Cyclotron Motion

Consider an electron ($m_e = 9.11 \times 10^{-31}$ kg, $q_e = -1.6 \times 10^{-19}$ C) moving perpendicular to a magnetic field with a magnitude $B = 3.574$ T. The magnetic field will cause the electron to travel in a circular path without changing its speed.

**Part 1.** If the electron is moving at $v_1 = 3.5 \times 10^5$ m/s, what is the radius $r_1$ of the cyclotron motion? In this case, how many times per second $f_1$ does the electron orbit?

**Solution:** Because $\vec{v} \perp \vec{B}$, $F = |q|vB = \frac{mv^2}{r}$, and solving for $r$ gives $r = \frac{mv}{|q|B} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.5 \times 10^5 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(3.574 \text{ T})} = 5.58 \times 10^{-7}$ m = 558 nm.

The number of orbits per second is $f = \frac{1}{\Delta t} = \frac{v}{C} = \frac{\nu}{2\pi r} = \frac{|q|B}{2\pi m} = 9.99 \times 10^9 \text{ Hz} = 99.9 \text{ GHz}$.

**Part 2.** If the electron is moving at $v_2 = 1 \times 10^5$ m/s, what is the radius $r_2$ of the cyclotron motion? In this case, how many times per second $f_2$ does the electron orbit?

**Solution:** Using the same formulas with a different $v$ gives $r_2 = 159$ nm and $f_2 = 99.9$ GHz. The frequency is unchanged!

**Part 3.** If the negatively-charged electron is travelling up, and the $\vec{B}$ field is out of the page, in what direction will the electron first be pulled? (Choose from left, right, up, down, into the page, out of the page.)

**Solution:** With motion up and $\vec{B}$ out of the page, the $\vec{v} \times \vec{B}$ term is to the right. Because this is an electron, the direction of the force is opposite, or **LEFT**.

**Equations**

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\vec{F} = q\vec{v} \times \vec{B} \quad \quad F = ma \quad \quad a = \frac{v^2}{r} \quad \quad C = 2\pi r \quad \quad v = \Delta s/\Delta t
\]