1 Problem K32.14

Parts A thru C. What are the magnetic field strengths at points 1, 2, and 3?

Solution: From the equation sheet, the strength of the magnetic field of a long straight wire carrying a current is \( B = \frac{\mu_0 I}{2\pi R} \) where \( R \) is the distance from the center of the wire. The direction of this field (needed for superposition) is found from the right-hand rule. The contribution of the top wire \( \vec{B}_{\text{top}}(1) \) will be out of the page \((+\hat{k})\), while \( \vec{B}_{\text{bot}}(1) \) will be into the page \((-\hat{k} \text{ direction})\), so they get subtracted. So, at point 1,

\[
\vec{B}_{\text{top}}(1) = \frac{\mu_0 I}{2\pi R(1)} \hat{k} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right) (10 \text{ A})}{2\pi (2 \times 10^{-2} \text{ m})} \hat{k} = 1 \times 10^{-4} \text{ T} \hat{k}
\]

\[
\vec{B}_{\text{bot}}(1) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(10 \text{ A})}{2\pi (6 \times 10^{-2} \text{ m})} \left(-\hat{k}\right) = 3.33 \times 10^{-5} \text{ T} \hat{k}
\]

\[
\vec{B}(1) = \left(1 \times 10^{-4} - 3.33 \times 10^{-5}\right) \hat{k} = 6.67 \times 10^{-5} \text{ T} \hat{k}
\]

At points 2 and 3 (don’t forget that the website asks for magnitudes only),

\[
\vec{B}(2) = -\frac{\mu_0 10 \text{ A}}{2\pi (2 \times 10^{-2} \text{ m})} \hat{k} - \frac{\mu_0 10 \text{ A}}{2\pi (2 \times 10^{-2} \text{ m})} \hat{k} = -2 \times 10^{-4} \text{ T} \hat{k}
\]

\[
\vec{B}(3) = -\frac{\mu_0 10 \text{ A}}{2\pi (6 \times 10^{-2} \text{ m})} \hat{k} + \frac{\mu_0 10 \text{ A}}{2\pi (2 \times 10^{-2} \text{ m})} \hat{k} = 6.67 \times 10^{-5} \text{ T} \hat{k}
\]

2 Problem K32.30

A proton moves in the magnetic field \( \vec{B} = 0.50 \hat{i} \text{ T} \) with a speed of \( 1.0 \times 10^7 \text{ m/s} \) in the directions shown in Figure 2. For each, what is magnetic force \( \vec{F} \) on the proton?

Part A. The vector \( \vec{v} \) for this part lies in the \( xz \) plane.

Solution: The magnetic force is \( \vec{F} = q\vec{v} \times \vec{B} \). In this case, \( \vec{v} = 1.0 \times 10^7 \left(\frac{\hat{i} + \hat{k}}{\sqrt{2}}\right) \), so

\[
\vec{F} = (1.6 \times 10^{-19} \text{ C}) \left(0.707 \times 10^7 \left(\hat{i} + \hat{k}\right) \text{ m/s}\right) \times (0.50 \hat{i} \text{ T})
\]

\[
= 5.656 \times 10^{-13} \hat{j} \text{ T} = 0, 5.656 \times 10^{-13}, 0 \text{ T}
\]

Part B. The vector \( \vec{v} \) for this part points in the negative \( x \) direction.

Solution: Since both \( \vec{v} \) and \( \vec{B} \) have only \( \hat{i} \) components, the cross product is zero.
3 Problem SP 27.19

A ball with a mass of $m$ which contains $N$ excess electrons is dropped into a vertical shaft with a height $h$. The ball falls in the $-z$ ($-\hat{k}$) direction. At the bottom of the shaft, the ball suddenly enters a horizontal uniform magnetic field (that is, in the $+y$ ($+\hat{j}$) direction) that has a magnitude of $B$.

**Parts A and B.** If air resistance is negligibly small, find the magnitude and direction of the force that this magnetic field exerts on the ball just as it enters the field.

**Solution:** The charge of the ball is $q = -eN$. The magnetic field is $\vec{B} = B\hat{j}$. To find the velocity, use conservation of energy. The change in potential energy as the ball falls is $-mgh$:

$$\Delta K = \frac{1}{2}mv^2 = mgh = -\Delta U$$

$$v = \sqrt{2gh}$$

$$\vec{v} = -\sqrt{2gh}\hat{k}$$

The magnetic force (remembering that $\hat{k} \times \hat{j} = -\hat{i}$) is:

$$\vec{F} = q\vec{v} \times \vec{B} = -(eN)\left(-\sqrt{2gh}\hat{k}\right) \times \left(B\hat{j}\right) = eN\sqrt{2gh}\hat{B}$$

The magnitude of the force is

$$|F| = eN\sqrt{2ghB}$$

And the direction is the $-\hat{i}$ direction.

4 Problem SP 27.8

A particle with charge $q = -5.4\text{ nC}$ is moving in a uniform magnetic field $\vec{B} = -B\hat{k} = -1.25\text{ T}\hat{k}$. The magnetic force on the particle is measured to be $\vec{F} = F_x\hat{i} + F_y\hat{j} = \left(-3.30 \times 10^{-7} \hat{i} + 7.60 \times 10^{-7} \hat{j}\right)\text{ N}$. (Notation: $q$ is the charge, which is negative. $F_x$ is the $x$ component of the force, which is negative. $B$ is the magnitude of the magnetic field, which is always positive.)

**Part A.** Are there components of the velocity that are not determined by the measurement of the force?

**Solution:** Yes! Any velocity component parallel to the $\vec{B}$ field will not contribute to the magnetic force. Notice how $v_z$ doesn’t appear in the force.

$$\vec{F} = q\vec{v} \times \vec{B} = q\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) \times \left(-B\hat{k}\right) = qv_xB\hat{j} - qv_yB\hat{i} = F_x\hat{i} + F_y\hat{j}$$

**Parts B and C.** Calculate the $x$ and $y$ components of the velocity of the particle.

**Solution:** Using the above calculation and matching $\hat{i}$ and $\hat{j}$ terms.

$$F_y = qv_xB \quad v_x = \frac{F_y}{qB} = \frac{7.60 \times 10^{-7}\text{ N}}{(-5.4 \times 10^{-9}\text{ C})(1.25\text{ T})} = -113\text{ m/s}$$

$$F_x = -qv_yB \quad v_y = -\frac{F_x}{qB} = -\frac{-3.30 \times 10^{-7}\text{ N}}{(-5.4 \times 10^{-9}\text{ C})(1.25\text{ T})} = -48.9\text{ m/s}$$

**Part D.** Calculate the scalar product $\vec{v} \cdot \vec{F}$. **Solution:**

$$\vec{v} \cdot \vec{F} = \left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) \cdot \left(F_x\hat{i} + F_y\hat{j}\right) = v_xF_x + v_yF_y = \frac{F_yF_x}{qB} - \frac{F_xF_y}{qB} = 0$$

**Part E.** What is the angle between $\vec{v}$ and $\vec{F}$? **Solution:** The angle is $90$ degrees. This is easily seen from $\vec{F} = q\vec{v} \times \vec{B}$ because the result of a cross product is perpendicular to both vectors that went into it.