Preface

This book is an introduction to group representation theory. With the exception of this preface, it presupposes no prior knowledge of the subject, not even what a representation is. The reader with no prior knowledge should have a look at Chapter 1, which presents an introduction to the theory.

In this preface we discuss our approach and objectives, so here the reader must have some prior knowledge to understand what is going on.

This is a text on the representation theory of finite groups. The theory can be divided into two parts: Ordinary (or semisimple) representation theory, the case where the characteristic of the field is 0 or prime to the order of the group, and modular (or non-semisimple) representation theory, the case where the characteristic of the field divides the order of the group.

In each case, our approach is to present the basic ideas of the theory in a reasonably general context. Thus we do not prove the basic results purely for group representations, but rather for the appropriate sort of rings and modules, deriving the results for group representations from them. This is the "Algebra" part of the subtitle; we do some general algebra motivated by its applications to representation theory. Also, particularly in our treatment of the characteristic 0 case, we do not just treat the case of algebraically closed fields, but rather pay quite a bit of attention to the question of the field of definition of a representation. This is the "Arithmetic" part of the subtitle.

Let us now be more specific. We begin, in Chapter 1, with an introduction to the representation theory of finite groups. Chapter 2 is a treatment of the theory of semisimple rings and modules. We begin Chapter 3 with some examples, but then derive the basic theorems of ordinary representation theory as immediate consequences of the results of Chapter 2. Of course, this theory is not just a special case of the more general theory, so we then develop the additional methods we need in this case, especially characters, the main (and powerful) computational tool here.

One of the most powerful methods in representation theory is that of induction, to which we devote Chapter 4. In the basics of induction, we are careful to adopt an approach that will generalize to the modular case. We conclude this chapter by proving Brauer's famous, and deep, theorem that all of the irreducible complex representations of a finite group of exponent u are defined over the cyclotomic field obtained by adjoining the u-th roots of unity to the field of rational numbers.

Next we turn to the modular theory, with a similar approach. We begin, in Chapter 5, with a number of examples. Then, in Chapter 6, we treat general rings and modules, and begin Chapter 7 by deriving the basic theorems of modular representation theory as immediate consequences of the results of Chapter 6. We then further develop modular representation theory and fully analyze the situation for some small groups.

The appendix contains several background results which we use and which the reader may not be familiar with.

The scope of this book can perhaps best be described by the following analogy. We explore widely in the valley of ordinary representations, and we take the reader over the mountain pass leading to the valley of modular representations, to a point from which (s)he can survey this valley, but we do not attempt to widely explore it. We hope the reader will be sufficiently fascinated by the scenery to further explore both valleys on his/her own.

This is a text, and we do not claim to have any new results here. Many of the proofs are standard, but part of the fun of writing this book has been the opportunity to think deeply about this beautiful subject, so many of the proofs are the author's own. This does not mean they are necessarily new, as the author may simply have rediscovered known proofs for himself, but, on the other hand, it does not mean that they are all necessarily already known, either.

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Algebra: An Approach via Module Theory, Graduate Texts in Mathematics volume 136, ©Springer-Verlag 1992 (corrected second printing 1999). Indeed, the author first started thinking seriously about group representation theory while writing [AW], which has a chapter on it, and has continued thinking about this subject over the past decade. Thus, section 7.1 and chapter 8 of [AW] are subsumed here, in some places taken over unchanged, but in most places reworked and deepened.

We have deliberately kept the prerequisites for this book to a minimum. The reader will require a sound knowledge of linear algebra and a good familiarity with basic module theory. As a text on module theory we naturally recommend [AW]. In addition, the reader will need to be familiar with some basic homological algebra: exact sequences, Hom and tensor product ([AW, sections 3.3 and 7.2]) and the definition and properties of a projective module ([AW, section 3.5]). The reader will also need to be familiar with the basics of field theory and algebraic number theory.