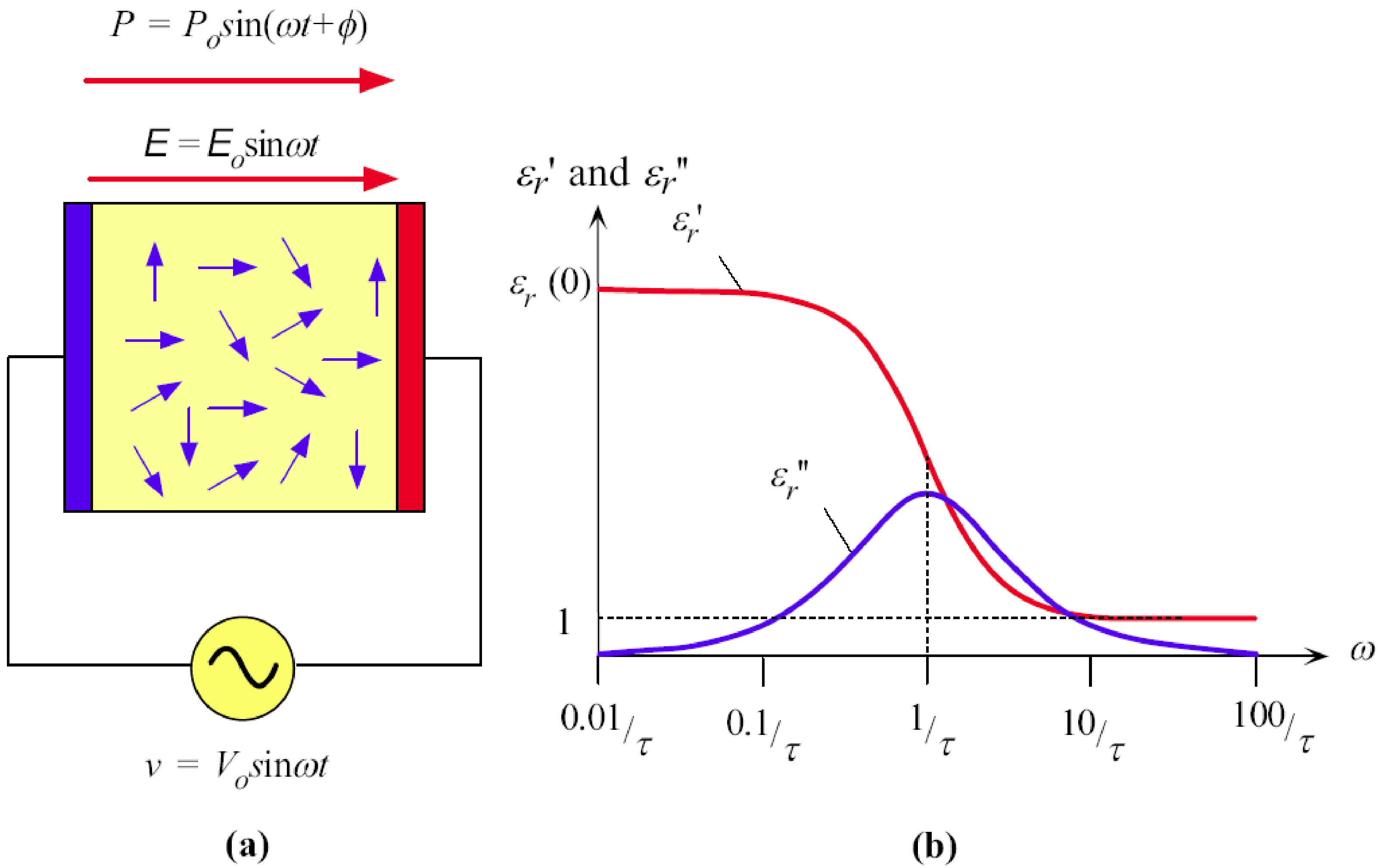
ε_r = dielectric constant (complex)

ε'_r = real part of the complex dielectric constant

ε''_r = imaginary part of the complex dielectric constant

ω = angular frequency of the applied field

τ = relaxation time



(a) An ac field is applied to a dipolar medium. The polarization P($P = Np$) is out of phase with the ac field.

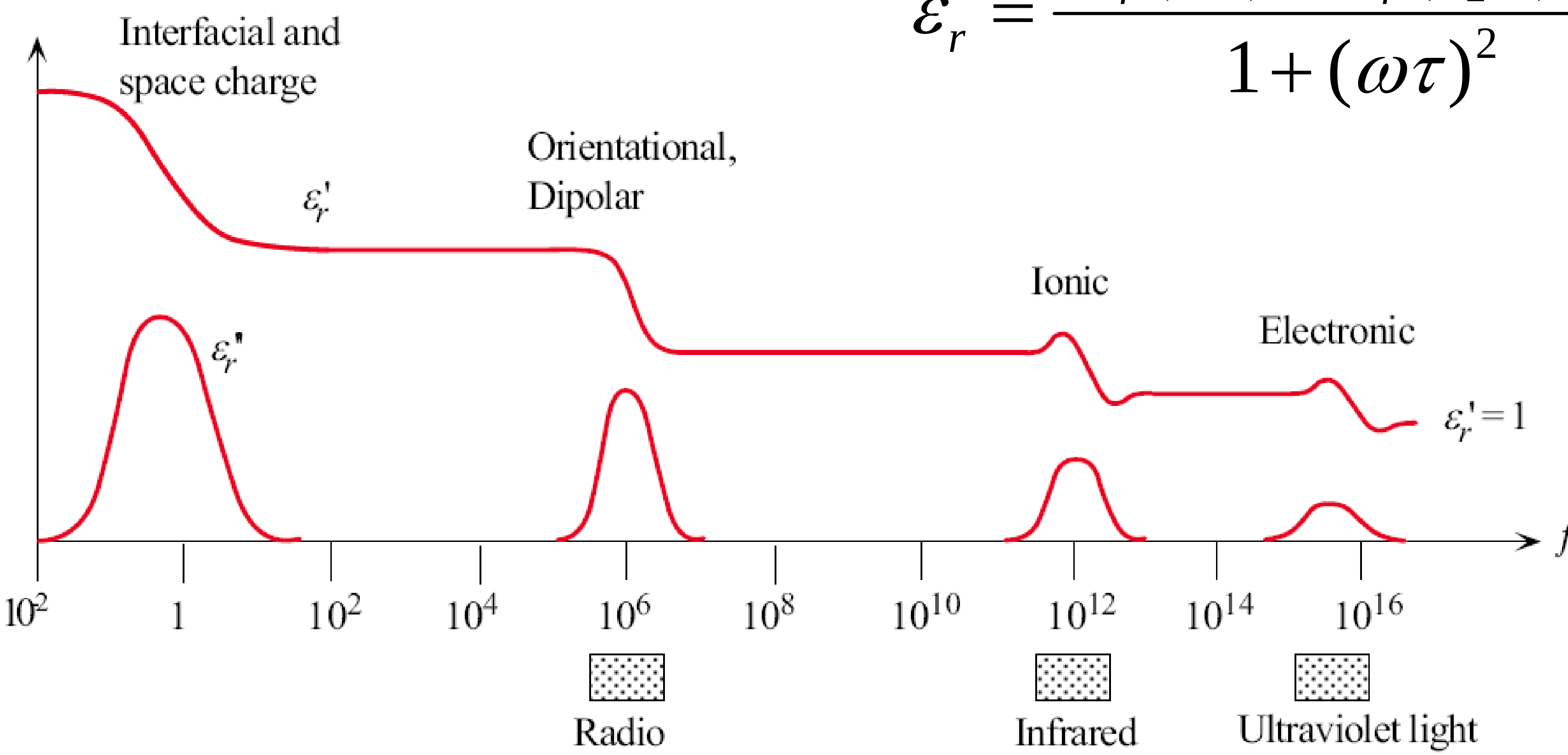
(b) The relative permittivity is a complex number with real (ε_r') and imaginary (ε_r'') parts that exhibit frequency dependence.

Dielectric constant over broad frequency range

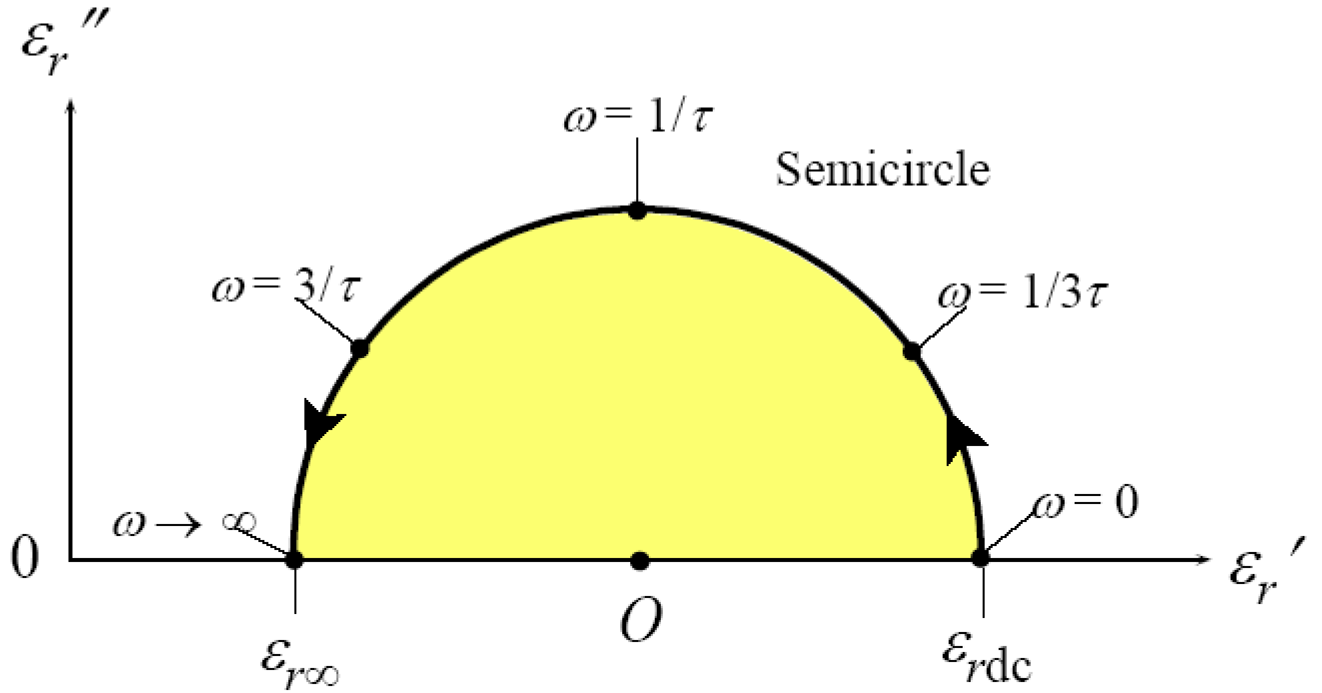
Dipolar contribution, typically below GHz range

$$\varepsilon_r'(\omega) = \varepsilon_r(opt) + \frac{[\varepsilon_r(dc) - \varepsilon_r(opt)]}{1 + (\omega\tau)^2}$$

$$\varepsilon_r'' = \frac{[\varepsilon_r(dc) - \varepsilon_r(opt)]\omega\tau}{1 + (\omega\tau)^2}$$

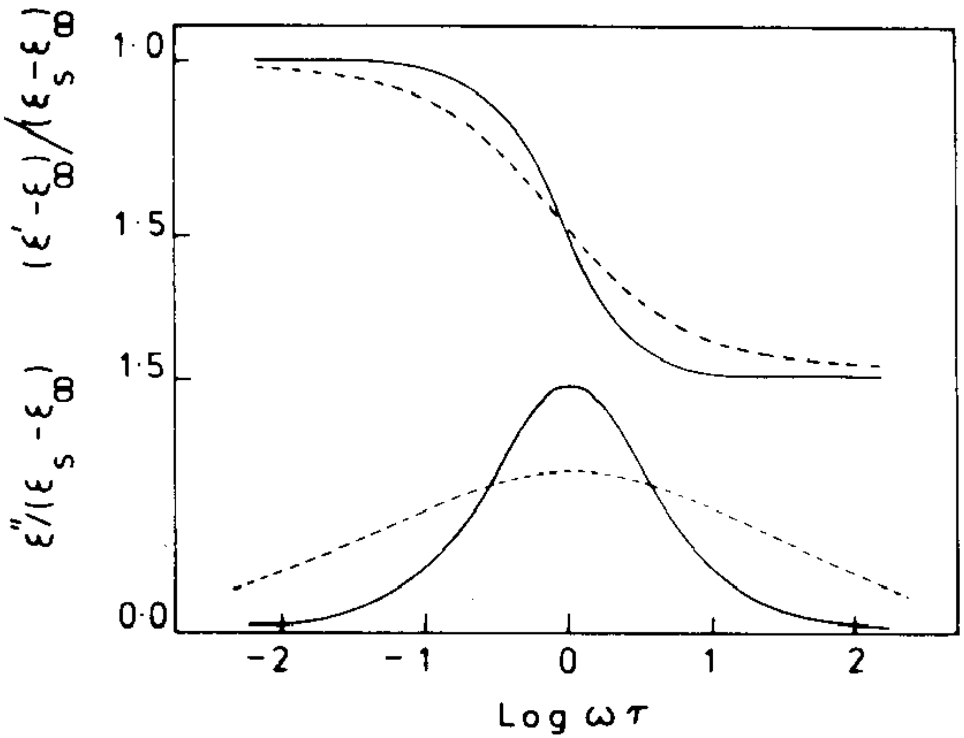


Cole-Cole plots



Cole-Cole plot is a plot of ε''_r vs. ε'_r as a function of frequency, ω. As the frequency is changed from low to high frequencies, the plot traces out a circle if Debye equations are obeyed.

Dipolar dielectric loss in complex systems



$\tau=\tau_0 \exp(Q/RT)$
where Q is activation energy for the reorientation of a dipole.

How would the loss peak change with increasing T?

Debye Eqs are valid when the dipole (ion) conc is small i.e. non-interacting dipoles, andε" vs log ω shows symmetric Debye peak at $\omega\tau = 1$

For high x, the dipoles interact causing distribution of τ ⇒ the loss peak is smeared.

$$\epsilon^* = \epsilon_\infty + (\epsilon_s - \epsilon_\infty)\int_0^\infty \frac{G(\tau)\,d\tau}{1+i\omega\tau}$$

where G(t) is an appropriate distribution function.

An example: $18Na_2O$-$10CaO$-$72SiO_2$ glass

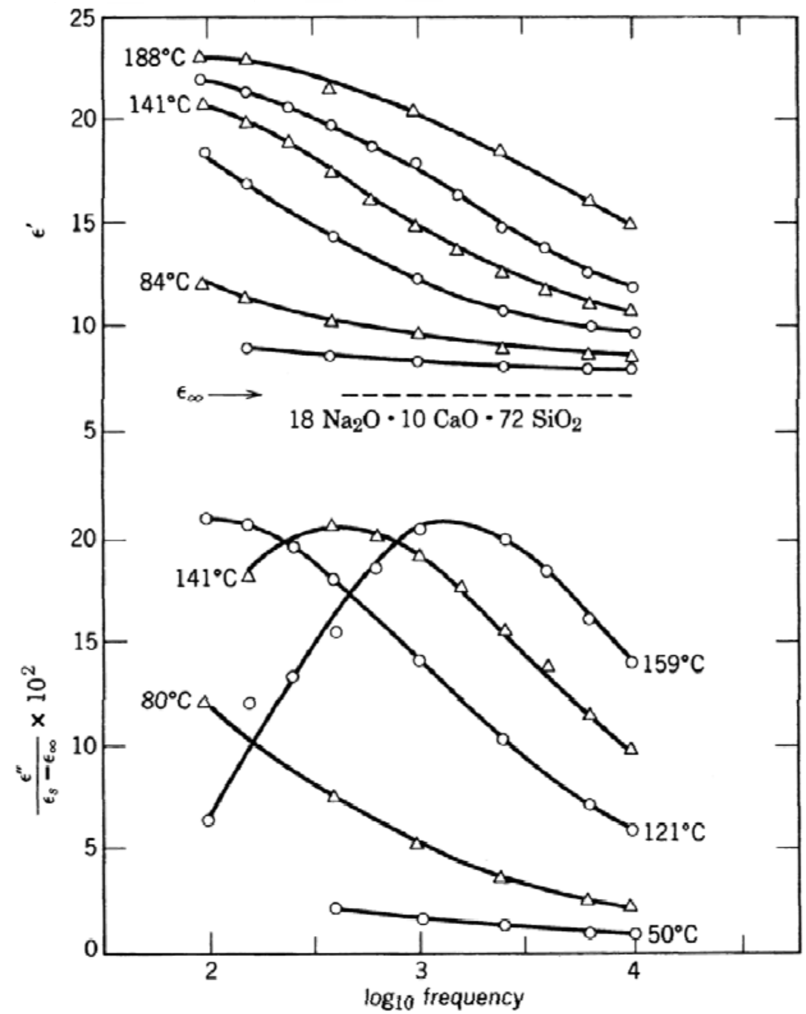


Fig. 18.18. Dielectric dispersion and absorption curves, corrected for dc conductivity, for a typical soda-lime-silicate glass. From H. E. Taylor, *J. Soc. Glass Technol.*, 43, 124 (1959).

Intro to Ceramics
Kingery et al.

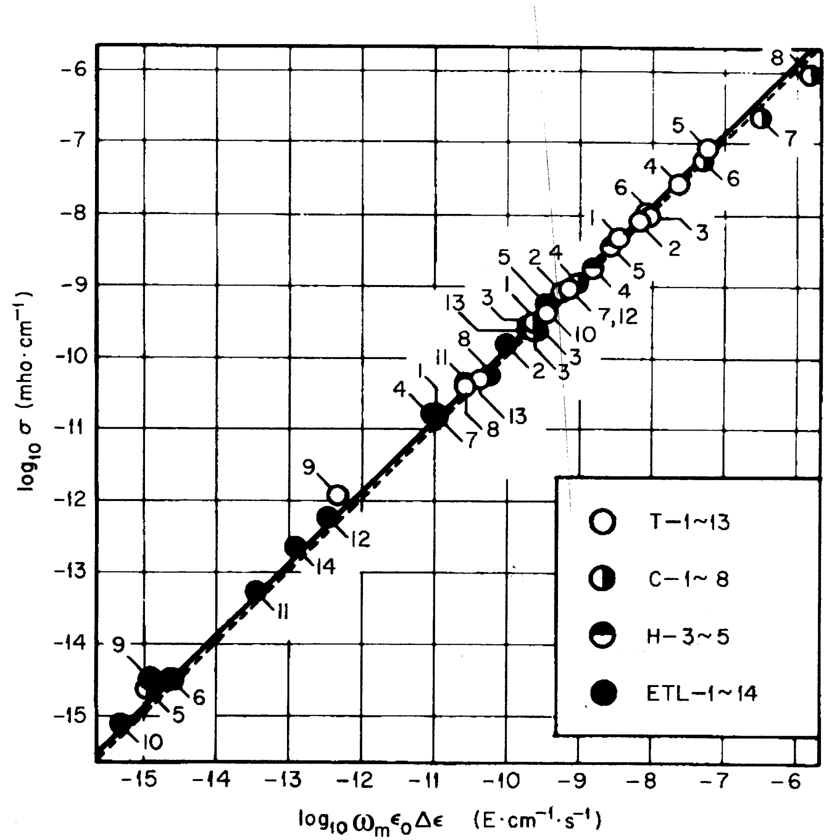
Barton-Nakajima-Namikawa (BNN) relation

$$\sigma_{dc} = p\varepsilon_0 \Delta\varepsilon\omega_m$$

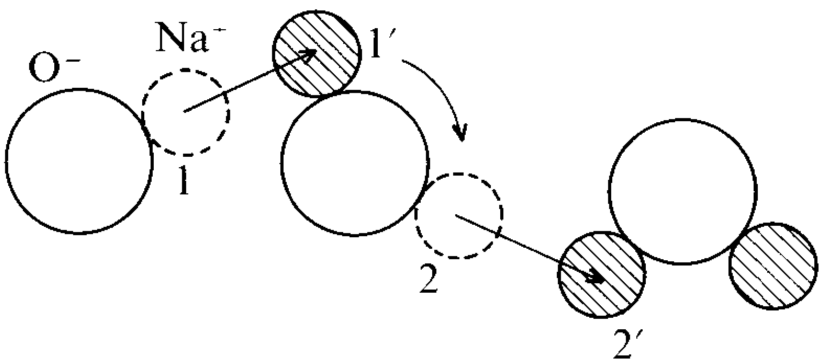
where p is a constant ~ 1. $\Delta\varepsilon$ is the step in ε' across the peak, ω_m is freq of ε'' maximum.

$$\omega_m = 1/\tau = \tau_0^{-1}\exp[-\Delta E_m/RT]$$

Dc conductivity and ε'' maximum have same activation energy⇒ common origin.



Correlation between conductivity and dielectric relaxation. ○ = Taylor (1957, 9); ◑ = Charles (1962; 1963); ◓ Heroux (1958); ● = measured at Electrotechnical Labo- ›ry, MITI, Japan. (After Nakajima, 1972.)

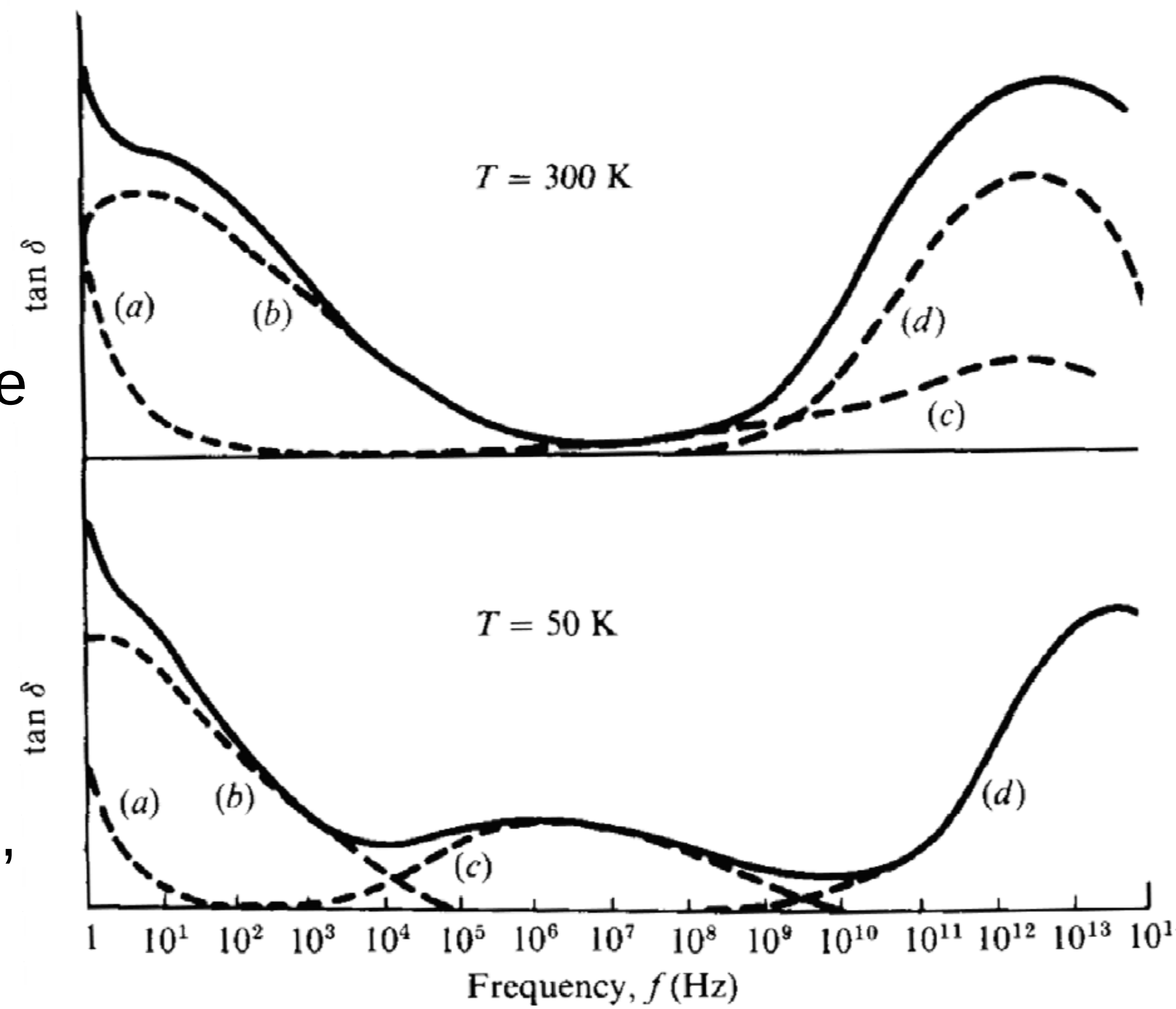


re 15-8. Confirmation of the BNN relationship. The symbols T, C, H, ETL correspond to measurements at four different laboratories cited by Tomozawa[11].

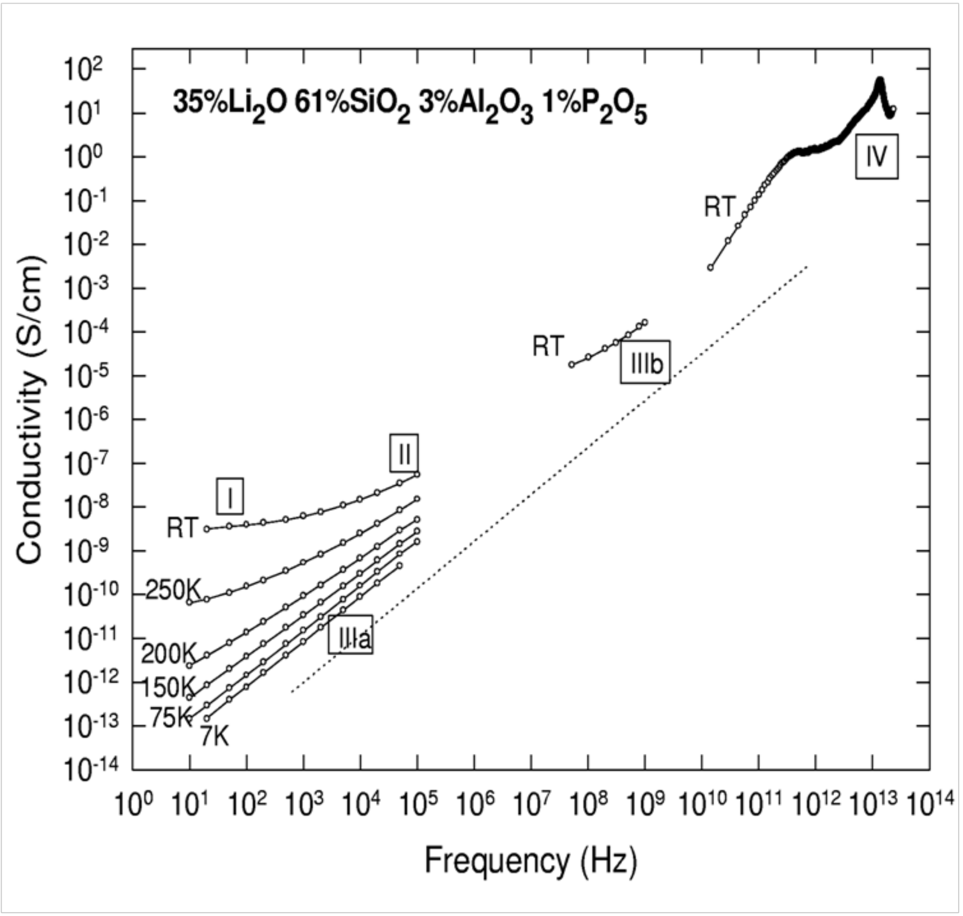
Loss tangent over a wide frequency range

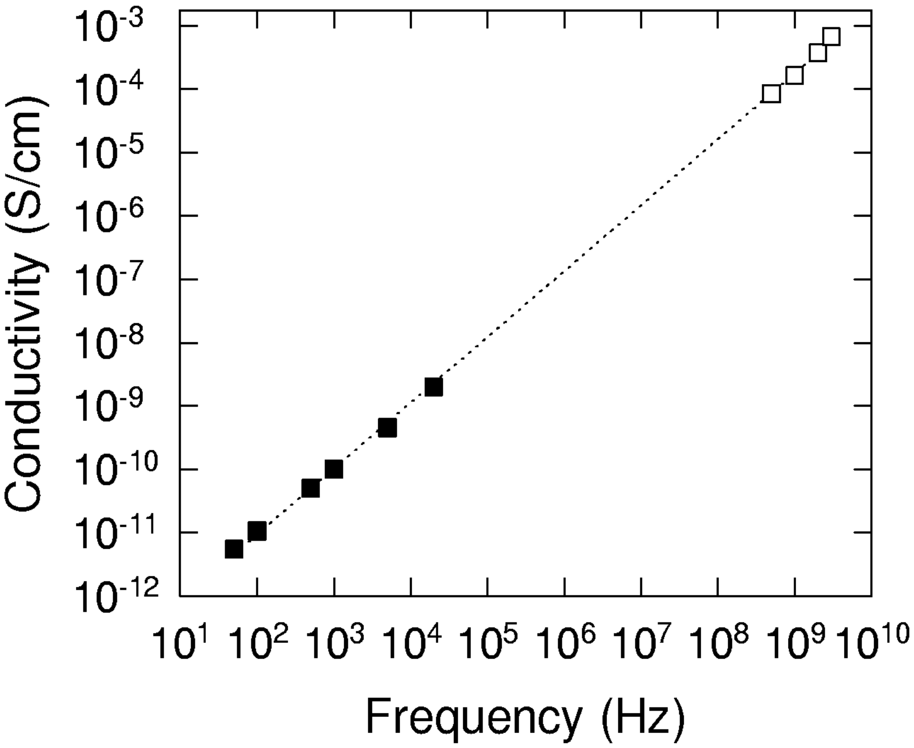
Below the visible frequencies, there are at least four different mechanisms that are responsible for dielectric loss in glass: (a) dc conduction, (b) dipole, (c) deformation/jellyfish, (d) vibration

Fig. 11.8. Various types of dielectric losses in a glass: (*a*) conduction, (*b*) dipole relaxation, (*c*) deformation, (*d*) vibration. After Ref. 404.



Frequency-temperature interchange





The source of dielectric loss (ac conductivity) at low T – low ω and high T – MW ω has a common underlying origin.

The jellyfish mechanism

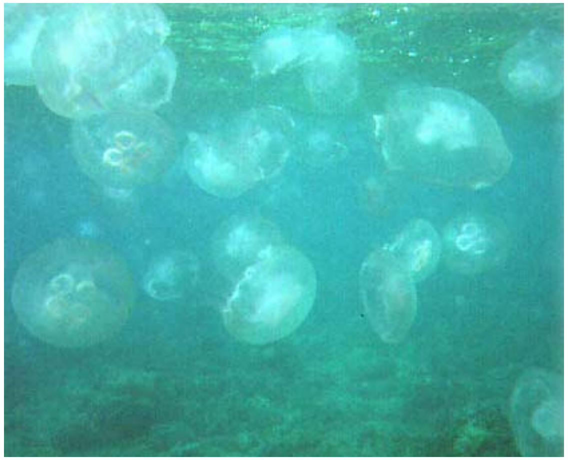
** It is a group of atoms which collectively move between different configurations, much like the wiggling of a ***jellyfish*** in ***glassy ocean***.

** There is no single atom hopping involved.

** The fluctuations are much slower than typical atom vibrations.

** The exact nature of the 'jellyfish' (ADWPC) depends on the material.

** In the same material more than one 'jellyfish' might exist and be observed in different T and f ranges.



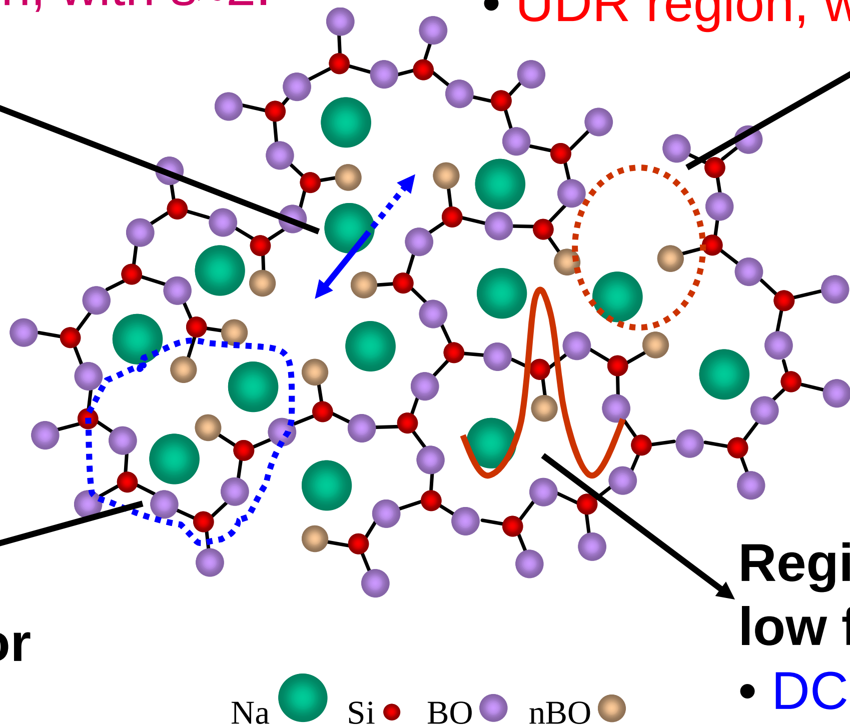
Broad view of the structural origin of conductivity

Regime IV: Very high f

- Vibrational loss region, with s≈2.

Regime II: High T - Intermediate f

- UDR region, with s≈0.6.



Regime III: MW f or Low T

- Jellyfish region, with s ~ 1.0.

Regime I: High T - low f

- DC conductivity region, with s=0.

***Random network structure of a sodium silicate glass in two-dimension* (after Warren and Biscoe)**