Homework #6
Math 205

Due Thursday, Mar 24 (TuTh) or Friday, Mar 25 (MWF).

You must show your work in order to receive full credit; correct answers with no, or insufficient work, may not receive credit.

1. (6 points) Let \( B = \{(3, 5), (2, -1)\} \) be a (ordered) basis for \( \mathbb{R}^2 \), and let \( E = \{(1, 0), (0, 1)\} \) be the standard basis.
   (a) Find the change-of-basis matrix \( P_{E \leftarrow B} \).
   (b) Find the change-of-basis matrix \( P_{B \leftarrow E} \).
   (c) If \( \vec{v} = (3, -1) \), find \( [\vec{v}]_B \).

2. (6 points) The nullspace of the matrix

\[
A = \begin{bmatrix}
5 & -6 & 8 & -8 \\
-3 & 8 & 4 & -4 \\
2 & 2 & 12 & -12
\end{bmatrix}
\]

has basis \( B = \{(-4, -2, 1, 0), (4, 2, 0, 1)\} \)
   (a) Is \( \vec{v} = (3, -1, 3, 2) \) in the nullspace of \( A \)? If so, find \( [\vec{v}]_B \).
   (b) Is \( \vec{w} = (-12, -6, 4, -1) \) in the nullspace of \( A \)? If so, find \( [\vec{w}]_B \).

3. Consider the matrix

\[
A = \begin{bmatrix}
1 & -3 & 1 & -1 & 0 & -1 \\
-1 & 3 & 0 & -1 & 1 & 3 \\
0 & 0 & 1 & -2 & 0 & 0 \\
2 & -6 & 0 & 2 & 1 & 0
\end{bmatrix}
\]

   (a) (6 points) Find a basis for the nullspace of \( A \).
   (b) (3 points) Find a basis for the rowspace of \( A \).
   (c) (3 points) Find a basis for the columnspace of \( A \). Explain how you know that the vectors you give are a basis. Did you compute something? If not, how did you know which vectors to pick?

4. (6 points) The following problems are to be solved using the Rank-Nullity Theorem.
   (a) If \( A \) is a 6-by-4 matrix with \( \text{Rank}(A) = 1 \), what is the dimension of the null space of \( A \)? Justify your answer.
   (b) If \( A \) is a 6-by-7 matrix, is it possible that \( \text{Rank}(A) = 4 \) and the dimension of the null space of \( A \) is 3? Justify your answer.