What is a Markov Chain?

- Discrete time stochastic process
- Markovian Property:
  \[ P(X_{t+1}|X_t, X_{t-1}, ..., X_0) = P(X_{t+1}|X_t) \]
- Transition probabilities, \( p_{ij} \)
- Stationarity Assumption:
  \[ p_{ij} = P(X_{t+1}=j|X_t=i) \text{ independent of } t \]

Definitions

- Transitions
  - Path
  - Reachable
  - Communicate
- Types of states:
  - Absorbing
  - Transient
  - Recurrent
Maintenance Policy

- Transition probabilities

<table>
<thead>
<tr>
<th>state</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7/8</td>
<td>1/16</td>
<td>1/16</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3/4</td>
<td>1/8</td>
<td>1/8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

More Definitions

- Ergodic Markov chain:
  - Recurrent
  - Aperiodic
  - Communicate
MC’s in Economic Analysis

- Markov processes can be used to evaluate:
  - Profitability of business venture.
  - Value of certain business.
  - Long run expected income.
  - Market share analysis.

Example: Willy’s

- Willy’s Rentals rents Big Wheels for recreational use at East Coast Parkway.
- Willy’s estimates that its profit is a function of the weather conditions.
- The weather is classified into 3 states:
  - Sunny
  - Cloudy
  - Rainy.
Willy’s: Weather Conditions

<table>
<thead>
<tr>
<th>Today’s weather</th>
<th>Sunny</th>
<th>Cloudy</th>
<th>Rainy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunny</td>
<td>0.75</td>
<td>0.20</td>
<td>0.05</td>
</tr>
<tr>
<td>Cloudy</td>
<td>0.45</td>
<td>0.40</td>
<td>0.15</td>
</tr>
<tr>
<td>Rainy</td>
<td>0.35</td>
<td>0.45</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Willy’s: Profitability

- Daily profit strongly related to weather.
- Daily profit is estimated to be:
  - $120 on a sunny day
  - $40 on a cloudy day
  - $200 loss on a rainy day
- If today is sunny, what is the expected profit tomorrow? What is Willy’s long-term expected profit?
Example: Traffic

- Highways 10 and 60 run between Atherton and downtown Woodside.
- Without congestion, travel time is about same.
- Traffic congestion:
  - 70% chance on Highway 10.
  - 80% chance on highway 60.
- Probability of choosing highway depends on experience.

Traffic continued ...

- To select expansion plan for the two highways, determine long-run percentages of drivers on each Freeway.

<table>
<thead>
<tr>
<th>Last trip was on Freeway...</th>
<th>Road Conditions</th>
<th>Next trip is on Freeway...</th>
<th>...with probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>No congestion</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>10</td>
<td>Congestion</td>
<td>10</td>
<td>0.3</td>
</tr>
<tr>
<td>60</td>
<td>No congestion</td>
<td>60</td>
<td>0.8</td>
</tr>
<tr>
<td>60</td>
<td>Congestion</td>
<td>60</td>
<td>0.2</td>
</tr>
</tbody>
</table>