White light (400-700 nm) is incident on a 600 line/mm diffraction grating. What is the width of the first-order rainbow on a screen 2.0 m behind the grating? Give your answer in cm. Do not make small angle approximations.

**Solution:** The two extreme wavelengths of light will diffract to different angles according to

\[ m\lambda = d \sin \theta \]

The distance between the lines of the grating is \( d = (1/600) \text{ mm} = 1.667 \times 10^{-6} \text{ m} = 1667 \text{ nm} \). Since this is the first-order rainbow, \( m = 1 \) for both waves. So, the two angles are

\[ \begin{align*}
\theta_{\text{red}} &= \sin^{-1} \left( \frac{\lambda}{d} \right) = \sin^{-1} \left( \frac{700 \text{ nm}}{1667 \text{ nm}} \right) = 24.834^\circ \\
\theta_{\text{violet}} &= \sin^{-1} \left( \frac{400 \text{ nm}}{1667 \text{ nm}} \right) = 13.887^\circ 
\end{align*} \]

The actual distance \( y \) from the center of the screen to the diffraction line comes from \( \tan \theta = \frac{y}{L} \), where \( L \) is the distance from the grating to the screen. So, the two distances and the width of the rainbow are:

\[ \begin{align*}
y_{\text{red}} &= L \tan \theta_{\text{red}} = (2.0 \text{ m}) \tan (24.834^\circ) = 0.925 \text{ m} \\
y_{\text{violet}} &= (2.0 \text{ m}) \tan (13.887^\circ) = 0.494 \text{ m} \\
\Delta y &= 0.431 \text{ m} = 43.1 \text{ cm}
\]

**2 Problem K22.46**

How many lines per millimeter does the grating have? (Do not make small angle approximations).

**Solution:** The first-order line is hitting at \( y = 0.436 \text{ m} \), so the angle is \( \theta = \tan^{-1} (0.436) = 22.557^\circ \). Using \( m\lambda = d \sin \theta \),

\[ d = \frac{m\lambda}{\sin \theta} = \frac{(1) (600 \text{ nm})}{\sin (22.557^\circ)} = 1501 \text{ nm} = 1.501 \times 10^{-3} \text{ mm} \]

Therefore, the density of the grating is 666 lines/mm.

(Notice that unlike the cases where the small angle approximation applies, here the different orders of the diffraction pattern get further and further apart.)

**3 Problem YF 35.29**

What is the thinnest film of a coating with an index of refraction of \( n = 1.35 \) on glass (with an index of refraction \( n_g > n \)) for which destructive interference of the red component (a wavelength of 630 nm) of an incident white light beam in air can take place by reflection?

**Solution:** The thinnest destructive interference comes with a phase difference of \( \Delta \phi = \pi \) between the wave reflected off the first surface and the one reflected off the back surface of the film. Since \( n > 1 \) (as is true for all materials), the wave from the front surface is inverted (given a base phase difference of \( \pi \)). But, the
wave reflected from the back surface is also inverted because \( n < n_g \), so in the end, both waves start out in phase. The only difference is the pathlength difference.

The wave from the back surface has to traverse the film twice, travelling a distance of \( \Delta \ell = 2y \), where \( y \) is the thickness of the film. The wavelength in the film is \( \lambda_n = \lambda / n = 630 \text{ nm}/1.35 = 466.7 \text{ nm} \) (remember that the frequency is constant for a given wave). Take \( \Delta \ell / \lambda_n \) (the fraction of a cycle difference between the two waves) and multiply it by \( 2\pi \) to get \( \Delta \phi \), which should be equal to \( \pi \)

\[
\Delta \phi = 2\pi \left( \frac{\Delta \ell}{\lambda_n} \right) = 2\pi \frac{2y}{\lambda / n} = \pi
\]

\[
y = \frac{\lambda}{4n} = 116.7 \text{ nm}
\]

4 Problem YF 36.29

Visible light passes through a diffraction grating that has 900 slits/cm and the interference pattern is observed on a screen that is 2.36 m from the grating. In the first-order spectrum, maxima for two different wavelengths are separated on the screen by 3.20 nm. What is the difference in these wavelengths (in meters)? (Here you need to make small angle approximations.)

**Solution:** We’ll make the small angle approximation that \( \sin \theta \approx \tan \theta \approx \theta \) (with \( \theta \) in radians). That means that

\[
\frac{y}{L} = \tan \theta \approx \theta \quad \text{and} \quad \Delta \theta \approx \frac{\Delta y}{L} = \frac{0.0032 \text{ m}}{2.36 \text{ m}} = 0.00136 \text{ rad}
\]

The grating spacing is \( d = (90000 \text{ lines/m})^{-1} = 1.11 \times 10^{-5} \text{ m} \). In the diffraction formula, the small-angle approx. with \( m = 1 \) gives:

\[
m\lambda = d \sin \theta \approx d \theta \quad \rightarrow \quad \Delta \lambda \approx d \Delta \theta = 15.1 \text{ nm}
\]

5 Problem K36.33

Plane monochromatic waves with wavelength 540 nm are incident normally on a plane transmission grating having 350 slits/mm. **Part A.** Find the angle of deviation in the first order.

**Solution:** The slit separation is \( d = (350000 \text{ slits/m})^{-1} = 2.857 \times 10^{-6} = 2857 \text{ nm} \). First order means \( m = 1 \) in the formula \( m\lambda = d \sin \theta \).

\[
\theta_1 = \sin^{-1} \left( \frac{m\lambda}{d} \right) = \sin^{-1} \left( \frac{540}{2857} \right) = 10.89^\circ
\]

**Part B.** Find the angle of the second order. **Solution:** Second order means \( m = 2 \), so

\[
\theta_2 = \sin^{-1} \left( \frac{m\lambda}{d} \right) = \sin^{-1} \left( \frac{2 \cdot 540}{2857} \right) = 22.21^\circ
\]

**Part C.** Find the angle of the third order. **Solution:**

\[
\theta_3 = \sin^{-1} \left( \frac{m\lambda}{d} \right) = \sin^{-1} \left( \frac{3 \cdot 540}{2857} \right) = 34.54^\circ
\]