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AUCTION-THEORETIC COORDINATION OF PRODUCTION PLANNING IN THE SUPPLY CHAIN

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Abstract

Most planning and optimization methods in manufacturing logistics assume centralized or hierarchical decision making using monolithic models. Motivated by the increasing needs to coordinate diverse decision processes and systems, we investigate an auction- theoretic mechanism for production coordination in a supply chain. Our efforts focuses on structural mappings between mathematical decomposition and iterative auction mechanisms wherein agents compete based on their local utilities, announced conflict pricing, and production targets. Building upon the rich literature in optimization and auction-theoretic analysis, we investigate the advantages and limitations of this distributed decision scheme on a large set problems in supply chain production planning. Experimental results show that the proposed auction mechanism provides impressive improvement over traditional monolithic method without significant degradation to the solution quality.

1. Introduction

Manufacturing competitiveness has escalated globally in the past two decades. Manufacturing firms experience increasing pressure to improve production efficiency, responsiveness to market changes, and substantial cost reduction. This increasing pressure is particularly evident in industries that have short product life cycles and complex supply structures, as is the case in automotive, electronics and semiconductor manufacturing. While the productivity and efficiency of manufacturing systems have been studied extensively, broader logistical issues spanning multiple levels of manufacturing facilities or multiple firms are not well understood. Manufacturing logistics is an emerging area of research that refers to "all planning, coordination and service functions required to carry out manufacturing activities between the point where end-item customer demands are determined, and the point where they are fulfilled" (Wu, et al. 1998). Among various topics in manufacturing logistics are planning and coordination of production across multiple facilities, and integration issues between manufacturing and other functional areas such as marketing, transportation, distribution and warehousing.

This paper focuses on the study of a distributed decision paradigm for operational coordination among manufacturing facilities. As a manufacturing entity seeks coordination with their internal or external customers and suppliers, it is quickly confronted with difficulties associated with different operational conventions, locally specific constraints, conflicting objectives, and misaligned incentives. If some form of centralized coordination is to be formed, significant time and resources must be first devoted to resolve these differences. However, as the level and the scope of coordination increase, the notion of centralized coordination breaks down at a point where the system complexity reaches its limit, and some form of decentralized coordination with local autonomy become unavoidable. Hierarchical decision making has been suggested to cope with the system complexity through decomposition, aggregation and feedback mechanisms. We propose a different approach to this problem using the notion of auction and market equilibrium. We believe that most decision entities in manufacturing have their own unique perspectives and economic incentives. Rather than forcing all decision entities into some unified decision structure, it may be helpful to view them as autonomous agents acting on their own behalf. Through the use of competitive market mechanisms these decision agents may be coordinated based on a much simpler set of policies while their long-term behavior can be predicted and modeled by various equilibrium conditions.

A main advantage of the new approach is the drastic simplification in information management. A basic paradigm in conventional Enterprise Resource Planning (ERP) system is one that seek "total visibility" of system details in a top-down, hierarchical manner. This is accomplished by maintaining painfully detailed information of all perceivable aspects of the organization using sophisticated information and database management systems. This information must be kept up-to-date since it serves as a basis for decision making throughout the organization. In a distributed system, since agents make locally autonomous decisions based on privately owned information and local preference/constraints, centralized information management can be decoupled and the monitoring and maintenance of information can be segmented and manipulated at a far more efficient manner. Analogous to the fast growing World Wide Web platform, this new paradigm facilitates interconnected software agents to communicate and to reconciliate their decisions through a universally agreed upon domain of information exchange.

A Multi-Facility Production Coordination Model

To submit the notion of distributed production coordination under rigorous testing, we reformulate a well-known production planning model in the literature to demonstrate the effects and implications of this new decision paradigm. The model we use is the multi-level, multi-item capacitated lot sizing problem (MLCLSP). MLCLSP can be defined in a multiple tier supply chain context as follows: given external demand for end items over a time horizon, a bill-of-material structure for each end item where the production of subassemblies may be spread across multiple facilities, find a production plan over multiple facilities that minimizes total inventory holding and setup costs. The main restrictions are that (1) items can be only produced after all their predecessor components (within or outside the facility) are available, (2) resources within each facility have limited capacity, and (3) no backlogging is allowed for the end items. To envision this mutli-facility production environment, Figure 2 presented later for computational testing may be useful.

MLCLSP is a monolithic model as it implicitly assumes that each production facility in the supply chain is willing to reveal its local constraints and cost parameters, and is ready to implement a centrally imposed solution. This assumption can be unrealistic especially when the production facilities are each owned by a different firm. Even in the case where all facilities are owned by the same company, the communication requirement and intra-company politics may render the use of monolithic models impractical. In this paper, we will show that regardless of its limited assumptions MLCLSP is useful for the studying of decentralized production planning, especially if we are interested in the performance of such system in correspondence to "global" optimal. For this purpose, we reformulate MLCLSP as a coordination problem where facilities are viewed as supplier-customer pairs who negotiate with one another mutually agreeable production plans. Using the viewpoint of the monolithic model, this defines a facility-based decomposition where each facility is responsible for their own production while relying on or supplying to other facilities according to the bill-of-material structure. We study a pricing mechanism designed to eliminate inconsistencies between local solutions of each supplier-customer pair.

In the next section, we summarize related literature in both production coordination and production planning. In Section 3, we present the MLCLSP formulation and two reformulations. We point out some of the fundamental issues of the traditional model and propose a new performance measure. In Section 4, we explain why Lagrangian decomposition does not work as a price-directive approach and introduce an auction-theoretic coordination mechanism. Section 5 presents main experimental results, and Section 6 concludes the paper.

2. Related Literature

Bhatnagar et al. [1] present a survey on multi-plant coordination. They divide coordination into two broad categories: coordination among different functional areas such as production planning, distribution, and marketing, and coordination of the same function across multiple layers of the organization. Our treatment of production coordination falls into the latter category.

McAfee and McMillan [2] define *auction* as a market institution where an explicit set of rules determine, based on "bids" from market participants, the ultimate resource allocation and payoff. The generic sequence of events in an iterative resource-allocation auctions is as follows: the market participants, or agents, disclose their specific requests for the shared resources at the announced prices so as to optimize their local utilities. The center, or the auctioneer, determines a new price for the shared resources using a price update mechanism based on current demands. The goal for the center is to interactively resolve conflicts among all agents, which lead to an equilibrium state where no agent can be better-off without worsening some other agent's utility. (Rassenti et al. [3]), (Banks et al. [4]), and (Kutanoglu and Wu [5]) have all proposed auction mechanism to complex resource allocation problems. To the best of our knowledge, there is no auction-theoretic mechanism suggested for multiple facility coordination in the framework of MLCLSP. The proposed coordination mechanism are motivated by the work of Jose and Ungar [6] where they suggest a so called "slack resource auction" mechanism for the coordination of interacting process units within a plant.

Another line of relevant research is that of the solution methods for multilevel lot sizing models. This line of work provides fundamental insights on the interdependency among production decisions in the context of monolithic optimization. If social welfare, collective cost efficiency, or global optimality is of interest, these models provide useful mathematical insights as well as performance benchmarks. MLCLSP is a difficult combinatorial problem. It has been shown that even finding a feasible solution to the problem is NP complete (Maes et al. [7]). A few exact methods existing in the literature (Pocket et al. [8], Chapman, [9]) are quite limited in problem sizes. A majority of research has been concentrated on heuristic approaches.

A wide range of heuristic methods have been proposed for multilevel lot sizing. Zahorik et al. [10] describe an optimization-based heuristic, employing a 3-period network flow formulation of the problem with no setup cost or time. Billington et al. [11] introduce a branch and bound heuristic using Lagrangian relaxation. It is assumed that the capacity restriction exists at a bottleneck resource in the BOM structure. Maes et al. [7] explore the complexity of finding feasible solutions to MLCLSP and present three similar heuristics for the solution. The three heuristics differ in the way they round the binary setup variables, which are obtained from LP relaxation. Roll and Karni [12] present a heuristic approach which consists of the application of eight different subroutines. These subroutines either convert an infeasible solution to a feasible one or improve a given solution. Maes and Van Wassenhove [13] extend their ABC algorithm (Maes and Van Wassenhove [14]) for capacitated single-level lot-sizing problems to multilevel problems with serial BOM structure. Kuik et al. [15] use simulated annealing and tabu search methods where the search neighborhood is defined on the setup variables. They show computationally that these heuristics are effective when compared to Maes et al. [7]. Tempelmeier and Helber [16] address four variants of a two-phase heuristic approach for the problem. Another work later on propose an effective Lagrangian relaxation heuristic (Tempelmeier and Derstroff [17]). In this method, they relax the inventory and the capacity constraints to obtain single-item uncapacitated lot-sizing subproblems. Since efficient *O(nlogn)* algorithm is available for this subproblem, they are able to develop efficient subgradient search algorithm for the overall problem.

A majority of the literature takes a monolithic view of the production planning problem which severely restricts its implementability in multi-facility environments. In the following, we will examine the notion of multilevel multi-facility coordination using this classical model as a basis.

3. Problem Formulation and Reformulations

There have been several formulations for MLCLSP in the literature (Stadtler [18]). We give a formulation of the problem similar to that presented in Tempelmeier and Derstroff [17]. Without loss of generality, we exclude production costs from our formulation.

γ_{kt} : binary setup variable for item k in period t. $\gamma_{kt} = 1$ if $q_{kt} > 0$ and $\gamma_{kt} = 0$ otherwise.

a_{ki} : number of units of item k required to produce one unit of item i.

b_{jt} : available capacity of resource j in period t.

d_{kt} : external demand for item k in period t.

h_k : inventory holding cost for item k.

s_k : setup cost for item k.

J : number of resources.

r_k : the resource where item k is produced

K : number of items.

K_j : set of items that are produced by resource j.

T : number of periods.

M : a large number.

N_k : set of items that are immediate successors of item k in the product structure.

q_{kt} : lot size for item k in period t.

tb_k : production time per unit of item k.

tr_k : setup time of item k.

y_{kt} : inventory of item k at the end of period t.

$z(k)$: deterministic minimal lead time for item k.

The monolithic production formulation is as follows:

(MP)

$$Min \sum_{t=1}^{T} \sum_{k=1}^{K} (h_k y_{kt} + s_k \gamma_{kt})$$

$subject\,to$

$$y_{kt-1} + q_{kt-z(k)} - \sum_{i \in N_k} a_{ki} q_{it} - y_{kt} = d_{kt} \qquad k = 1,2,..,K;\;\; t = 1,2,..,T \qquad (1)$$

$$\sum_{k \in K_j} (tr_k \gamma_{kt} + tb_k q_{kt}) \le b_{jt} \qquad j = 1,2,..,J;\;\; t = 1,2,..,T \qquad (2)$$

$$q_{kt} - M\gamma_{kt} \le 0 \qquad k = 1,2,..,K;\;\; t = 1,2,..,T \qquad (3)$$

$$q_{kt} \ge 0,\; y_{kt} \ge 0,\; \gamma_{kt} \in \{0,1\} \qquad k = 1,2,..,K;\;\; t = 1,2,..,T \qquad (4)$$

The above model is quite common in the lot-sizing literature. The object is minimize inventory holding and setup costs. Constraint set (1) describes the mass-balance relationships between item inventories in the system over time, capacity restriction is forced by constraint set (2), while the production and setup relationship is represented by constraint set (3). This model captures several important aspects of the production planning problems: the BOM structure as characterized by a_{ki} which defines the supply structure required to produce the end-item, the fundamental trade-off between setup and inventory, and the complicating factor of limited capacity.

To explore multi-facility coordination, we reformulate the above model by introducing a new variable, x_{kit}^{r}, as depicted in Figure 1.

We define x_{kit}^{r} as the production output (input) from (to) facility r associated with the product (BOM) link (k,i) in period t. Further, we introduce an *a priori* determined production target $parx_{kit}$ into the model as a parameter. The model including these additional elements is as follows:

(MP$'$)

$$Min \sum_{t=1}^{T} \sum_{k=1}^{K} (h_k y_{kt} + s_k \gamma_{kt})$$

$$subject\,to$$

$$y_{kt-1} + q_{kt} - \sum_{i \in N_k} a_{ki} x_{kit}^{r_k} - y_{kt} = d_{kt} \qquad k = 1,2,..,K; \quad t = 1,2,..,T \qquad (5)$$

$$x_{kit}^{r_i} = q_{it} \qquad \forall (k,i) \in L; \quad t = 1,2,..,T \qquad (6)$$

$$x_{kit}^{r_k} = parx_{kit} \qquad \forall (k,i) \in L; \quad t = 1,2,..,T \qquad (7)$$

$$x_{kit}^{r_i} = parx_{kit} \qquad \forall (k,i) \in L; \quad t = 1,2,..,T \qquad (8)$$

$(2), (3),\ and\ (4).$

Without loss of generality, we assume lead time $z(k)$ to be zero for all items k. In the above reformulation, constraint sets (5) and (6) together represent the mass-balance relation between item inventories in the system. Constraint set (6) represents a just-in-time policy where no component is sent before needed. A simplifying assumption here is that any component that is produced in period t must be available for use in the upper layer during the same period. Parameter $parx_{kit}$ represents a placeholder for the to-be-agreed on flow on product link (k, i) in period t by the associated facilities, i.e. r_k and r_i. We will refer $parx_{kit}$'s as the *target parameters*.

To further explore the essence of production coordination we now introduce the notion of fairness as perceived by facility-centric agents. We define the *fairness of a solution* as how evenly the facilities share the burden of compromising their respective *facility-best solutions* for the *system-feasible solution (any* solution feasible for MP). With these ideas in mind, we introduce the following definitions:

$ddem_{kt}$: Dependent demand for item *i* in period *t* (demand for item *k* corresponding to the demands for end items in period t assuming no initial inventory in the system), where

$$ddem_{kt} = \sum_{i \in N_k} a_{ki} ddem_{it}$$

Dependent demand constraint : Total amount that a facility sends out by period t_m has to be greater than or equal to the total dependent demand by period t_m. Let R be the set of facilities, this constraint is as follows ;

$$\sum_{t=1}^{t_m} \sum_{(k,i) \in L} x_{kit}^{r_k} \geq \sum_{t=1}^{t_m} ddem_{kt} \quad \forall r \in R, \forall k \in K_r, t_m = 1, 2, 3, .., T$$

Facility-best solution: The solution of a facility-based submodel that satisfies its own dependent demand constraints without taking into consideration its supplier or customer facilities' production schedules. Leaving the precise formulation for the facility sub model to later, we define Δ_r for each facility *r* as the value deviation of a *system-feasible solution* from its facility-best solution:

$$\Delta_r = \sum_{t=1}^{T} \sum_{k \in K_r} (h_k y_{kt} + s_k \gamma_{kt}) - (facility - best\, solution\, for\, facility\ r)$$

We then define the average deviation $\overline{\Delta}$ across all facilities :

$$\overline{\Delta} = (\sum_i \Delta_i)/No.\, facilities$$

Given the above definitions, we now define a benchmark "maximum fairness problem" (MFP), which has the goal of minimizing unfairness across facilities.

(MFP)

$$Min \sum_r \delta_r$$

$$s.t.$$

$$\delta_r \geq \Delta_r - \overline{\Delta} \qquad \forall r \in R \qquad (9)$$

$$\delta_r \geq -(\Delta_r - \overline{\Delta}) \qquad \forall r \in R \qquad (10)$$

$$and\, (1), (2), (3),\ and\, (4)$$

In this problem, constraint sets (9) and (10) ensure that the condition $\delta_r = |\Delta_r - \overline{\Delta}|$ is satisfied. Thus, the model basically minimizes the total absolute deviation from $\overline{\Delta}$. Defining a ratio $\alpha_r = \frac{\Delta_r}{\sum_r \Delta_r}$ we can see that MFP also minimizes $\sum_r |\alpha_r - \frac{1}{|R|}|$ where $|R|$ is the cardinality of the facility set. Thus, $\frac{1}{|R|}$ is the value that $\frac{\Delta_r}{\sum_r \Delta_r}$ should take for each facility *r* if a solution is perfectly fair. A related question is what should be the maximum achievable unfairness a solution can ever have. This question can be formulated as follows:

(MFP$'$)

$$Max \sum_r |\alpha_r - \frac{1}{|R|}|$$

$s.t.$

$$\sum_r \alpha_r = 1 \qquad (11)$$

$$\alpha_r \geq 0 \qquad \forall r \in R \qquad (12)$$

As it turns out, for problem (MFP$^{'}$) there is a rather straight forward answer. This is explained in the following proposition.

Proposition: $The\, optimal\, solution\, to\, MFP^{'} is\; one\, where\, one\, of\, the\, \alpha_r's$, $r \in R,\ is\, 1$ $and\, others\, are\, 0$. $The\, value\, of\, the\, optimal\, solution\, is\ (|R|-1) * \frac{1}{|R|} + (1 - \frac{1}{|R|})$.

Proof : Let $b_r = 1$ if $\alpha_r - \frac{1}{|R|} > 0$ and $b_r = -1$ if $\alpha_r - \frac{1}{|R|} \leq 0$. Then we can write the objective function of MFP$^{'}$ as follows ;

$$Max \sum_r b_r(\alpha_r - \frac{1}{|R|}) = Max\Big[\sum_r b_r\alpha_r\Big] + \Big[-\frac{1}{|R|}\sum_r b_r\Big]$$

The maximum value of the first term in the expression above is 1 where one of the α_r's is 1 and the rest of them are 0, which is also the solution that maximizes the second term. This is the optimal solution defined in the proposition. The value of the solution is trivial to figure out.☐

The proposition states that the maximum unfair case is the one where only one of the facilities suffers in deviating from its facility-best solution while all of the remaining facilities achieve their respective facility-best solutions. Following is an obvious result of the proposition .

Corollary: $Value\,of\,the\,optimal\,solution\,to\,MFP'$, $or\;the\,maximum\,unfairness\,a\,solution\,can\,achieve,\;is\;monotonically\,increasing\,in\,|R|\,and\,bounded\,by\,2,\;that\,is$

$$\frac{\partial\left[(|R|-1)*\frac{1}{|R|}\;+(1-\frac{1}{|R|})\right]}{\partial|R|}=\frac{2}{|R|^2}>0$$

and

$$\lim_{|R|\to\infty}(|R|-1)*\frac{1}{|R|}\;+(1-\frac{1}{|R|})=2$$

Instead of using $\sum_r|\alpha_r-\frac{1}{|R|}|$ as the fairness measure, one might consider using relative cost increases above $facility\text{-}best\,solutions$ directly. However, this will lead into the fact that the facility with relatively high-value objective will tend to be favored in the final solution, since we can expect that the cost increase of the facility with the high-value objective will be proportional to its objective value and we will be particularly trying to reduce high deviations in the final solution.

Another alternative one can consider is replacing $\sum_r|\alpha_r-\frac{1}{|R|}|$ with a value representing the deviation of α_r's from one another. For instance, the difference between maximum and minimum α_r values across all facilities or the variance of α_r's. But one might see that if we use such a deviation value to direct the solution process, there is no guarantee to not to end up with a solution that has all the Δ_r values being high, a solution where each facility deviates from its best solution significantly although the deviation is equal across facilities.

4. Problem Decomposition

In this section we examine two different decompositions of the reformulated MLCLSP. The decomposition reveals properties which relate the mathematical structure of monolithic optimization, decentralized decision making, and auction-theoretic coordination. We first examine a more standard Lagrangian decomposition approach for (MP$'$).

Lagrangian Decomposition: Lagrangian decomposition is first proposed by Guignard and Kim [19] as an alternative to Lagrangian relaxation. Here, we first replace constraints (7) and (8) in (MP$'$) by the following constraint:

$$x_{kit}^{r_k} = x_{kit}^{r_i} \qquad \forall (k,i) \in L; \quad t = 1,2,..,T \qquad (7')$$

We then Lagrangian relax this linking constrain (7$'$) between facilities, thus (MP$'$) becomes facility-separable. Denote f_r the objective function of facility r in the facility submodel, and C_r the subset of the remaining constraints (2),(3),(4), and (5) associated with facility r. Then the Lagrangian decomposed model of the problem is expressed as follows:

(LD): $Min\,\{\sum_r f_r \mid C_r,\, r \in R\}$

where $f_r = \sum_{t=1}^{T}\left[\sum_{k\in K_r}(h_k y_{kt} + s_k\gamma_{kt}) + \sum_{\{(k,i)\in L|r^k=r\}}\lambda_{kit}\, x_{kit}^{r_k} - \sum_{\{(k,i)\in L|r^i=r\}}\lambda_{kit}\, x_{kit}^{r_i}\right]$

and the Lagrangian dual problem is as follows:

(LDD) : $\underset{\lambda}{Max}\,\{\,Min\,\{\sum_r f_r \mid C_r,\, r \in R\}\,\}$

Standard method to solve the Lagrangian dual problem (LDD) is subgradient search or dual ascent. From the viewpoint of decentralized decision making, one may draw a connection between the subgradient search method and a price-directive decision making scheme known as *adaptive auction tâtonnement with regular payment function* (Kutanoglu and Wu [5]). Under this auction scheme the center announces the conflict pricing λ in each iteration, while decision makers from each facility solve a facility-subproblem. However, this auction scheme is not without problems. First, as discussed in (Kutanoglu and Wu [5]), a market clearing condition may never be achievable since a non-zero duality gap is likely at convergence. This requires the center to impose a market clearing (feasibility restored) solution. Worse, with linear local objective f_r, each facility subproblem converges to its own extreme point solution regardless of conflict pricing λ. This results in the oscillation where the auction does not even converge to a lower bound solution but oscillate from one facility optimal to another.

To explore this particular auction scheme we implemented a *adaptive auction tâtonnement* similar to that of subgradient search for (LDD). We indeed encounter the problem of oscillation for a significant portion of the test cases. After examining more closely this decentralized implementation of (MP$'$), some addition insights can be offered as to what causes the oscillation: first, the peer-facility imposed demand x_{ijt}'s are not hard constrained except for the facilities producing end items (assuming there is no external component demand). In fact, x_{ijt}'s are determined solely based on the costs and conflict pricing λ_{kit}. As a result, variable x_{ijt}'s take either zero or the maximum value possible. For instance, when x_{ijt}'s represent the outflow of a facility, they would be zero if $\lambda_{kit} \geq h_k$ since sending item k out will be more expensive than holding it. When the converse is true, x_{ijt}'s take the maximum value possible. In conclusion, while Lagrangian decomposition defines a mathematical structure amenable for decentralized decision making at the facility level, its inherent problems concerning duality gap and oscillation suggest that such scheme may be unstable.

Auction-Theoretic Decomposition: To overcome inherent problems associated with the above decision scheme, we develop a new decomposition amenable to an *adaptive auction tâtonnement with augmented payment function*, according to the categorization in

(Kutanoglu and Wu [5]). We first define *inconsistency* as the total mismatch between the solutions of facility submodels *i* and *k*, i.e., $\sum_{(k,i)\in L}\sum_{t}|x_{kit}^{r_k}-x_{kit}^{r_i}|$.

We then redefine the facility objective and constraints, $f_r^{'}$ and $C_r^{'}$, respectively, as follows:

$$f_r^{'} = \sum_{t=1}^{T}\Bigg[\sum_{k\in K_r}(h_k y_{kt}+s_k\gamma_{kt}) + \sum_{\{(k,i)\in L|r^k=r\}}\lambda_{kit}^{r_k}\,|x_{kit}^{r_k}-parx_{kit}| + \sum_{\{(k,i)\in L|r^i=r\}}\lambda_{kit}^{r_i}\,|x_{kit}^{r_i}-parx_{kit}|\Bigg]$$

Define the facility constraint set $C_r^{'}$ as the original set C_r and the *dependent demand constraints* associated with facility r (as defined in Section 3). With this definition of $f_r^{'}$ and $C_r^{'}$, we state the auction-theoretic decomposition as follows:

$$\text{(AD): } \underset{\lambda,\, parx}{Min}\ \{\sum_r f_r^{'} \mid C_r^{'},\ r\in R\}$$

Note that (AD) is facility-separable. Further, any solution to AD with zero *inconsistency* is a feasible solution to the original problem MP. In other words, if an auction is to be designed based on the mathematical structure of (AD), there is now a market clearing condition via price update. There is no longer the need for the center to interfere and impose market clearing solutions. We will now show that (AD) defines a mathematical structure amenable to an auction like coordination mechanism for decentralized decision making.

An Auction-Theoretic Coordination Mechanism: Consider a supply chain environment where production facilities need to coordinate their production planning so as to satisfy end-item customer demand. Consider further that each facility is represented by a self-interest (but truthful) agent who computes its local production plan, x_{kit}^{r}, given the currently announced conflict pricing λ_{kit}^{r}'s and production targets $parx_{kit}$'s. The facility agent negotiate their production plan with one another using an iterative auction scheme. In each iteration, the auctioneer updates the conflict pricing λ_{kit}^{r}'s and *target parameters,*

$parx_{kit}$'s, using current λ_{kit}^{r}'s, $parx_{kit}$'s and current production plan (x_{kit}^{r}). The updated parameters are used by the facility agents in the following iteration.

In addition to concerns on convergence and feasibility, an important aspect for the auction mechanism is that of fairness as characterized in Section 3. Since agents are self-interested, it is no longer reasonable to assume that they will accept just any solution for the "common good." Indeed, the global optimal solution may not be fair to all agents relative to their *facility-best solutions*, and therefore corresponds unstable agreements or simply non-implementable in reality. The fairness consideration is an integral part of the following coordination mechanism.

(The Auction-Theoretic Coordination Mechanism):

1. Each facility agent finds its *facility-best solution* given the current information
2. The center initialize $parx_{ijt}$ to the averages of the *facility-best solutions* as follows:

$$parx_{ijt} = \frac{x_{kit}^{r_k} + x_{kit}^{r_i}}{2}$$

   set all $\lambda_{kit}^{r_k}$'s and $\lambda_{kit}^{r_i}$'s to zero.
3. Each facility agent solves its facility subproblem as defined by (AD) using the current set of $parx_{ijt}$'s, $\lambda_{kit}^{r_k}$'s, and $\lambda_{kit}^{r_i}$'s. Report the production plan x_{kit}^{r}.
4. If the current *inconsistency* is less then a prespecified *threshold* or the *iteration limit* is exceeded, go to 9.
5. The center compute a price scale as follows:

$$price\,scale = [penalty\,ratio] \text{ x } \frac{\sum_{r} f_r}{\sum_{(k,i)\in L} |x_{kit}^{r_k} - parx_{ki}| + |x_{kit}^{r_i} - parx_{ki}|}$$

6. The center updates $parx_{kit}$'s as follows:

$$parx_{kit} = [\,parx_{kit} + (\frac{\alpha_{r_k}^n x_{kit}^{r_k} + \alpha_{r_i}^n x_{kit}^{r_i}}{\alpha_{r_k}^n + \alpha_{r_i}^n})\,]/2 \qquad \forall (k,i) \in L\,;$$

$$t = 1, 2, .., T$$

7. The center updates $\lambda_{kit}^{r_k}$ as follows:

$$\lambda_{kit}^{r_k} = \lambda_{kit}^{r_k} + [price\, scale] \text{ x } |x_{kit}^{r_k} - parx_{kit}| \text{ and}$$

$$\lambda_{kit}^{r_i} = \lambda_{kit}^{r_i} + [price\, scale] \text{ x } |x_{kit}^{r_i} - parx_{kit}|$$

$$\forall (k,i) \in L\,;\, t = 1, 2, .., T$$

8. Adjust the *penalty ratio* depending on the rate of convergence, which is defined as the percentage change in *inconsistency* in the last M number of iterations. Increase iteration counter by one and go to 3.
9. The center has the option of accepting the current solution as is, or alternatively, the center may fix the non-zero setup variables γ_{kt} in the current solution and solve the LP-relaxation of problem (MP), then announce the resulting solution, STOP.

In the coordination mechanism, each agent first finds its *facility-best solution* as a starting point for negotiating the production plan. In each iteration, the auctioneer forces the facility solutions to reduce overall *inconsistency* by updating *conflict pricing* and *target parameters*. The amount of increment in *prices* is proportional to total current inconsistency (step 5) and to individual deviations from *target parameters*, i.e. $|x_{kit}^r - parx_{kit}|$ in step 7.

In updating the *target parameters*, we use both current parameters and current associated solutions in Step 6. Note that we weigh the current solutions according to $\alpha_r = \dfrac{\Delta_r}{\sum_r \Delta_r}$, which measures the amount of "suffering" facility *r* experiences as compared to the average. Thus, the facilities that "suffers" more than the average would have more influence on the new *target parameters.* To further control the degree of this effect, we take the nth power of the weight α_r in step 6.

To allow the auction mechanism to tune its convergence rate, we introduce the *penalty ratio* as a leverage. In step 8, the mechanism may increase or decrease the *penalty ratio* by a fixed amount by observing the convergence up-to-date.

5. Computational Experiments

To submit the above coordination mechanism under rigorous testing, and to gain insights on overall solution quality and fairness across facilities, we conduct intensive computational testing on 300 standard test problems for MLCLSP.

Test problems: We use a subset of standard test problems (set B) in Tempelmeier and Derstroff [17], kindly provided by the authors. This subset consists of 150 problems, 75 of which has the non-cyclic product structure and the other 75 with (potentially) cyclic product structure. Each problem involves 10 items, three facilities A,B, and C, and four periods.

The product structure and the assignment of items to facilities is depicted in Figure 2. The 75 test problems are generated using three end-item demand structures with different coefficient of variation, five combinations of time-between-orders (a factor for the determination of setup costs), and five capacity utilization profiles. The problem generation details can be found in Tempelmeier and Derstroff [17]. Our tested problems are the ones with the "first setup time profile" in their paper.

Comparing different coordination schemes

We implemented four different coordination schemes for comparison: *optimal*, $maximum\, fair$, *coordinated*, and *center-imposed*, these schemes result in four types of production planning solutions. *Optimal* corresponds to the optimal solution of (MP), $maximum\, fair$ is the optimal solution of the model (MFP), *coordinated* corresponds to the solution that the auction-theoretic mechanism converges to (without center-imposed LP solution in Step 9), and *center-imposed* is the center-imposed LP solution in Step 9. To establish comparison, we have define the following two performance measures:

Percent deviation from optimal:

$$dfo = 100 \text{ x} \frac{A\, solution\ -\ Optimal\, solution}{Optimal\, solution}$$

Fairness of a solution across facilities:

$$fos = \sum_r |\alpha_r - 1/No.\, facilities|$$

Clearly the second performance measure is of interest from the viewpoint of implementable production coordination, while the first measure provides benchmarks for the quality of the production plan. Two sets of experiments are run: the first set of experiments is carried out using the regular 150 problems described above. The second set of experiments involve the solution of the same problems except the unit setup and inventory holding costs are multiplied by 5 for items produced in facility B. We will refer this second set of problems as the high-cost problems. This second set of experiments are designed to study the effect of a high-cost facility on the performance of the coordination mechanism. Of particular interest is the effects on solution fairness fos. In both sets of experiments we solved each problem by setting the parameter n=3 in step 6. We use the following parameter setting in step 8 throughout the experiments:

Initial *penalty ratio* = .001

Step size for increasing or decreasing *penalty ratio* = .0015

Iteration interval to check convergence rate = 4 iterations

Upper limit for convergence rate = .15

Lower limit for convergence rate = .02

These parameters are set based on experiences from a small number of pilot problems. The general tendency is that lowering the values of these parameters increases the solution quality while increasing the number of iterations needed for convergence.

Results and analyses

We summarize the computational results in Tables 1-3. Our main focus is on the *coordinated* solution while the other alternatives provide benchmarks from different perspectives of the coordination. The *coordinated* results represent an auction-theoretic coordination scheme for production decisions.

Table 1 provides the deviations from optimal in terms of objective value (dfo). Each cell represents the average over the corresponding 75 problems. The most interesting result the table shows is how far away the maximum fair solution can be from optimal, which averaged 37.15 % for the 300 problems. This also shows the potential gap between classical optimization approach and the distributed decision paradigm. dfo of maximum fair solutions surprisingly does not seem to be dependent on the problem structure even though there are much less number of inter-facility product links for cyclic problems compared to non-cyclic ones. The coordination mechanism performs much better for cyclic problems for the simple reason that there are less number of inter-facility product links to coordinate for these problems compared to non-cyclic ones.

Table 2 gives the fairness measure of the four different schemes. The maximum unfairness for our three-facility problems is $\frac{4}{3}$ according to the Proposition in Section 2. The smaller the figures in this table are, the better the fairness is achieved.

As evident from the table, the *coordinated* scheme outperforms both *center-imposed* and *optimal* in terms of the fairness measure fos in all problem types. Although center-imposed solutions are the solutions that are partially determined by the auction mechanism (the setup variables), they operate similar to that of *optimal*. Again, we see a significant gap between *optimal* and *maximum fair* solutions. It is interesting to note that percentage wise the gap is much wider than the optimality gap observed in Table 1. If we look at the fairness of the optimal solution we see that the optimal is less fair for high cost problems compared to regular ones. This is intuitive since in high cost case, the facility with high costs is more dominant in the optimal solution. There is no significant difference in this manner for the *coordinated* solutions which is in line with how the coordination mechanism is design to accomplish.

In Table 3, we present a comparison between *optimal* and the three coordination schemes in terms of the fairness measure fos. We give the average percentage improvement over the optimal, and the number of cases (out of 75 problems) in which the coordination scheme outperforms the optimal. The *coordinated* scheme outperforms the *optimal* on the average 56.75 out of 75 cases, or 75% of the time. The average improvement of the *coordinated* scheme over the optimal is 28.09%. To put this into perspective, this is accomplished with the sacrifice of 9.95% in solution quality (from optimality) as given in Table 1. As for the *maximum fair* solutions, there is a 95.83% improvement in fairness together with the sacrifice of 37.15% in solution quality. Performance of the *coordinated* scheme appears to be independent of the problem structure. Average improvement of the *coordinated* scheme is more for high cost problems than regular ones.

If we compare *coordinated* solutions and *center-imposed* solutions in Table 3, the former significantly outperform the latter with respect to fairness. This confirms the potential of the auction theoretic coordination scheme since the full effect of the coordination comes without the needs for significant center intervention toward the end.

6. Conclusion

The distributed decision paradigm discussed in this paper has potential to enjoy broad acceptance and success in today's increasingly distributed business environments. Large manufacturing supply chains presents a typical example for such planning environment. In this study, we attacked a classical production planning problem directly related to the issues of supply chain coordination. Using this model, we demonstrated that an auction theoretic mechanism can be constructed to coordinate production planning in such environment. This coordination scheme provides significantly more implementable (fair) solutions then what monolithic optimization is able to provide, paying the price of some solution quality degradation.

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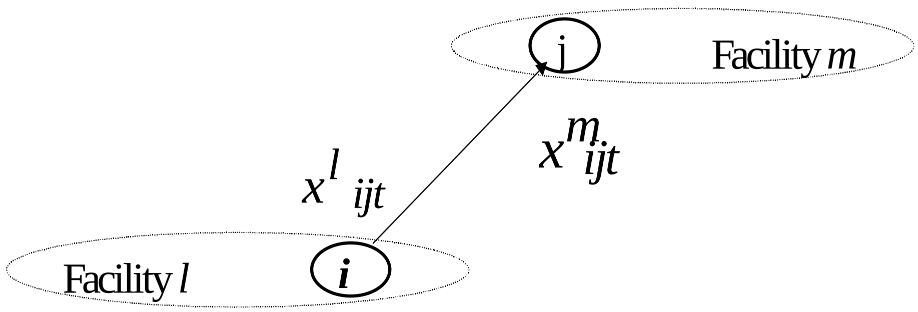

Figure 1. New variable x^r_{kit}.

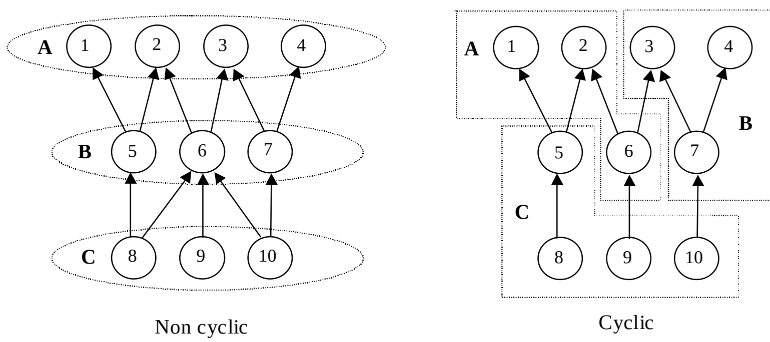

Figure 2 : Problem structures.

Table 1: Deviations from optimal (dfo).

		Center-Imposed sol.	Coordinated sol.	Max. Fair sol.
Non-cyclic	**Regular**	8.19	17.66	39.89
	High cost	9.33	14.93	33.17
Cyclic	**Regular**	2.80	4.24	44.50
	High cost	1.65	2.99	31.04
Averages		5.49	9.95	37.15

Table 2: Fairness of the solutions (fos)

		Center Imposed	Coordinated	Max. Fair	Optimal
Non-cyclic	**Regular**	.6124	.4473	.0041	.5807
	High cost	.5503	.4568	.0924	.6225
Cyclic	**Regular**	.8700	.6977	.0152	.8591
	High cost	.8884	.6778	.0139	.9163
Averages		.7303	.5699	.0314	.7446

Table 3: Comparison with optimal (minimal cost solution) in fos measure.

		Center-Imposed		**Coordinated**		**Max. Fair**	
		Better-than-optimal cases	% Average improve-ment	Better-than-optimal cases	% Average improve-ment	Better-than-optimal cases	% Average improve-ment
Non-cyclic	**Regular**	31	3.05	57	23.36	75	99.28
	High cost	42	25.52	51	29.58	75	87.81
Cyclic	**Regular**	26	10.11	54	31.09	75	98.15
	High cost	21	16.42	65	28.32	75	98.09
Averages		**30**	**13.77**	**56.75**	**28.09**	**75**	**95.83**

Biographies

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