1 Problem K33.19

An FM radio station broadcasts at a frequency of 100 MHz. What inductance should be paired with a 5.00 pF capacitor to build a receiver circuit for this station?

**Solution:** The resonant frequency of a capacitor-inductor pair determines the inductance:

$$\omega^2 = \left(2\pi f\right)^2 = \frac{1}{LC} \quad \Rightarrow \quad L = \frac{1}{(2\pi f)^2 C} = \frac{1}{(2\pi (100 \times 10^6 \text{ Hz}))^2 (5 \times 10^{-12} \text{ F})} = 5.07 \times 10^{-7} \text{ H} = 0.507 \mu\text{H}$$

2 Problem K33.17

How much energy is stored in a 2.70 cm diameter, 14.0 cm long solenoid that has 160 turns of wire and carries a current of 0.750 A?

**Solution:** The inductance is

$$L = \mu_0 N^2 A/\ell = \mu_0 (160)^2 \left(\pi \left((2.70 \times 10^{-2}/2) \text{ m}\right)^2\right)/(0.14 \text{ m}) = 1.316 \times 10^{-4} \text{ H}$$

The energy of an inductor is

$$U_L = \frac{1}{2} LI^2 = \frac{1}{2} (1.316 \times 10^{-4} \text{ H})(0.750 \text{ A})^2 = 3.70 \times 10^{-5} \text{ J}$$

3 Problem OH 15-6-21

For this problem, you must determine the magnetic flux through a square loop of side $a$ if one side is parallel to, and a distance $a$ from, a straight wire that carries a current $I$.

**Solution:** The $\vec{B}$ field curls around the wire, so it is directed perpendicular to the loop. The flux is

$$\Phi = \int \vec{B} \cdot d\vec{A} = \int_0^a \int_{-a}^{a} \frac{\mu_0 I}{2\pi} \frac{dz}{dy} dy dx = \frac{\mu_0 I a}{2\pi} \ln 2$$

Notice that the integrals separated. The $y$ integral just became $a$, the height of the loop, and the $r$ integral was:

$$\int_0^{2a} \frac{1}{r} dr = [\ln r]_{r=a}^{2a} = \ln 2a - \ln a = \ln \frac{2a}{a} = \ln 2$$

![Figure 1: A loop near a current.](image)
4  Induced Current in a Pair of Solenoids

For each of the actions depicted, determine the direction of the current induced to flow through the resistor in the circuit containing the secondary coil.

Solution A: The $\vec{B}$ field (and hence the flux) will be increasing to the left. The induced current must try to counter this increase by creating a $\vec{B}_{\text{ind}}$ to the right, opposing $\vec{B}$. By the RHR, $\vec{B}_{\text{ind}}$ to the right is created by a current going down the front of the solenoid, which would be going to the right in the resistor.

Solution B: The $\vec{B}$ field (and hence the flux) will be decreasing to the left. The induced current will try to boost the flux by creating a $\vec{B}_{\text{ind}}$ to the left, supporting $\vec{B}$. By the RHR (see above), this means the current in the resistor is to the left.

Solution C: This is exactly the same as Part A. Swapping the positions of the coils makes no difference since they are still wound in the same direction. The current in the resistor is to the right.

Solution D: By moving the primary coil to the left, some of the magnetic flux will escape between the coils, and the flux felt by the secondary coil will decrease. The $\vec{B}$ field is still pointed toward the left. So this is the same as Part B with a decreasing flux to the left, which leads to (see above) a current to the left in the resistor.

5  Induced EMF and Current in a Shrinking Loop

A circular loop of flexible iron wire has an initial circumference of 160 cm, but its circumference is decreasing at a constant rate of 15.0 cm/s due to a tangential pull on the wire. The loop is in a constant uniform magnetic field of magnitude 0.700 T, which is oriented perpendicular to the plane of the loop. Part A. Find the EMF $\mathcal{E}$ induced in the loop after exactly time 9.00 s has passed since the circumference of the loop started to decrease. Part B. Find the direction of the induced current in the loop, as viewed along the direction of the magnetic field.

Solution: The EMF is the time derivative of the flux. The flux is

$$\Phi = \int \vec{B} \cdot d\vec{A} = BA = \pi r^2 B \quad \Rightarrow \quad |\mathcal{E}| = \left| \frac{d\Phi}{dt} \right| = \pi B 2r \left| \frac{dr}{dt} \right| = BC(t) \left| \frac{dr}{dt} \right|$$

Note that the information given gives us an equation for the circumference of the circle $C$ and the derivative of $r$:

$$C(t) = C_0 + \frac{dC}{dt} t = (1.6 \text{ m}) - (0.15 \text{ m}) t \quad \text{and} \quad \frac{dr}{dt} = \frac{1}{2\pi} \frac{dC}{dt} = -\frac{0.15}{2\pi} t$$

Plug this into the equation for the EMF, and the answer is

$$\mathcal{E} (t = 9.0) = B \left( C_0 + \frac{dC}{dt} t \right) \left| \frac{dr}{dt} \right| = (0.7 \text{ T}) (1.6 \text{ m} - 0.15 \text{ m/s} (9.0 \text{ s})) \left( \frac{0.15 \text{ m}}{2\pi} \right) = 4.18 \times 10^{-3} \text{ Tm}^2$$

Figure 2: Four scenarios for a pair of solenoids for Problem 4.
For the direction, the flux is decreasing into the loop (remember we’re looking along the $\vec{B}$ field). Therefore the induced current must try to boost the flux and hence $\vec{B}_{\text{ind}}$ is parallel to the field. By the RHR, that means the current is \textbf{clockwise}.