Capital Budgeting

Risk and Uncertainty

Risk – the possibility that actual returns will deviate from expected returns
Risk – situations in which a probability distribution of possible outcomes can be estimated
Uncertainty – worse, not enough information available

Probability distribution of expected outcomes
Initial measure of risk

Standard deviation of expected cash flows $\sigma$

$$\sigma = \sqrt{\sum_{j=1}^{m} (CF_j - \overline{CF})^2 P_j}$$

$$\overline{CF} = \sum_{j=1}^{m} CF_j P_j$$

$m = \text{number of possible outcomes}$

$\overline{CF}_j = \text{the } j^{\text{th}} \text{ possible outcome}$

$P_j = \text{probability of } \overline{CF}_j \text{ occurring}$

Improved measure of risk

Coefficient of variation (cv) puts dispersion on a relative basis

$$cv = \frac{\sigma}{\overline{CF}}$$

Consider $\sigma_1 = 300 \text{ and } \overline{CF}_1 = 1000 \text{ versus } \sigma_2 = 300 \text{ and } \overline{CF}_2 = 4000$

Intuitively $x$ is riskier. Need to show that.

$$cv_1 = \frac{300}{1000} = .300 \text{ while } cv_2 = \frac{300}{4000} = .075$$

Forecasted cash flows

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>$CF_j$</th>
<th>$P_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>j = 1 Recession</td>
<td>100</td>
<td>30%</td>
</tr>
<tr>
<td>j = 2 Normal</td>
<td>300</td>
<td>50%</td>
</tr>
<tr>
<td>j = 3 Boom</td>
<td>800</td>
<td>20%</td>
</tr>
</tbody>
</table>
Computing coefficient of variation

\[
\overline{CF} = 0.30(100) + 0.50(300) + 0.20(800) = 340
\]
\[
\sigma = \sqrt{(100 - 340)^2 + (300 - 340)^2 + (800 - 340)^2} = 245.76
\]
\[
CV = \frac{\sigma}{\overline{CF}} = \frac{245.76}{340} = 0.72
\]

Required hurdle rate \( k' \)

\[\]
\[
k' = f(\text{risk}) = f\left(\frac{\sigma}{\overline{CF}}\right)
\]

Required rate of return \( k' \) is a function of the forecasted risk of the project.

"Penalize" a riskier project by requiring a higher hurdle rate for it to be acceptable.

Alternate methods for incorporating risk into capital budgeting

(1) Risk-adjusted discount rate

(2) Certainty equivalents
Risk-adjusted discount rate schedule

4% is the risk-free rate
Curve is a risk-return trade-off function
Curve is an indifference curve
Firm is indifferent to a \( cv=0.5 \) and \( k'=7\% \) or a \( cv=1.0 \) and \( k'=10\% \)
Select \( k' \) based on risk from a predetermined schedule and compute NPV

Certainty Equivalents

Convert the expected cash flows to their certainty equivalents
Discount the certainty equivalents at the risk-free rate of interest
Risk-free rate is the yield on a US Treasury bond of the same maturity as the project
Certainty equivalents coefficients

$\alpha_t$ is the certainty equivalent coefficient for the cash flow at time $t$

$0 < \alpha_t < 1$  $\alpha_t$ falls as risk increases

$\alpha_t$ is determined subjectively by the firm or obtained from a predetermined schedule

$\hat{CF}_t = \alpha_tCF_t$  $\hat{CF}_t$ is the certainty equivalent period t

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NPV w/ Certainty Equivalents

Let's say $\alpha_1 = 1.00, \alpha_2 = 0.95, \alpha_3 = 0.82, ..., \alpha_{10} = 0.50$ and $i = 4\%$

$NPV = -CE_0 + \frac{\alpha_1(\hat{CF}_1)}{1.04^1} + \frac{\alpha_2(\hat{CF}_2)}{1.04^2} + \frac{\alpha_3(\hat{CF}_3)}{1.04^3} + \cdots + \frac{\alpha_{10}(\hat{CF}_{10})}{1.04^{10}}$

$NPV = -26000 \left( \frac{1.005800}{1.04} + \frac{1.095800}{1.04^2} + \frac{1.0825800}{1.04^3} + \cdots + \frac{1.05019800}{1.04^{10}} \right)$

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Risk in a portfolio context

Consider the potential investment, not in isolation (as we have been doing), but in a portfolio context

Look at the relationship between the investment and the firm's existing assets and other potential investments
Two projects and the firm

Which is the more attractive project?
Project A is cyclical like the overall firm
Project B is counter cyclical
In isolation $\sigma_A = \sigma_B$ but for this firm, B is the more attractive project
Project B is highly negatively correlated with the firm’s other assets so addition of Project B reduces overall risk

Correlation and diversification
Difficult to find projects with high negative correlation
However, if projects whose returns are uncorrelated are combined, overall risk can be reduced and even eliminated
Firms seek to diversify into other areas
Firms try to build a portfolio of assets
Portfolio definitions

Portfolio  ➔ combination of assets
Optimal portfolio  ➔ maximum return for a given
degree of risk - or - minimum risk for a given
rate of return
Opportunity set  ➔ all possible portfolios
Efficient frontier  ➔ locus of all optimal portfolios
Efficient frontier  ➔ dominates all other portfolios of
the opportunity set
Risk of an individual asset in a portfolio context

yield on security $j$ = \frac{\text{capital gain} + \text{dividend}}{\text{original price}}

$$\bar{k}_j = \frac{(P_{j,t+1} - P_{j,t}) + D_{j,t+1}}{P_{j,t}}$$

Risk of an individual asset in a portfolio context

Excess return (or risk premium) on security $j$ is the difference between the yield on the security and the yield on risk-free treasury securities ($R_f$):

risk premium on security $j$ = $(\bar{k}_j - R_f) = \frac{(P_{j,t+1} - P_{j,t}) + D_{j,t+1}}{P_{j,t}} - R_f$

The market's risk premium can be defined similarly:

risk premium on the market = $(\bar{k}_m - R_f) = \frac{(P_{m,t+1} - P_{m,t}) + D_{m,t+1}}{P_{m,t}} - R_f$

Plot the last 60 months of observations

<table>
<thead>
<tr>
<th>Month</th>
<th>$\bar{k}_1$</th>
<th>$\bar{k}_2$</th>
<th>$\bar{k}_3$</th>
<th>$\bar{k}_4$</th>
<th>$\bar{k}_5$</th>
<th>$\bar{k}_6$</th>
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<tbody>
<tr>
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<tr>
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<td>01</td>
<td>02</td>
<td>05</td>
<td>06</td>
<td>07</td>
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</tr>
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Characteristic Line

![Characteristic Line Graph]

Equation of Characteristic Line

\[
\bar{k}_j - R_f = \alpha + \beta (k_m - R_f)
\]

If \( \bar{\alpha} = 0 \),

\[
\bar{k}_j = R_f + \beta (k_m - R_f)
\]

Security \( j \)'s expected return is equal to the risk-free rate plus a risk premium. This risk premium is equal to the market risk premium times \( j \)'s beta coefficient.

Beta and systematic risk

Beta is an indicator of systematic risk or market risk: interest rate risk, inflation, panics.

\( \beta > 1 \): stock is aggressive, more volatile than the overall market, e.g., airlines, steel, tires.

\( \beta < 1 \): stock is defensive, less volatile than the overall market, e.g., utilities.
Unsystematic risk

Variations in security j’s return not due to market forces
Unique to the firm, e.g., financial and operating leverage, managed by crooks
Eliminated by diversification
Only systematic risk matters to a firm with a diversified portfolio

What’s a firm to do?

McDonald's and Sears are contemplating going into the pizza business
McDonald's is only in fast foods - not diversified;
∴ they must be concerned with total risk

\[ k_{pizza} = f \left( \frac{\sigma}{\beta} \right) \] userisk-adjusted discount method

or certainty equivalent method to find NPV
Sears is diversified into department stores, auto parts, insurance, real estate, etc.;
∴ they are concerned only with systematic risk

\[ k_{pizza} = R_f + \beta_{pizza} [k_m - R_f] \] use “Beta Model”