Capacitors and Dielectrics

1. (2 points) A dielectric capacitor with a capacitance of \( C_0 = 20 \mu F \) is charged up to \( V_0 = 60 \text{ V} \). What is the energy \( U_0 \) stored in the capacitor?

**Solution:** The energy comes straight from the formula.

\[
U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} (20 \mu F) (60 \text{ V})^2 = 36000 \mu J = 36 \text{ mJ} = 0.036 \text{ J}
\]

2. (2 points) The capacitor is then disconnected from everything else. The dielectric is removed, changing the capacitance to \( C_1 = 10 \mu F \). What was the dielectric constant \( \kappa \) of the dielectric?

**Solution:** The effect of the dielectric is to increase the air-filled capacitance by a multiplicative factor. \( \kappa \) is always greater than 1. (Note, here \( C_1 \) is the low air-filled value, while \( C_0 \) is the higher dielectric-filled value.)

\[
\kappa = \frac{C_{\text{dielectric}}}{C_{\text{air}}} = \frac{C_0}{C_1} = \frac{20 \mu F}{10 \mu F} = 2
\]

3. (4 points) The capacitor is still disconnected. What are the new charge \( Q_1 \), voltage \( V_1 \), and energy \( U_1 \) stored in the capacitor?

**Solution:** Since the capacitor is disconnected, the charge \( Q \) cannot change. So first calculate the old \( Q_0 \) then calculate new parameters.

\[
Q_0 = C_0 V_0 = (20 \mu F) (60 \text{ V}) = 1200 \mu C
\]
\[
Q_1 = Q_0 = 1200 \mu C
\]
\[
V_1 = \frac{Q_1}{C_1} = \frac{1200 \mu C}{(10 \mu F)} = 120 \text{ V}
\]
\[
U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \frac{Q_1^2}{C_1} = \frac{1}{2} \frac{(10 \mu F) (120 \text{ V})^2}{10 \mu F} = 72 \text{ mJ}
\]

4. (2 points) Did the energy of the capacitor increase or decrease? Where did the energy come from/go to?

**Solution:** The energy increased. Removing the dielectric requires a force over a distance, which is work.