Cost of Capital

The $F$ in $V_{firm} = f(I, F, D)$

Big picture

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities and Net Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>$k_i$</td>
</tr>
<tr>
<td>Preferred stock</td>
<td>$k_p$</td>
</tr>
<tr>
<td>Equity</td>
<td></td>
</tr>
<tr>
<td>Retained earnings</td>
<td>$k_e$</td>
</tr>
<tr>
<td>Common stock</td>
<td>$k_n$</td>
</tr>
<tr>
<td>Weighted average</td>
<td>$k_o$</td>
</tr>
</tbody>
</table>

Cost of Capital

$k_o$ is the required rate of return used in capital budgeting (NPV/IRR) calculations

$k_o$ is the cut-off rate for the allocation of capital

$k_o$ is the rate of return on a project that will leave the market price of the firm's common stock unchanged
Component costs

Explicit cost of capital is that discount rate that equates the present value of the funds received by the firm, net of underwriting and other flotation costs, with the present value of the expected outflows. Concerned only with future, marginal costs. Historical costs are irrelevant.

Component cost equation

Solve this equation for $k$

$$I_0 = \frac{C_1}{(1+k)^1} + \frac{C_2}{(1+k)^2} + \cdots + \frac{C_n}{(1+k)^n} = \sum_{t=1}^{n} \frac{C_t}{(1+k)^t}$$

$I_0$ = net amount received by the firm at $t = 0$ after subtracting underwriting and flotation costs.

$C_t$ = outflow at end of period $t$ (coupon, principal, dividend).

$n$ = maturity of the security.

$k$ = component cost of capital.

Cost of debt $k_i$

$I_0$ = net proceeds from bond issue.

Solve the previous equation for $k$.

Adjust $k$ for the tax-deductibility of interest.

$k_i = k(1 - t)$ where $t$ is firm's marginal tax rate.

Tends to make after-tax cost of debt substantially below before-tax cost.
**Cost of debt example**

maturity=20 years annual coupon =8% comp a. 
investment banker buys for $980 and resells to public for$1000

tax rate = 40%

Last chapter: find yield to the investor

\[ P = \frac{\text{1000}}{(1+i)^{15}} \left( \frac{40}{(1+i)^{15}} + \frac{40}{(1+i)^2} + \frac{40}{(1+i)^3} + \cdots + \frac{1000}{(1+i)^{15}} \right) \]

\[ i = 4\% \text{ period or } 8\% \text{ yr comp a.} \]

This chapter: find cost to the firm

\[ I_t = \frac{980}{(1+i)^{15}} \left( \frac{40}{(1+i)^{15}} + \frac{40}{(1+i)^2} + \frac{40}{(1+i)^3} + \cdots + \frac{1000}{(1+i)^{15}} \right) \]

\[ k = 4.1\% \text{ period or } 8.2\% \text{ yr comp a.} \]

\[ k_v = 0.08(1-4) = 4.92\% \text{ yr comp a.} \]

---

**Cost of preferred stock**

Preferred stock is perpetual with a constant dividend

A share of $100 par preferred stock with 7.5% dividend rate is sold to an underwrite for $98.50 who then resells it to the public for $110

Last chapter: find the yield to the investor

\[ \text{yield} = \frac{\text{D}}{\text{P}} = \frac{7.50}{110} = 6.82\% \]

This chapter: find the cost to the firm

\[ k_v = \frac{\text{D}}{\text{Pv}} = \frac{7.50}{98.50} = 7.61\% \]

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**Cost of equity**

Most difficult to measure

New equity comes from:
- Retained earnings (internal)
- New common stock (external)

Need to discuss cost of equity in general
Cost of equity

\[ P_0 = \lim_{i \to \infty} \frac{D_1}{(1+k_i)} + \frac{D_2}{(1+k_i)^2} + \cdots + \frac{D_n}{(1+k_i)^n} = \lim_{i \to \infty} \sum_{n=1}^{\infty} \frac{D_n}{(1+k_i)^n} \]

\( k_i \) is the required rate of return in our dividend valuation model and it is also the cost of equity

\( k_i \) is the rate of return required by investors on equity of the given risk class

Normal growth model

Assume dividends will grow at a constant, normal rate \( g \) for indefinite future

Assume that \( k_i > g \)

We already know \( P_0 = \frac{D_1}{k_i - g} \)

So after rearranging \( k_i = \frac{D_1}{g} + g \)

Cost of equity is dividend yield plus growth rate

If \( D_1 = 2.00 \), \( g = 4\% \) and \( P_0 = 40/\)share, then

\( k_i = \frac{2.00}{40} + 04 = 9\% \)

Beta model method

An alternative way to compute \( k_i \) is using CAPM

\[ k_i = R_e + \beta (k_m - R_f) \]

\( \beta \) is the firm's beta coefficient

We will use the normal growth model
Cost of retained earnings

Not zero: retained earnings are not free
Opportunity cost: if firm cannot generate projects with expected returns of at least \( k_e \) then it should pay out profits as dividends
Investors can find stocks of similar risk that do earn \( k_e \) and they can invest their dividends in these other firms
Cost of retained earnings is \( k_e \)

Let's revisit an old slide

Net proceeds \( (I_0) < \text{Current Price} (P_0) \)
- Under pricing of the new shares
  - Public won’t pay the current price for new shares so price must decline to attract sufficient new buyers
- Flotation costs
  - Investment bankers and brokerage firms need to be compensated for underwriting and selling the new shares

Cost of new common stock

\( I_o \) is the net proceeds per share and \( I_o < P_0 \)
But firm pays same infinite dividends stream to new shareholders

\[
P_0 = \sum_{t=0}^{\infty} \frac{D_t}{(1+k_e)^t} \quad I_o = \sum_{t=0}^{\infty} \frac{D_t}{(1+k_e)^t}
\]

\( k_e \) is the cost of new common stock
Since \( I_o < P_0 \) it must be that \( k_e > k_c \)
If we again assume a normal growth model,

\[
k_e = \frac{D_0}{I_o} + g
\]

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Meaningful digression

• You want to buy a new $40,000 car
• Sources of the funds:
  – Parents: $10,000 @ 5%
  – Brother: $4,000 @ 15%
  – Bank: $10,000 @ 10% (they get the title)
  – Your portfolio:
    • $26,000 @ 20% in McDonald's
    • $10,000 @ 25% in Microsoft

Cost of financing the car

<table>
<thead>
<tr>
<th>Source</th>
<th>Wt.</th>
<th>Amount</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parents</td>
<td>.25</td>
<td>10,000</td>
<td>.05</td>
</tr>
<tr>
<td>Brother</td>
<td>.10</td>
<td>4,000</td>
<td>.15</td>
</tr>
<tr>
<td>Bank</td>
<td>.25</td>
<td>10,000</td>
<td>.10</td>
</tr>
<tr>
<td>Stocks</td>
<td>.40</td>
<td>16,000</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40,000</td>
<td></td>
</tr>
</tbody>
</table>

.1325 = .25(.05)+.10(.15)+.25(.10)+.40(.20)

Weighted Average Cost of Capital

Given:  $k_e=4%  $k_d=9%  $k_p=15%  $k_n=18%$
Given:  firm has $60 million from retained earnings available for investment
Given:  firm will raise funds using weights:
  debt 30%  pf'd 20%  equity 50%  "optimal"
Given:  firm wants to raise $100 million total
Assume:  firm finances all of its equity from RE until they are exhausted and then switches completely over to new common stock for its equity
Let's revisit the big picture

**Assets** | **Liabilities and Net Worth**
--- | ---
Debt (30%) | $k_d = 4\%$
Preferred stock (20%) | $k_p = 9\%$
Equity (50%) |
  - Retained earnings | $k_e = 15\%$
  - Common stock | $k_n = 18\%$
Weighted average | $k_w = ???$

Weighted Average Cost of Capital

<table>
<thead>
<tr>
<th>Source</th>
<th>Wt.</th>
<th>Amount</th>
<th>Avg. Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>30</td>
<td>30</td>
<td>4%</td>
</tr>
<tr>
<td>Preferred</td>
<td>20</td>
<td>20</td>
<td>9%</td>
</tr>
<tr>
<td>Equity</td>
<td>50</td>
<td>50</td>
<td>15%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td></td>
<td><strong>10.5%</strong></td>
</tr>
</tbody>
</table>

$k_w = .30(4\%) + .20(9\%) + .50(15\%) = 10.5\%$

One more dollar

- The average cost of raising funds from 0 to $100$ million is $k_w = 10.5\%$.
- What if the firm now decides it wants one more dollar – where does it come from?
- 30¢ from debt, 20¢ from preferred stock and 50¢ from equity
- What type of equity? Retained earnings
- Compute the cost of that marginal dollar
Marginal and average costs

- Sorry, but both symbolized by $k_o$
- To clear up any confusion, let's expand our example problem
- Instead of $100 million, firm now decides it wants to raise $150 million
- Compute average cost of capital

Weighted Average Cost of Capital

<table>
<thead>
<tr>
<th>Source</th>
<th>$W_t$</th>
<th>Amount</th>
<th>$k_{avg}$</th>
<th>$k_{marg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>.30</td>
<td>30</td>
<td>4%</td>
<td>4%</td>
</tr>
<tr>
<td>Preferred</td>
<td>.20</td>
<td>20</td>
<td>9%</td>
<td>9%</td>
</tr>
<tr>
<td>Equity</td>
<td>.50</td>
<td>50</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td>$k_o = 10.5%$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$k_p = 10.5%$</td>
</tr>
</tbody>
</table>

$k_o = .30(4\%) + .20(9\%) + .50(15\%) = 10.5\%$

Weighted Average Cost of Capital

<table>
<thead>
<tr>
<th>Source</th>
<th>$W_t$</th>
<th>Amount</th>
<th>$k_{avg}$</th>
<th>$k_{marg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>.30</td>
<td>45</td>
<td>4%</td>
<td>4%</td>
</tr>
<tr>
<td>Preferred</td>
<td>.20</td>
<td>30</td>
<td>9%</td>
<td>15%</td>
</tr>
<tr>
<td>Equity</td>
<td>.50</td>
<td>75</td>
<td>15.6%</td>
<td>15.6%</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
<td></td>
<td>$k_o = 10.8%$</td>
</tr>
</tbody>
</table>

$k_{equity} = (.60)(15\%) + (.15)(18\%) = 15.6\%$
One more dollar

- The average cost of raising funds from 0 to $150 million is $k_o = 10.8\%$
- What if the firm now decides it wants one more dollar – where does it come from?
- 30¢ from debt, 20¢ from preferred stock and 50¢ from equity
- What type of equity? Common stock
- Compute the cost of that marginal dollar

Weighted Average Cost of Capital

<table>
<thead>
<tr>
<th>Source</th>
<th>Wt.</th>
<th>Amount</th>
<th>Avg. Cost</th>
<th>Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>30</td>
<td>45</td>
<td>4%</td>
<td>4%</td>
</tr>
<tr>
<td>Preferred</td>
<td>20</td>
<td>30</td>
<td>9%</td>
<td>9%</td>
</tr>
<tr>
<td>Equity</td>
<td>50</td>
<td>75</td>
<td>15.6%</td>
<td>18%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15.6%</td>
<td>18%</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>150</td>
<td>10.8%</td>
<td>12.0%</td>
</tr>
</tbody>
</table>

$k_o = .30(4\%) + .20(9\%) + .50(18\%) = 12.0\%$

Weights

- Where do the weights come from?
- Use the firm’s current capital structure only if it is optimal (always assumed in textbook and exam problems)
- If firm is changing its attitude toward leverage, cannot use the current capital structure for weights – use marginal weights
Give me a break

- Critical question in every cost of capital problem: where is (are) the break point(s)?
- Restated: how much total funds from all three sources can be raised without the firm having to issue new common stock?
- 50% of funds comes from equity and firm has $60 million in retained earnings
- \( 0.50X = 60 \Rightarrow X = 120 \) (million)

Cost of Capital Schedule

Lots of breaks

- 30% of funds is debt
  - 0 to $15 mil @ 5%
  - $15 to $36 mil @ 6%
  - Beyond $36 mil @ 8.5%
  - Debt causes a break at \( 0.30W=15 \Rightarrow W=50 \text{ mil} \) and another at \( 0.30X=36 \Rightarrow X=120 \text{ mil} \)
- 70% of funds is equity
  - 0 to $14 mil in RE @ 16%
  - 0 to $35 mil in CS @ 19%
  - Beyond $35 mil in CS @ 21%
  - Equity causes a break at \( 0.70Y=14 \Rightarrow Y=20 \text{ mil} \) and another at \( 0.70Z=(14+35) \Rightarrow Z=70 \text{ mil} \)
**Break breakdown**

- Breaks at $50 and $120 mil due to debt
- Breaks at $20 and $70 mil due to equity

**Old slide revisited**

**Investment and Financing Decisions**
Investment and Financing Decisions

Investment and financing decisions are interrelated and determined simultaneously.
Cost of capital depends upon amount of funds being raised.
Amount of funds being raised depends upon acceptability of the projects.
Acceptability of the projects depends upon the cost of capital (IRR=k_o)

Capital Structure

Optimal capital structure is the set of weights that:
- minimizes k_o, the weighted average cost of capital.
- maximizes share price.
If debt is so much cheaper than equity (k_i<k_e) why doesn't the firm use more and more debt? How can 30 or 40% be optimal?

How much debt is optimal?

What happens as leverage is increased?
- Cost of equity (k_e)
- Cost of debt (k_i)
- Weighted average cost (k_o)
Cost of equity \( k_e \)

- Investors perceive an increase in leverage as being risky and "penalize" the stock by requiring a higher rate of return
- \( k_e = R_f + \beta + F \)
- \( R_f \) is the risk-free rate of interest
- \( \beta \) is a premium reflecting degree of business risk
- \( F \) is a premium reflecting the degree of financial risk; \( F \) rises with leverage

Financial risk

- Higher debt \( \rightarrow \) higher fixed interest charges \( \rightarrow \) higher probability of default and bankruptcy
- Higher debt \( \rightarrow \) greater volatility of net income and EPS \( \rightarrow \) magnifies a downturn

Cost of debt \( k_i \)

- Firm cannot continue to borrow more and more at a constant rate
- Firm must first justify the use of the capital through profitable investment
- If the firm continues to borrow, lenders will require higher interest rates to compensate for the risk
Average cost of capital  $k_o$

The cost of equity $k_e$ and the cost of debt $k_i$ both rise as firm moves to higher levels of debt. Since $k_o$ is simply an average of $k_e$ and $k_i$, what happens to $k_o$ as the level of debt increases?

Falls then rises?

- How can the average fall if the only two components are both rising?
- Remember, the weights are changing in favor of the relatively cheaper debt as leverage increases

Optimal Capital Structure Example

<table>
<thead>
<tr>
<th>Debt</th>
<th>Assets</th>
<th>$k_e$</th>
<th>$k_i$</th>
<th>$k_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100%</td>
<td>3.6%</td>
<td>10.0%</td>
<td>6.0%</td>
</tr>
<tr>
<td>10%</td>
<td>90%</td>
<td>3.7%</td>
<td>10.1%</td>
<td>6.4%</td>
</tr>
<tr>
<td>30%</td>
<td>70%</td>
<td>4.2%</td>
<td>11.5%</td>
<td>7.0%</td>
</tr>
<tr>
<td>40%</td>
<td>60%</td>
<td>4.8%</td>
<td>13.0%</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

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Why not use more and more debt?

• Need to consider the implicit, as well as, the explicit component costs
• Implicit cost of debt is that it makes the firm riskier and the component costs rise — when you financed your car, the bank loaned you money at 10% because you, your parents and your brother put up 75% of the money
• What if you borrowed 95% of the cost?

Why not use more and more debt?

• The implicit cost of equity is really a benefit and not a cost
• The use of retained earnings and common stock is necessary to build the firm's equity base
• The firm is able to borrow at attractive rates only if its owners supply a safe percentage of the total funds

Optimal capital structure

• The optimal capital structure will vary from industry to industry
• Consider a public utility vs. an alien owned cloning company
• Firm will strive for its optimal combination of debt and equity over time