1 Problem K32.34

The aurora is caused when electrons and protons, moving in the earth’s magnetic field of \( \approx 5 \times 10^{-5} \) T, collide with molecules of the atmosphere and cause them to glow.

**Part A.** What is the radius of the cyclotron orbit for an electron with speed \( 1.0 \times 10^6 \) m/s?

**Solution:** The cyclotron radius is found by combining the magnetic force with \( \vec{F} = m\vec{a} \) and the circular motion equation \( a = v^2/r \).

\[
qvB = F = ma = \frac{mv^2}{r} \implies r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31} \text{ kg}) (1.0 \times 10^6 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C}) (5 \times 10^{-5} \text{ T})} = 0.114 \text{ m}
\]

**Part B.** What is the radius of the cyclotron orbit for a proton with speed \( 5.0 \times 10^4 \) m/s?

**Solution:** This is the same as Part A with a different mass and speed.

\[
r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg}) (5 \times 10^4 \text{ m/s})}{(1.6 \times 10^{-19} \text{ kg}) (5 \times 10^{-5} \text{ T})} = 10.4 \text{ m}
\]

2 Problem K32.54

What is the *magnitude* of the magnetic field at the center of the loop in Figure 1?

**Solution:** Break the problem into two parts, (1) the field due to the straight part of the wire and (2) the field due to the single loop of wire. In both parts, use the proper form of the right-hand rule to determine the direction of the \( \vec{B} \) field at the requested point. Define \( \hat{k} \) as out of the page. The field due to the straight wire will point in the \( -\hat{k} \) direction at the center of the loop.

\[
\vec{B}_{\text{straight}} = \frac{\mu_0 I}{2\pi r} (\hat{\mathbf{r}}) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (5 \text{ A})}{2\pi (0.01 \text{ m})} (-\hat{k}) = -0.0001 \text{ T} \hat{k}
\]

The field due to the loop is also in the \( -\hat{k} \) direction. (Example 32.5 leading to Eq. 32.7.)

\[
\vec{B}_{\text{loop}} = \frac{\mu_0 I}{2r} (-\hat{k}) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (5 \text{ A})}{2 (0.01 \text{ m})} (-\hat{k}) = -0.000314 \text{ T} \hat{k}
\]

The total field is

\[
\vec{B}_{\text{tot}} = -(0.0001 + 0.000314) \hat{k} \text{ T} = -0.000414 \hat{k} \text{ T}
\]
3 K32 Electromagnetic Velocity Filter

When a particle with charge $q$ moves across a magnetic field of magnitude $B$, it experiences a force to the side. If the proper electric field $E$ is simultaneously applied, the electric force on the charge will be in such a direction as to cancel the magnetic force with the result that the particle will travel in a straight line. The balancing condition provides a relationship involving the velocity $\vec{v}$ of the particle. In this problem you will figure out how to arrange the fields to create this balance and then determine this relationship.

**Part A.** Consider the arrangement of ion source and electric field plates shown in the figure. The ion source sends particles with velocity $\vec{v}$ along the positive $x$ axis. They encounter electric field plates spaced a distance $d$ apart that generate a uniform electric field of magnitude $E$ in the $+y$ direction. To cancel the resulting electric force with a magnetic force, a magnetic field (not shown in the first panel) must be added in which direction? Using the right-hand rule, you can see that the positive $z$ axis is directed out of the screen. What should be the direction of $\vec{B}$?

**Solution:** The electric field $\vec{E}$ points from the positive plate toward the negative plate. For a positive charge $q$ this is also the direction of the force. Using the right-hand rule, point the hand in the direction of $\vec{v}$ (right) with the thumb in the direction of the desired force $\vec{F}$ (down), and bent fingers will point in the necessary direction of $\vec{B}$, which is out of the page, i.e. the $k$ direction. Note that the component of $\vec{B}$ in a direction parallel to $\vec{v}$ is not determined by this procedure.

**Part B.** Now find the magnitude of the magnetic field that will cause the charge to travel in a straight line under the combined action of electric and magnetic fields. Write an expression for the magnitude of the magnetic field $B_{\text{bal}}$ that will just balance the applied electric field in terms of some or all of the variables $q$, $v$, and $E$.

**Solution:** The component of $\vec{B}_{\text{bal}}$ perpendicular to $\vec{v}$ will cause a force

$$F = qvB_z$$

$$B_z = \frac{F}{qv} = \frac{(qE)}{(qv)} = \frac{E}{v}$$

Figure 2: The electric and magnetic fields of a velocity filter.