Outline

Probabilistic Dynamic Programming
Dealing with **UNCERTAINTY**
- Current stage costs uncertain, but next period’s state is certain:
  - RESOURCE ALLOCATION EXAMPLE
- Next period’s state is uncertain:
  - INVENTORY EXAMPLE

Resource Allocation Example
- 6 gallons of milk available, $1 each
- Selling price is $2 per gallon
- Salvage cost is $0.50 per gallon
- 3 stores, uncertain demand
- Want to maximize expected net profit

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<tr>
<th>STORE 1</th>
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Inventory Example

- At Period 1, firm has 1 unit.
- Production cost for $x$ units is $c(x)=3+2x$, $c(0)=0$, $x<5$.
- Demand is random and equal to 1 w.p. 0.5, 2 w.p. 0.5.
- Holding cost $1/unit, inventory at end cannot exceed 3.
- Salvage cost $2/unit.

Example: Sunco Oil

- D dollars to allocate
- Sites 1, 2, ..., T
- $q_t(x) =$ probability that oil will be found on site $t$ given $x$ dollars allocated
- $r_t =$ worth of oil found at site $t$
- Goal: maximize $E$(value) of oil found on all sites.
Example: Catching Bass

- Currently
  - Lake contains 10,000 bass
- During year t
  - \( p_t = \) unit price of bass
  - \( c_t(x|b) = \) cost of catching x bass | lake contains b bass
- Between time year t bass are caught and year t+1 begins
  - Bass in lake multiply by factor D, \( P(D=d) = q(d) \)
- Goal: Maximize net profit over next 10 years.

Example: ATM

- Sally has 30 minutes for lunch break
- If she makes it to head of the line at the ATM, her reward is r
- Cost per minute waiting time, c
- \( p(x|n) = \) probability that x people will complete service in one minute if n people are ahead of Sally
- Currently, 20 people are ahead.
- Goal: maximize \( E(\text{net revenue}) \)
**Example: Cash Management**

- Demand for cash
  - \( P(D=d) = p(d) \)
- Demand met by
  - Previous day’s cash
  - Money from bank
- Shortage cost, \( s \)
- Holding cost, \( i \)
- Day 1: $10,000 on hand, $100,000 in bank
- Time horizon: 30 days. Goal: \( \min E(\text{cost}) \)

**Example: Parking**

- Approach from west
- Nearsighted
- Cannot return to a spot that’s been passed
- \( p_t = P(\text{space } t \text{ is empty}) \)
- \( M = \text{cost of no parking} \)
- \( |t| = \text{cost of parking in } t \)
- Decision: to park?
Example: Safecracker Dirk

- Begin with $50,000
- Time horizon, 1 – 60
- $d_t$ = payment for job
- $p_t$ = $P$(capture)
  - All is lost
- Goal: max $E$(asset)