27.4. The electric field is constant over each face of the cube shown in the figure. Does the box contain positive charge, negative charge, or no charge?

**Solution:** Use Gauss’s Law.

\[ \oint E \cdot dA = \frac{Q}{\epsilon_0} \]

For the integrals, since \( E \) is constant, just multiply by some area \( A \). Remember that the contribution is positive for outward fields and negative for inward fields.

\[ \oint E \cdot dA = (-15 - 15 - 15 + 10 + 20 + 20)A = 5A > 0 \]

Therefore, the charge inside the box is **positive**.

27.12. A 2.0 cm \times 3.0 cm rectangle lies in the \( xz \)-plane. What is the electric flux through the rectangle if:

- **Part A:** \( \vec{E} = (50\hat{i} + 100\hat{k}) \) N/C
- **Part B:** \( \vec{E} = (50\hat{i} + 100\hat{j}) \) N/C

**Solution:** The electric flux is the integral: \( \oint \vec{E} \cdot d\vec{A} \) In this case, \( E \) is constant so the integral can be performed simply as \( \vec{E} \cdot \hat{A} \). With the rectangle in the \( xz \) plane, the area vector points in the \( y \)-direction, so \( \hat{A} = 6 \times 10^{-4} \hat{j} \) m\(^2\).

- **Part A:** The dot product is zero (no \( y \)-component of \( E \)), so the flux is zero.
- **Part B:** Only the \( y \)-component of \( E \) contributes, so:

\[ \Phi = (50\hat{i} + 100\hat{j}) \cdot (6 \times 10^{-4} \hat{j}) = 100(6 \times 10^{-4}) = 0.06 \text{ Nm}^2 / \text{C} \]
27.38. A hollow metal sphere has inner radius $a$ and outer radius $b$. The hollow sphere has charge $+2Q$. A point charge $+Q$ sits at the center of the hollow sphere.

Part A: Determine the magnitude of the electric field in the region $r \leq a$.

Part B: Determine the magnitude of the electric field in the region $a < r < b$.

Part C: Determine the magnitude of the electric field in the region $b \leq r$.

Part D: How much charge is on the inside surface of the hollow sphere?

Part E: How much charge is on the outside surface of the hollow sphere?

Solution: Use Gauss’s Law. In each region, the flux is

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = 4\pi r^2 E = \frac{Q}{\varepsilon_0}.$$  

Part A: There is only the inner charge $+Q$ inside the integral, so

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}.$$  (This should look familiar.)

Part B: In electrostatics, there is no electric field inside the bulk of a conductor. $E = 0$.

Part C: The total charge inside the integration region is $+3Q$, so

$$E = \frac{3Q}{4\pi \varepsilon_0} \frac{1}{R^2}.$$  

Part D: Since the electric field in part B was zero, the charge inside that integration region must be zero. This includes the charge in the middle and the charge on the inner surface. Therefore, the charge on the inner surface is $-Q$.

Part E: We know the total charge on the metal sphere is $+2Q$, and this charge is divided between the inner and outer surfaces. Therefore, the charge on the outer surface is $+3Q$.

27.50. A long, thin straight wire with linear charge density $\lambda$ runs down the center of a thin, hollow metal cylinder of radius $R$. The cylinder has a net linear charge density $2\lambda$. Assume $\lambda$ is positive.

Part A: Find expressions for the magnitude of the electric field strength inside the cylinder, $r < R$.

Part B: Find expressions for the magnitude of the electric field strength outside the cylinder, $r > R$.

Solution: Use Gauss’s Law. Make your integration region a cylinder with unit length and ignore the end caps, so that

$$\oint \mathbf{E} \cdot d\mathbf{A} = 2\pi r E = \lambda' / \varepsilon_0$$ (where $\lambda'$ is the charge per unit length inside your integration region).

Part A: Inside the cylinder, only the wire is in the integration region, so $\lambda' = \lambda$, and $E = \frac{\lambda}{2\pi \varepsilon_0 r}$.

Part B: Outside the cylinder, both the wire and the cylinder are in the integration region, so $\lambda' = 3\lambda$ and $E = \frac{3\lambda}{2\pi \varepsilon_0 r}$.
Charged Insulating Spheres (Note that this problem uses randomly-generated numbers). Two small insulating spheres with radius $3.00 \times 10^{-2}$ m are separated by a large center-to-center distance of 0.545 m. One sphere is negatively charged, with net charge $-2.40 \, \mu C$, and the other sphere is positively charged, with net charge $4.05 \, \mu C$. The charge is uniformly distributed within the volume of each sphere.

Part A: What is the magnitude $E$ of the electric field at the point midway between the spheres?

Part B: What is the direction of the electric field midway between the spheres?

Solution: Because of Gauss’s law, this is basically a regular electric field problem. Think of the positive charge as being along the positive $x$-axis and the negative charge on the negative $x$-axis. The $\mathbf{r}$ vector is zero, and $\mathbf{r}'$ is $\pm 0.545 \hat{i}$ depending on which charge is being examined.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{4.05 \times 10^{-6}}{(0.545/2)^2} (-0.545\hat{i}) + \frac{1}{4\pi\varepsilon_0} \frac{-2.40 \times 10^{-6}}{(0.545/2)^2} (0.545\hat{i}) = -7.81 \times 10^5 \hat{i} \, \text{N/C}$$

With this single-component vector, the magnitude is the absolute value of the number and the direction is in the negative $x$-direction, or toward the negative charge.