Capacitors and Dielectrics

1. (2 points) A dielectric capacitor with a capacitance of $C_0 = 40 \, \mu F$ is charged up to $V_0 = 20 \, V$. What is the energy $U_0$ stored in the capacitor?

**Solution:** The energy comes straight from the formula.

$$U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2} (40 \, \mu F) (20 \, V)^2 = 8000 \, \mu J = 8 \, mJ = 0.008 \, J$$

2. (2 points) The capacitor is then disconnected from everything else. The dielectric is removed, changing the capacitance to $C_1 = 5 \, \mu F$. What was the dielectric constant $\kappa$ of the dielectric?

**Solution:** The effect of the dielectric is to increase the air-filled capacitance by a multiplicative factor. $\kappa$ is always greater than 1. (Note, here $C_1$ is the low air-filled value, while $C_0$ is the higher dielectric-filled value.)

$$\kappa = \frac{C_{\text{dielectric}}}{C_{\text{air}}} = \frac{C_0}{C_1} = \frac{40 \, \mu F}{5 \, \mu F} = 8$$

3. (4 points) The capacitor is still disconnected. What are the new charge $Q_1$, voltage $V_1$, and energy $U_1$ stored in the capacitor?

**Solution:** Since the capacitor is disconnected, the charge $Q$ cannot change. So first calculate the old $Q_0$ then calculate new parameters.

$$Q_0 = C_0V_0 = (40 \, \mu F) (20 \, V) = 800 \, \mu C$$

$$Q_1 = Q_0 = 800 \, \mu C$$

$$V_1 = Q_1/C_1 = (800 \, \mu C) / (5 \, \mu F) = 160 \, V$$

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} (800 \, \mu C) (160 \, V)^2 = 64 \, mJ$$

4. (2 points) Did the energy of the capacitor increase or decrease? Where did the energy come from/go to?

**Solution:** The energy increased. Removing the dielectric requires a force over a distance, which is work.