Market Value

Market value of any asset, whether it be a bond, a share of preferred or common stock, a rare painting or a classic car, is theoretically the discounted value of the expected cash flows.

Valuation Models - Bonds

AT&T has a bond issue outstanding:
- Coupon rate = 8%/yr comp semiannually
- Matures in 20 years
- Par value = face value = principal = 1,000
Calculate its market value
One additional piece of information is needed
Synonyms

• interest rate
• yield to maturity on comparable securities
• market rate of return or market yield
• going rate of return

Given \( i \), find \( P_0 \)

• Let’s assume that the yield to maturity or interest rate on similar bonds is 10%/yr compounded semiannually
• \( P_0 \) is the discounted value of the expected cash flows
• \( P_0 \) is the discounted value of the annuity of coupon payments and the return of principal at maturity

Value of the AT&T bond

\[
C = \frac{.08 \times 1000}{2} = 40 \text{ period, } n = 20 \times 2 = 40 \text{ periods and } i = \frac{10}{2} = .05 \text{ period}
\]

\[
P_i = \frac{C}{(1 + i)^1} + \frac{C}{(1 + i)^2} + \cdots + \frac{1000}{(1 + i)^{220}}
\]

\[
P_r = \frac{40}{(1.05)^1} + \frac{40}{(1.05)^2} + \cdots + \frac{1000}{(1.05)^{219}}
\]

\[
P_i = 40(PVIFu - .05 \times 40) + 1000(1.05)^{219}
\]

\[40 \Rightarrow \text{PMT} \Rightarrow 1 \text{ yr} \Rightarrow 40 \Rightarrow n \Rightarrow 1000 \Rightarrow \text{FV}\]

\[
PV = 828.41 < 1000 \quad \text{Bond sells at a discount}
\]
Yield to Maturity on the AT&T bond

\[ C = \frac{\text{88} \times 1000}{2} = 40 \text{periods}, \quad n = 20 \times 2 = 40 \text{ periods} \quad \text{and} \quad P = $828.41 \]

\[
P\quad = \left( \frac{C}{(1+i)^1} \right) + \left( \frac{C}{(1+i)^2} \right) + \cdots + \left( \frac{C}{(1+i)^{40}} \right) + \left( \frac{1000}{(1+i)^{40}} \right)
\]

\[828.41 = 40 \left( \frac{C}{(1+i)^1} \right) + \left( \frac{C}{(1+i)^2} \right) + \cdots + \left( \frac{C}{(1+i)^{40}} \right) + \left( \frac{1000}{(1+i)^{40}} \right)
\]

\[ i = 0.05/\text{period} \quad \text{or} \quad i = 0.10/\text{yr} \text{ compounded semiannually} \]

Internal Rate of Return (IRR)

\[ CF_t = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_t}{(1+r)^t} = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} \]

\[ CF_t = \text{cash flow, end of period} \quad t \]

\[ n = \text{life of the project} \]

\[ r = \text{IRR} \]

Why the discount?

- New 20 year securities being issued today probably are paying a coupon of $100/yr
- If it did sell for $1,000 it would yield only 8% which is less than other similar bonds
- Price must adjust to bring the yield or interest rate into line with similar bonds
Back in time

• It’s common for a new bond to be issued at a price close to its par value of 1,000
• 5 years ago AT&T issued our bonds with a maturity of 25 years and an annual coupon of 8%. Let’s assume that the interest rate at that time was 8.2%/yr, compounded semiannually. What was the issuing price?

Issuing price of AT&T bonds

\[ C = \frac{.08 \times 1000}{2} = 40 \text{ period, } \quad n = 25 \times 2 = 50 \text{ periods and } i = \frac{.082}{2} = .041 \text{ /period} \]

\[ P_v = \frac{C}{(1 + i)^1} + \frac{C}{(1 + i)^2} + \ldots + \frac{C}{(1 + i)^n} + \frac{1000}{(1 + i)^n} \]

\[ P_v = \frac{40}{(1.041)^1} + \frac{40}{(1.041)^2} + \ldots + \frac{40}{(1.041)^n} + \frac{1000}{(1.041)^n} \]

\[ P_v = 40(\text{PVIF}_{i = .041, n = 50}) + 1000(\text{PVIF}_{i = .041, n = 50}) \]

40 \Rightarrow \text{PMT} \quad 4.1 \Rightarrow i/\text{yr} \quad 50 \Rightarrow n \quad 1000 \Rightarrow \text{FV}

\[ PV = 978.88 \]

Can you say “capital loss?”

• What about the investor who bought these very safe bonds 5 years ago and now wants to sell?
• Can she recover her $978.88?
• No, only $828.41 because interest rates have risen
• Let’s see an old slide again
Incredibly important relationships

\[ i \uparrow \iff P_{\text{bonds}} \downarrow \]

\[ i \downarrow \iff P_{\text{bonds}} \uparrow \]

Why the inverse relationship?

\[ P_a = \frac{C}{(1 + i)^1} + \frac{C}{(1 + i)^2} + \cdots + \frac{1000}{(1 + i)^t} \]

With the numerators fixed (bonds are called “fixed income” securities), if the denominator changes, the only thing that can give is that the left side of the equation has to move in the opposite direction.

What if interest rate had fallen?

\[ C = \frac{.08 \times 1000}{2} = 40 \text{/period, } n = 20 \times 2 = 40 \text{ periods and } i = \frac{.05}{2} = .025 \text{/period} \]

\[ P_a = \frac{C}{(1 + 0.025)^1} + \frac{C}{(1 + 0.025)^2} + \cdots + \frac{1000}{(1 + 0.025)^t} \]

\[ P_a = \frac{40}{(1.025)^1} + \frac{40}{(1.025)^2} + \cdots + \frac{1000}{(1.025)^{40}} \]

\[ P_a = 40 \left( \frac{PVIF_1 - 2.5 \% - 40}{(1.025)^{40}} - \frac{1000}{(1.025)^{40}} \right) \]

\[ 40 \Rightarrow \text{PMT} 2.5 \Rightarrow \text{I/yr} 40 \Rightarrow n \Rightarrow 1000 \Rightarrow \text{FV} \]

\[ PV = 1376.54 > 1000 \text{ Bond sells at a premium} \]
Why the premium?

• New 20 year securities being issued today probably are paying a coupon of $50/yr
• If it did sell for $1,000 it would yield 8% which is more than other similar bonds
• Price must adjust to bring the yield or interest rate into line with similar bonds

Important observations

• The longer the period to maturity, the more sensitive is a bond’s price to a given change in the interest rate
• Maturity is one factor affecting a bond’s yield
• Long-term bonds are inherently riskier than short-term bonds and generally carry higher yields to maturity
• Especially true when rates are low and expected to rise
Maturity vs. yield example

- Bond A: \( i_A = 5\%, C_A = 5\%, n_A = 2, P_A = 1000 \)
- Bond B: \( i_B = 5\%, C_B = 5\%, n_B = 20, P_B = 1000 \)
- Instantaneously change \( i_A = i_B = 8\% \)
- Verify that \( P_A = 945.55 \) and \( P_B = 703.11 \)
- 3 percentage point rise in interest rate produces a much bigger decrease in price of long-term bond B than in short-term bond A

Interest rate risk

- Interest rate risk is the possibility that the interest rate will rise and the price of bonds will fall.
- The price of long-term bonds will fall more than the price of short-term bonds for a given change in the interest rate
- Relationship between maturity and yield is shown by the term structure of interest rates

Term Structure or Yield Curve

![Term Structure or Yield Curve](image-url)
Preferred Stock

- Perpetual – infinite maturity
- Constant fixed dividend – never changes
- Par value usually $50 or $100/share
- If dividend rate=8% and par=$50, 
  \[ D = 0.08 \times 50 = 4.00 \text{ per share} \]
- \( k_p \) → market capitalization or discount rate
  for a share of preferred stock of the given risk class

Preferred Stock Valuation

\[
P_{pfd} = \frac{D_1}{1+k_p} + \frac{D_2}{(1+k_p)^2} + \cdots + \frac{D_n}{(1+k_p)^n} + \sum_{t=n}^{\infty} \frac{D_t}{(1+k_p)^t}
\]

For preferred stock, \( D_1 = D_2 = \cdots = D_n \)

\[
P_{pfd} = \frac{D}{k_p}
\]

Common Stock

- Why buy a share of common stock?
  - Capital gains
  - Dividend stream

- Let's consider an arbitrary 10 year holding period
Common Stock Valuation

\[ P_n = \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \cdots + \frac{D_n}{(1 + k)^n} + \frac{P_{n+1}}{(1 + k)^{n+1}} \]

\[ P_n = \text{price per share paid by second investor at end of year 10} \]

\[ P_n = \frac{D_1}{(1 + k)^1} + \cdots + \frac{D_n}{(1 + k)^n} \]

could have a third investor, but assume \( n \to \infty \)

\[ P_{\infty} = \frac{D_1}{(1 + k)^{\infty}} + \cdots + \frac{D_n}{(1 + k)^n} + \frac{1}{(1 + k)^{\infty}} \left[ \frac{D_1}{(1 + k)^1} + \cdots + \frac{D_n}{(1 + k)^n} \right] \]

\[ P_{\infty} = \frac{D_1}{(1 + k)^{\infty}} + \cdots + \frac{D_n}{(1 + k)^n} + \frac{D_1}{(1 + k)^1} + \cdots + \frac{D_n}{(1 + k)^n} \]

\[ P_{\infty} = \sum_{k=1}^{\infty} \frac{D_k}{(1 + k)^k} \]

This equation is basis for all common stock valuation

Common Stock Growth Models

- Need to make assumptions regarding the behavior of the dividend stream
- Normal growth model – describing large majority of firms
- “Super” growth model – describing the exceptional firms

Normal Growth Model

Assumed dividend will grow at an annual rate of \( g \) for an infinite duration

\[ D_1 = D_0(1 + g)^1 \]

\[ D_2 = D_0(1 + g)^2 = D_0(1 + g)^2 \]

\[ D_n = D_0(1 + g)^n = D_0(1 + g)^n \]

\[ P_n = \lim_{n \to \infty} \left[ \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \cdots + \frac{D_n}{(1 + k)^n} \right] \]

\[ P_{\infty} = \lim_{n \to \infty} \left[ \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \cdots + \frac{D_n}{(1 + k)^n} \right] \]

\[ n \to \infty \quad \text{and} \quad k > g \]

\[ P_{\infty} = \frac{D_0}{(k - g)} \]

Copyright ©2003 Stephen G. Buell
Normal Growth Model

If \( n \to \infty \) and \( k_r > g \)

\[
P_0 = \frac{D_1}{(k_r - g)}
\]

Example: \( D_1 = 2.00, k_r = 14\%, g = 4\% \)
Then \( P_0 = \frac{2.00}{(0.14 - 0.04)} = \$20.00 \text{ per share} \)

“Super” Growth Model

\[
P_n = \sum_{t=1}^{N} \frac{D_t(1+g)^{t-n}}{(1+k_r)^t}
\]

\[
P_n = \sum_{t=0}^{n-1} \frac{D_0(1+g)^{t-n}}{(1+k_r)^t} + \sum_{t=n}^{N} \frac{D_0(1+g)^{t-n}}{(1+k_r)^t}
\]

\[
P_0 = \sum_{t=0}^{N} \frac{D_0(1+g)^{t-n}}{(1+k_r)^t}
\]

Assumption:\n\( g \geq k_r \) for \( N \) years before declining to a normal growth rate of \( g \) for indefinite future

Analyzes if stock is held for \( N \) years then sold for \( P_n \)
“Super” Growth example

\[ P = \sum_{k=1}^{N} \left( \frac{D_k}{1+k} \right)^{1/2} + \frac{D_k}{1+k} \left( \frac{P}{1+k} \right)^{1/2} \]

Example: \( D_1 = 3000 \), \( c_g = 20\% \), \( g_n = 5\% \), \( k = 1\% \), \( N = 3 \)

\[ P = \frac{1.00120^3}{1.170} + \frac{1.00120^3}{1.170} \left( \frac{3.2}{1.170} \right)^{1/2} \]

\[ P = \frac{1.00120^3}{1.170} \cdot 15.05 = 18.14 \]

\[ P = 3.2 \cdot 18.14 = 57.24 \]

Copyright ©2003 Stephen G. Buell

“Super” Growth exam question

Find \( P_t \) and \( P_{30} \)

\[ P_t = \frac{D_1}{(1+k)^t} + \frac{D_2}{(1+k)^2} + \frac{P}{(1+k)^2} \]

\[ P_t = 1.44 + 1.73 + 18.14 \]

\[ P_t = 18.14 \]

\[ P_t = 18.14 \cdot 1.05 = 19.14 \]

\[ P_{30} = \frac{D_{30}}{k - g_s} = \frac{1.00(1.20)^{10}(1.05)^{28}}{0.15 - 0.05} = 67.74 \]

Copyright ©2003 Stephen G. Buell