1 Transformers

**Learning Goal:** To understand the concepts explaining the operation of transformers. (Note, the “Printable Solutions” on MasteringPhysics have a lot of information about transformers in the introduction to this problem.) The ratio of the EMF’s in the primary and secondary is the same as the ratio of the number of turns. The EMF in an ideal coil is also the voltage across it.

\[
\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1}
\]

Also, the transformer doesn’t add or take away energy, so the power in the primary and secondary is equal:

\[V_1I_1 = V_2I_2\]

**Part A.** The primary coil of a transformer contains 100 turns; the secondary has 200 turns. The primary coil is connected to a size AA battery that supplies a constant voltage of 1.5 volts. What voltage would be measured across the secondary coil?

**Solution:** There would be no voltage across the secondary. A DC current does not cause an EMF.

**Part B.** A transformer is intended to decrease the RMS value of the alternating voltage from 500 volts to 25 volts. The primary coil contains 200 turns. Find the necessary number of turns \(N_2\) in the secondary coil.

**Solution:** To step the voltage down from 500 V to 25 V, the ratio of the number of turns would be 500:25 = 20:1. Thus, \(N_2\) is:

\[N_2 = \frac{E_2}{E_1}N_1 = \frac{25\text{ V}}{500\text{ V}}\cdot 200 = 10\]

**Part C.** A transformer is intended to decrease the rms value of the alternating current from 500 amperes to 25 amperes. The primary coil contains 200 turns. Find the necessary number of turns \(N_2\) in the secondary coil.

**Solution:** Since the power going in equals the power going out, the ratio of the currents is the inverse of the ratio of the voltages.

\[N_2 = \frac{I_1}{I_2}N_1 = \frac{500\text{ A}}{25\text{ A}}\cdot 200 = 4000\]

**Part D.** In a transformer, the primary coil contains 400 turns, and the secondary coil contains 80 turns. If the primary current is 2.5 amperes, what is the secondary current \(I_2\)?

**Solution:** This is a step-down transformer with a ratio of 5:1. Assuming the primary current is AC, the secondary current would be

\[I_2 = \frac{N_1}{N_2}I_1 = \frac{400}{80}\times 2.5\text{ A} = 12.5\text{ A}\]

**Part E.** The primary coil of a transformer has 200 turns and the secondary coil has 800 turns. The power supplied to the primary coil is 400 watts. What is the power generated in the secondary coil if it is terminated by a 20-ohm resistor?

**Solution:** Since the power in the primary and secondary is the same, the power supplied to the resistor will be 400 W.
Part F. The primary coil of a transformer has 200 turns, and the secondary coil has 800 turns. The transformer is connected to a 120-volt (rms) ac source. What is the (rms) current \( I_1 \) in the primary coil if the secondary coil is terminated by a 20-ohm resistor?

Solution: The RMS voltage in the secondary will be \( V_2 = (N_2/N_1) V_1 = (800/200) \cdot 120 = 480 \text{ V} \)

With the 20 \( \Omega \) resistor, the current in the secondary will be \( I_2 = V_2/R = (480 \text{ V})/(20 \Omega) = 24 \text{ A} \).

In order to produce this secondary current, the primary current would have to be larger

\[
I_1 = \frac{N_2}{N_1} I_2 = \frac{800}{200} 24 \text{ A} = 96 \text{ A}
\]

Part G. A transformer supplies 60 watts of power to a device that is rated at 20 volts (rms). The device is connected to the secondary coil. The primary coil is connected to a 120-volt (rms) ac source. What is the current \( I_1 \) in the primary coil?

Solution: The power is the same on both sides of the transformer, so the current is

\[
I_1 = \frac{P}{V_1} = \frac{60 \text{ W}}{120 \text{ V}} = 0.5 \text{ A}
\]

Part H. The voltage and the current in the primary coil of a nonideal transformer are 120 V and 2.0 A. The voltage and the current in the secondary coil are 19.4 V and 11.8 A. What is the efficiency \( e \) of the transformer? The efficiency of a transformer is defined as the ratio of the output power to the input power, expressed as a percentage: \( e = 100 \% \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) \).

Solution: The efficiency is

\[
e = 100 \% \left( \frac{19.4 \text{ V}}{120 \text{ V}} \right) \left( \frac{11.8 \text{ A}}{2.0 \text{ A}} \right) = 95.4 \%
\]

2 Problem K34.19

A square parallel-plate capacitor 5.30 cm on a side has a 0.540 mm gap. What is the displacement current in the capacitor if the potential difference across the capacitor is increasing at 500,000 V/s? (Displacement current is defined on page 1100 of the book. It is part of the extra term in Ampere’s Law.)

Solution: The displacement current is defined as \( I_D = \epsilon_0 \frac{d\Phi_E}{dt} \), where \( \Phi_E = \int \vec{E} \cdot d\vec{A} \) is the electric flux. Just like magnetic flux, usually \( \Phi_E = EA \). Taking the time derivative, \( I_D = \epsilon_0 A \frac{dE}{dt} \). The area is \( A = (0.0530 \text{ m})^2 \) and the electric field and voltage in the capacitor are related by \( V = Ed \), so

\[
I_D = \epsilon_0 \frac{A \, dV}{dt} = \epsilon_0 \frac{(0.0530 \text{ m})^2}{(0.000540 \text{ m})} (500,000 \text{ V/s}) = 2.30 \times 10^{-5} \text{ A}
\]

3 Problem K35.23

What capacitor in series with a 100 \( \Omega \) resistor and a 23.0 mH inductor will give a resonance frequency of 1010 Hz?

Solution: The resonant frequency depends only on the inductor and the capacitor.

\[
\omega^2 = (2\pi f)^2 = \frac{1}{LC} \quad \Rightarrow \quad C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi 1010 \text{ Hz})^2 (23 \times 10^{-3} \text{ H})} = 1.08 \times 10^{-6} \text{ F}
\]
4 Problem K35.48

A series RLC circuit consists of a 100 Ω resistor, a 0.10 H inductor, and a 100 μF capacitor. It is attached to a 120 V/60 Hz power line.

**Part A.** What is the peak current $I_0$?

**Solution:** The loop equation for a series RLC circuit is

$$\mathcal{E}(t) - I(t)R - L\frac{dI}{dt} - \frac{1}{C}Q(t) = 0$$

Where the charge on the capacitor is $Q(t) = \int I(t)\,dt$ with the arbitrary constant set so the average $Q$ is zero. The current as a function of time is

$$I(t) = I_0 \cos \omega t + \phi$$

Because of the sin's and cos's, to find $I_0$, we draw the phasor diagram. The lengths of the vectors are

- $V_R = I_0 R$
- $V_L = I_0 X_L = I_0 \omega L$
- $V_C = I_0 X_C = \frac{I_0}{\omega C}$

$$\mathcal{E}_0 = V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

The reactances are $X_L = \omega L = 2\pi (60\text{ Hz}) (0.1 \text{ H}) = 37.7 \Omega$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60\text{ Hz})100\times10^{-6}\text{ F}} = 26.5 \Omega$.

By solving for $I_0$, noting that $\mathcal{E}_0 = \sqrt{2}V_{\text{RMS}}$, we get

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{\sqrt{2}120 \text{ V}}{\sqrt{(100 \Omega)^2 + (37.7 \Omega - 26.5 \Omega)^2}} = 1.69 \text{ A}$$

**Part B.** What is the phase angle $\phi$ in degrees?

**Solution:** The phase angle is the angle between the total voltage $V_T$ and the total current $I_0$ in the phasor diagram. By inspection, it has the value

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{37.7 \Omega - 26.5 \Omega}{100 \Omega}\right) = 0.111 \text{ radians} = 6.39^\circ$$

**Part C.** What is the average power loss?

**Solution:** The only component with a power loss is the resistor. The instantaneous power is $P = IV$, and the average power is

$$P_{\text{ave}} = I_{\text{RMS}}V_{\text{RMS}} = (I_{\text{RMS}})^2 R = \frac{I_0^2 R}{2} = \frac{(1.69 \text{ A})^2 (100 \Omega)}{2} = 142 \text{ W}$$