Due-Date Coordination in an Internal Market via Risk Sharing

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Abstract

We investigates the problem of due-date coordination and negotiation between the marketing and manufacturing entities within a make-to-order or engineer-to-order firm. Marketing is concerned about satisfying customers who each have a preferred due-date for his/her order but is willing to compromise in return for price discounts. Manufacturing is concerned about the efficient utilization of capacity and is not willing to offer any given order a higher service levels unless the incurred cost is reimbursed. This reimbursement rewards manufacturing for risk sharing with marketing. Operating in an environment of dynamic order arrivals, we design a Nash game between marketing and manufacturing. Each party quotes a due-date based on a utility function defined by local cost structure, a belief function of job completion times, and agreed penalty when the quoted due-date is missed. We identify properties for each agent's utility function and show that with belief functions such as uniform, Weibull, gamma and Pearson Type V distributions, a unique Nash equilibrium exists. Nevertheless, due to double marginalization the solution (due-date quotation) achieved at Nash equilibrium is never the system optimum. We develop an incentive scheme for marketing and manufacturing in such a way that the system optimum can be achieved at equilibrium. We conduct sensitivity analysis on the transfer payments such that they could be tailored for alternative utilities.

1. Introduction

The escalating pressure for competent and improved customer service has fettered make-to-order manufacturers to investigate ways to satisfy customer demand quickly at a lower cost. The literature on due-date based planning and scheduling typically assume order due-dates to be exogenous and given. In most real life situations, however, due-date determination is negotiable and is typically part of the marketing and sales function. When negotiating due-dates, marketing must consider both the customer's preferences *and* internal constraints such as production capacity. The latter is controlled by the manufacturing division within the firm who sets the pace for production that would ultimately influence the order completion time. In order for the firm to properly integrate due-date quotation and capacity utilization decisions, the marketing and manufacturing decisions must be coordinated. However, managers are typically rewarded based on the performance of their local unit, thus both marketing and manufacturing have incentives to behave according to their own local cost structure. Since the best interests of individual departments rarely coincide with the firm's optimal (profit maximizing) policy, the firm must provide proper incentives for marketing and manufacturing to coordinated.

In this paper, we examine incentive scheme that operates in the above decentralized, cross-functional decision environment. We design a Nash game for due-date quotation where marketing determines the *external due-date* to be quoted for the customer, while manufacturing determines an *internal due-date* for marketing based on its prospect on capacity. The quoted due-dates define a risk-sharing agreement between the two decision entities, which in turn specifies the internal transfers between the two parties when the order is delivered. To determine proper incentives for the local units to align their local decision with the system optimal, we analyze the Nash game and derive a payment scheme between marketing and manufacturing such that system optimal is achieved at Nash equilibrium.

Two areas of literature are directly relevant to our study: marketing and manufacturing coordination, and due-date quotation/negotiation. In the following, we provide a brief overview the literature.

Marketing/Manufacturing Coordination

The need for coordinating marketing and manufacturing decisions has been recognized by researchers for more than two decades (c.f., Davis (1977), Shapiro (1977), Montgomery and Hausman (1986) and Karmarkar and Lele (1989)). In general these research defines the need for marketing and manufacturing coordination in companies producing industrial goods, and discuss the nature of the problem as "necessary cooperation but potential conflict." Areas of coordination includes capacity planning and allocation, forecasting, scheduling, delivery and distribution, quality assurance, cost control, product design and adjunct services. A broad survey of these approaches are reported by Eliashberg and Steinberg (1993).

Porteus and Whang (1991) propose a different approach to the problem by developing an incentive plan that would reward the division managers to act in a system-optimal way. They propose a plan where product managers receive all revenues from the sales, while paying manufacturing manager the realized marginal value of capacity. While this "internal market" induces optimal local behavior, the firm needs to provide subsidies. Kouvelis and Lariviere (2000) present a generalization of the internal market mechanism based on linear transfer payments between functions, i.e., a market maker buys from upstream managers and resell it to downstream managers, where the buying/selling prices are set in such a way that they lead to system optimal actions. Desai (1996) compares three different contracts in a marketing-manufacturing channel faced with seasonal demand. He discusses Stackelberg games under fixed retailer processing rate, fixed manufacturing price, and a general case without variable fixing. Kim and Lee (1998) study optimal coordination strategies for short-term production and marketing decisions. They propose a scheme where manufacturing determines the production volume based on the marginal revenue given by marketing using the previous demand rate. Celikbas *et al.* (1999) investigates coordination mechanisms based on different penalty schemes which enable the firm to match demand forecasts with production. They consider both centralized and decentralized organizational structures and show that by setting appropriate penalty levels the decentralized system could operate similar to the centralized one. In many cases, the work in the contracting and coordination between retailers and manufacturers can be applied to marketing/manufacturing coordination with little modifications. Analytical studies of such problems can be found in Blair and Lewis (1994), Agrawal and Tsay (1998) and Cachon and Lariviere (1999).

Due-Date Quotation Problems and Coordination

The importance of setting reliable job due-dates in make-to-order production system is well recognized in the literature for at least two decades. Most early work on due-date setting uses generalized but ad hoc decision rules. An extensive survey of research regarding traditional due-date setting problems is provided by Cheng and Gupta (1989). A vast majority of this literature do not consider customer preferences when setting due-dates, assuming that any due-date quoted will be accepted. However a recent survey of US manufacturing practices in make-to-order companies by Wisner and Siferd (1995) reveals that in over 60% of the cases customers' specifications and preferences are the main determinator in due-date quotation. The due-date setting problems are typically studied using centralized and monolithic models where the decision regarding due-dates are considered in conjunction with decisions such as capacity utilization, sequencing and scheduling, pricing etc. Some of the recent work include Wein (1991), Duenyas and Hopp (1995), Zijm and Buitenhek (1996), Spearman and Zhang (1999) and Weng (1999).

Lawrence (1994) finds flowtime distribution estimations as the most important factor in achieving competitive due-date quotation between manufacturing, marketing and the customers. He acknowledges that flow time distributions allow the construction of managerially useful tradeoff curves contrasting order completion probabilities and expected tardiness costs with order lead times. Van der Maijden *et al.* (1994) underlines the importance of setting goals as a result of negotiation between departments especially under demand uncertainty. Elhafsi and Rolland (1999) propose a due-date quotation model based on the congestion level of the manufacturing shop floor and the operating cost. Easton and Moodie (1999) discuss a procedure where the manufacturer bids the price and lead time for the customer, and the customer may accept, reject or modify the terms. As a hedging strategy, the manufacturer may bid on other projects. In case more customers accept the bid, some orders will be delayed. Thus, the hedging strategy must balanced potential profits with the tardiness penalty.

Weng (1999) studies the impact of quoted due-dates and order acceptance rates on expected profit. His results apply to cases where the flowtime follows a general phase-type distribution function of the order acceptance rate. Palaka *et al.* (1998) and So and Song (1998) consider customers who are sensitive to quoted due-dates and prices. They propose nonlinear optimization models to find the "jointly" optimal due-date, capacity utilization and price that maximize the firm's profit.

Competitive due-date quotation has been investigated to a limited extend in the literature. Lederer and Li (1997) investigates the competitive equilibrium between multiple buyers (customers) and suppliers (firms) over selecting prices, production rates and scheduling policies. Lead time (thus due-date) represents a function of production rate and scheduling policy, which specifies how arriving jobs are sequenced. In this setting, firms differ in operation costs, mean processing times and processing time variability, while customers are differentiated based on their delay costs. A competitive equilibrium is found when the Kuhn-Tucker conditions for the firm's optimization problem and the market clearing condition are simultaneously satisfied.

Customers who carry delay costs are also considered by Ha (1998). In this setting a G1/G1/1 service queue is assumed where the customers choose the service rates and linear delay costs while the firm sets a price for each customer served. It is shown that when customers choose the service rates based on their local cost structure, the resulting system service rate and arrival rate are always smaller than the optimum due to externalities. The author proposes incentive-compatible pricing consisting of a fixed admission fee, and a variable fee that is proportional to the actual service time. Grout (1996) proposes an incentive-inducing contract between a buyer and a supplier aiming at the timely delivery of orders. In his setting, the buyer dominates the supplier and moves first by selecting an incentive scheme consists of on-time delivery bonus and tardiness penalty. The optimal probability for on-time delivery can be ensured if the supplier responds to the incentive scheme by selecting a flow time allowance that would minimize his own expected cost.

2. Due-Date Quotation in an Internal Market: Model Description

We consider a due-date quotation process between the decision makers of marketing and manufacturing while the customer incentives are exogenous. We define the following sequence of events: (1) the customer places an order with a preferred due-date cd_i , (2) base on their local costs and an (system-imposed) incentive scheme, marketing announces an *external due-date* dd_i*,* and manufacturing announces a promised completion time pc_i (an *internal due-date)*, (3) if $dd_i > cd_i$, marketing must offers marginal discount g_i to the customer; if $pc_i > dd_i$, manufacturing must offers marginal discount n_i to marketing, (4) production occurs and the order is filled, tardiness costs are charged based on the actual completion time and the quoted due-dates dd_i, and pc_i.

The main essence of the due-date quotation procedure is to establish a risk-sharing policy between marketing and manufacturing via an internal market payment scheme. We will now provide further details of the payment scheme which is depicted in Figures 1 and 2. Suppose an initial completion time pc_o can be established for any given order assuming no adjustment to the plant capacity. pc_o provides the first time point at which θ service level is guaranteed (Figure 1). Given the customer preferred due dates (cd_i), marketing quotes a counter due-date dd_i (we assume that $dd_i \geq cd_i$) while offering a discount g_i to the customer for each unit of the quadratic difference $(dd_i - cd_i)^2$. g_i can be viewed as the *external negotiation cost*. Manufacturing quotes a promised completion time pc_i for the customer order while offering a unit discount n_i to marketing for the difference $(pc_i - dd_i)^2$. Manufacturing may choose to offer an improved completion time $(pc_i < pc_0)$, in which case he must pay a premium m_i to expand his capacity (for each unit of $(pc_o - pc_i)^2$). Equivalently, we may view the manufacturing as been *rewarded* n_i (proportional to $(pc_i - dd_i)^2$) for providing an earlier pc_i. Concerning the tardiness penalty, marketing and manufacturing have the understanding that if the order completion time falls before dd_i (region *A* in Figure 2), neither of them will pay additional penalty. If the completion time falls in between dd_i and pc_i, (region *B*), marketing is responsible for a per unit premium a_i. If the completion time falls after pc_i (region *C*), marketing and manufacturing split the responsibility with per unit penalty $(1-\gamma)a_i$ and γa_i, respectively (where $0 \leq \gamma \leq 1$). Marketing and manufacturing both make use of a pubic belief function $\phi(x, pc)$ on the order completion times to compute their expected tardiness penalty.

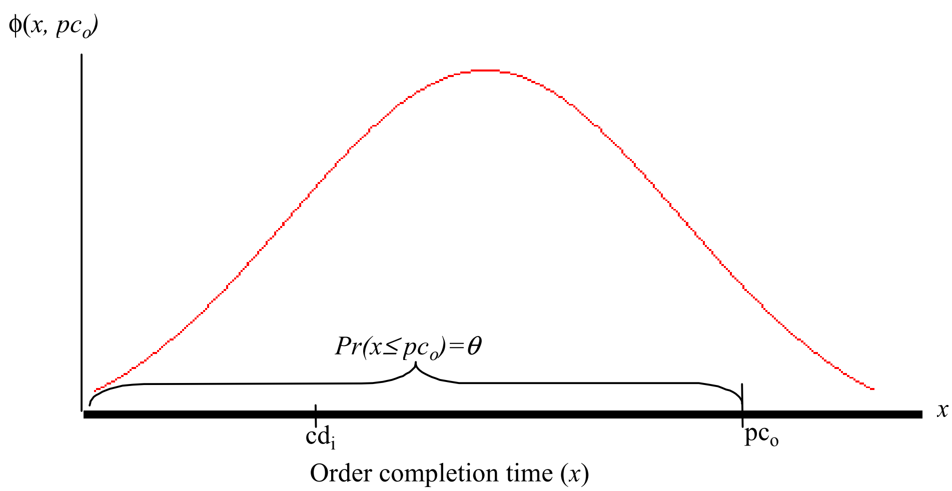

Figure 1. Initial stage and belief function before the actions of the marketing and the manufacturing

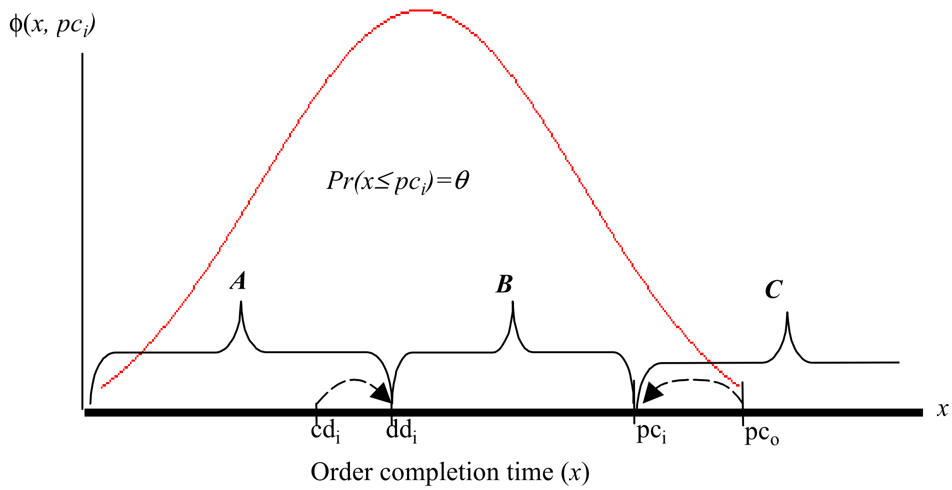

Figure 2. Belief function after parties act

By design, decisions regarding due-date quotation and capacity utilization are closely related. Whilst the due-dates quoted for customers hinges on the "normal" service level utilized by manufacturing, manufacturing is encouraged to make capacity adjustments based on a service quality shaped by the market's requirements for being competitive. This is modeled by the belief function. Let $\overline{c}_i$ be the realized completion time of order i and, ϕ and Φ be the density and distribution functions regarding the belief of the completion time. Both ϕ and Φ are also functions of pc_i. We assume $\Phi(x, pc_i)$ is continuous when $pc_i \geq 0$, 0 when $pc_i < 0$, increasing in x and decreasing in pc_i, and differential for both $x \geq 0$ and $pc_i \geq 0$. In general the scale parameter of the distribution, if exists, can be expressed as a function of pc_i. As pc_i decreases the capacity utilization increases and as capacity increases the value of the cumulative distribution function increases for any x $(0 < x < \infty)$. Moreover, $\Phi(0, *) = 0$, so that completion times are positive and $\Phi(pc_i, pc_i) = \theta$ for all $pc_i > 0$. It is assumed that the expected tardiness, and thus the expected completion time is up-slopping with pc_i in a convex fashion. Moreover, as shown in Figures 1 and 2 the current θ-service-level date (pc_o) is established prior to the game (Figure 1), while the new θ-service-level date is moved to pc_i as a result of change in the *belief function* due to capacity adjustments.

3. The System (Firm's) Optimization Model

The firm's objective is to minimize the total transaction costs due to the due-date concession, capacity adjustments, and expected tardiness costs from the loss of goodwill to the customer. The system cost function is given as follows;

$$G_o = g_i(dd_i - cd_i)^2 + m_i(pc_o - pc_i)^2 + a_i E[[(\overline{c}_i - dd_i)^+]^2]$$

where

$$E[\overline{(c}_i - dd_i)^+] = \int_{dd_i}^{\infty}(x - dd_i)\phi(x, pc_i)dx$$

Note that the penalty due to the deviation between dd_i and pc_i does not influence the system's model since it is an internal transfer between marketing and manufacturing. It is trivial to show that if the expected tardiness is a convex function then G_o is also convex. We can now define the system's optimization model as follows:

$$\boldsymbol{P(1)} \qquad \begin{array}{l} Min \quad G_o \\ \text{s.t.} \\ dd_i \geq cd_i \\ pc_i \leq pc_o \end{array} \tag{1}$$

The first constraint ensures that the customer will not be quoted for a due date that is earlier than the preferred due date and second constraint implies that the capacity utilization cannot be reduced. Let dd_i^o and pc_i^o be the external and internal due date values that minimize the *unconstrained* system cost function. Furthermore, assume pc_o is a positive number large enough so that pc_i is always greater than 0.

THEOREM 1. *Assuming that expected tardiness is convex, dd_i^o and pc_i^o will minimize $P(1)$.*

PROOF. Suppose dd_i^o is less than cd_i. By incrementing dd_i^o by ϵ units the center decreases the external due date negotiation cost since the gap between cd_i and dd_i is reduced. Moreover the expected tardiness cost will decrease since it decreases in dd_i. Because of this increment no additional cost will be observed. Hence dd_i^o cannot violate (1). The value of dd_i^o can be computed using the following equation;

$$dd_i^o = cd_i + \frac{a_i E[[\overline{c}_i - dd_i]^+]}{g_i} \tag{2}$$

Optimum value for pc_i^o can be found by computing the derivative of G_o with respect to pc_i after fixing dd_i and solving it for pc_i. The resulting equation will be as follows;

$$pc_i^o = pc_o - \frac{a_i}{2m_i}\frac{\partial E[([\overline{c}_i - dd_i]^+)^2]}{\partial pc_i} \tag{3}$$

Since the expected tardiness is convex, the last term in (3) is positive. Consequently, $pc_i^o \leq pc_o$ always holds. □

Without specifying a belief function, we can't generate the closed form expression for equations (2) and (3). Even with a given distribution function it may not be tractable to obtain a closed form definition. In this case a simple recursive search can be employed to approximate the closed form in a reasonable time given that G_o is strictly convex. As a result of the theorem, we can drop the constraints from $P(1)$ and use the closed form cost function G_o as the system's optimization model.

4. The Due Date Quotation Game

We now define a Nash game corresponding to the due-date quotation procedure described in Section 2. The game consists of marketing and manufacturing decision makers as independent players. The game, Ω, consists of a single move where the players simultaneously choose their strategies. The strategy space for marketing, σ_1, has a lower bound, cd_i and has no upper bound. Hence, $dd_i \in \sigma_1 = [cd_i, M]$ where M is a large arbitrary constant that will never constraint marketing in her decision. The strategy space for manufacturing, σ_2, is bounded with 0 and pc_o, i.e., $pc_i \in \sigma_2 = [0,\, pc_o]$. Both players have complete information about each others' cost functions and thus, all parameters in the model are common knowledge. The belief function over the order completion time is public (e.g., computed from historic information) and therefore identical for both players.

Let $H_j(dd_i, pc_i)$ denote the player j's expected cost when players adopt the joint strategy of (dd_i, pc_i), j is equal to 1 for marketing and 2 for manufacturing. The best response mapping for player j is a set-valued function corresponding each strategy of player $k\,(k \neq j)$, with a subset of σ_j and formally defined as follows for each player in this game;

$$r_1(pc_i) = \left\{ dd_i \in \sigma_1 \,|\, H_1(dd_i, pc_i) = \underset{x \in \sigma_1}{min}\; H_1(x, pc_i) \right\}$$

$$r_2(dd_i) = \left\{ pc_i \in \sigma_2 \,|\, H_2(dd_i, pc_i) = \underset{x \in \sigma_2}{min}\; H_2(dd_i, x) \right\}$$

In this setting a pure strategy Nash equilibrium is a pair of external and internal due-dates, (dd_i^e, pc_i^e), such that each player chooses a best reply to the other player's equilibrium decision, i.e., $dd_i^e \in r_1(pc_i^e)$, $pc_i^e \in r_2(dd_i^e)$.

4.1. Decision Models for the Players

Marketing is charged for the deviation of negotiated due-date from the customer preferred due-date and the tardiness. If manufacturing completes the order later than his promised date (internal due-date), he shares part of the tardiness penalty (specified by parameter γ) that marketing pays. Marketing also receives a discount from manufacturing for the deviation between pc_i and dd_i. We define the cost function of marketing as follows:

$$H_1(dd_i, pc_i) = g_i(dd_i - cd_i)^2 + a_i \int_{dd_i}^{\infty} (x - dd_i)^2 \phi(x, pc_i) dx$$
$$- \gamma a_i \int_{pc_i}^{\infty} (x - pc_i)^2 \phi(x, pc_i) dx - n_i (pc_i - dd_i)^2$$

where $cd_i < dd_i < M$.

LEMMA 1. *For any* $pc_i \geq 0$, $H_1(dd_i, pc_i)$ *is strictly convex in* dd_i.

PROOF. see the appendix □

Define dd_i^* as the only value that minimizes H_1 for a given pc_i. This value can be found using the equation $H_1^{(1)}(dd_i^*, pc_i) = 0$,

$$r_1(pc_i) = dd_i^* = max(cd_i, \frac{g_i cd_i - n_i pc_i + a_i E[[\overline{c}_i - dd_i]^+]}{g_i - n_i}) \tag{4}$$

Manufacturing is charged for the deviation of promised completion date from the negotiated due-date and the tardiness with respect to the promised completion date. Manufacturing is also charged for capacity increase. We define the cost function for manufacturing as follows:

$$H_2(dd_i, pc_i) = m_i(pc_o - pc_i)^2 + \gamma a_i \int_{pc_i}^{\infty} (x - pc_i)^2 \phi(x, pc_i) dx + n_i (pc_i - dd_i)^2$$

where $0 < pc_i \leq pc_o$. Let $K(y)$ be equal to the following;

$$K(y) = \int_y^{\infty} (x - y)^2 \phi^{(2)}(x, y) dx - 2E[[\overline{c}_i - y]^+]$$

LEMMA 2. *For any* dd_i, $H_2(dd_i, pc_i)$ *is strictly convex in* pc_i.

PROOF. See the appendix. □

Let pc_i^* be the value that minimizes H_2. The following equation gives pc_i^*;

$$r_2(dd_i) = min\left(pc_o, \frac{2m_i pc_o + 2n_i dd_i - \gamma a_i K(pc_i^*)}{2(m_i + n_i)}\right) \tag{5}$$

4.2. Analysis of Equilibria

To investigate the existence of equilibria for the above game, we first introduce a relaxed game $(\overline{\Omega})$ where the strategy spaces of the players are identical and $\overline{\sigma}_1 = \overline{\sigma}_2 = (-\infty, +\infty)$. Players' best response mappings for the relaxed game can be rewritten as follows:

$$\overline{r}_i(pc_i) = \overline{dd}_i^* = \frac{g_i cd_i - n_i pc_i + a_i E[[\overline{c}_i - dd_i]^+]}{g_i - n_i}$$

$$\overline{r}_2(dd_i) = \overline{pc}_i^* = \frac{2m_i pc_o + 2n_i dd_i - \gamma a_i K(\overline{pc}_i^*)}{2(m_i + n_i)}$$

The structure of the best response mappings enables us to reach at the following conclusion;

LEMMA 3. *If* $pc_i \leq cd_i$ *then* $\overline{r}_1(pc_i) > cd_i$

PROOF. see the appendix □

Observe that in the relaxed game marketing is charged for negative deviation of dd_i from cd_i and manufacturing is charged for positive deviation of pc_i from pc_o (this is the main difference of the relaxed game from the original version). The foregoing lemma implies that in the relaxed game if manufacturing promises an internal due-date that is smaller than the customer's preferred delivery date, marketing will respond with an external due-date that is greater than the customer preferred delivery date. This behavior (from marketing) is intuitive as manufacturing is charged by marketing based on the deviation between pc_i and dd_i. From the marketing's viewpoint, quoting a due-date smaller than cd_i in such a case would increase the price discount to the customer, but the expected tardiness cost would regulate this deviation and thus the amount of payment from manufacturing to marketing. Hence the cost will increase while the revenue decreases. A positive deviation from cd_i, on the other hand, will increase the payment from manufacturing (to marketing) and decrease the expected tardiness cost even though it will increase the price discount for the order. This result is valid for any belief function that satisfies the assumptions mentioned earlier.

Based on the implicit function theorem, the derivatives $\overline{r}_1^{(1)}(pc_i)$ and $\overline{r}_2^{(1)}(\mathrm{dd}_i)$ can be given as follows;

$$\overline{r}_1^{(1)}(pc_i) = -\left(\frac{\partial^2 H_1}{\partial dd_i \partial pc_i} / \frac{\partial^2 H_1}{\partial dd_i^2}\right) = \frac{a_i \frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i} - n_i}{g_i + a_i \overline{\Phi}(dd_i, pc_i) - n_i}$$

$$\overline{r}_2^{(1)}(dd_i) = -\left(\frac{\partial^2 H_2}{\partial pc_i \partial dd_i} \Big/ \frac{\partial^2 H_2}{\partial pc_i^2}\right) = \frac{2n_i}{2m_i + \gamma a_i K^{(1)}(pc_i) + 2n_i}$$

It is obvious that $0 < \overline{r}_2^{(1)}(dd_i) < 1$. As $g_i > n_i$, the denominator in the right hand side of the first equation is positive and since expected tardiness increases in pc_i for a given dd_i, $\frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i} > 0$ which implies that $\overline{r}_1^{(1)}(pc_i)$ is negative if $n_i > a_i \frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i}$ and non-negative otherwise. Moreover if

$$g_i / a_i > \frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i} - \overline{\Phi}(dd_i, pc_i) \tag{6}$$

then $\overline{r}_1^{(1)}(pc_i) < 1$. In general, if the upper bound for the right hand side of (6) is close to 0 then $\overline{r}_1^{(1)}(pc_i)$ will always be less than 1 unless the external negotiation cost is too small compared to the expected tardiness cost. Since the cost functions of the players are strictly convex, each player has a unique best response to the other's strategy. We now state the existence of a Nash equilibrium for the original due-date quotation game.

THEOREM 2. *There exists at least one Nash equilibrium in* $\overline{\Omega}$. *Furthermore, given that (6) holds for all positive values of* dd_i *and* pc_i, *the equilibrium is unique, and there exists a unique Nash equilibrium for the original game,* Ω.

PROOF. We know from Osborne and Rubinstein (1994) that if the set of actions of each player is a non empty compact convex subset of a Euclidean space and each player's cost function is continuous and quasi-convex, then there exists a pure strategy Nash equilibrium. By Lemmas 1 and 2 along with our assumptions regarding the belief function, these conditions are met for Ω and thus, for the relaxed game there is at least one equilibrium. Let $(\overline{dd}_i^e, \overline{pc}_i^e)$ be the strategy pair at any equilibrium for $\overline{\Omega}$ and $(\overline{dd}_i^n, \overline{pc}_i^n)$ another strategy pair at another equilibrium for the same game. If $\overline{dd}_i^e > \overline{dd}_i^n$ then $\overline{pc}_i^e > \overline{pc}_i^n$ and $\overline{dd}_i^e - \overline{dd}_i^n > \overline{pc}_i^e - \overline{pc}_i^n$ since $0 < \overline{r}_2^{(1)}(dd_i) < 1$. Assuming (6) holds we know that $-\infty < \overline{r}_1^{(1)}(pc_i) < 1$ which implies that when $-\infty < \overline{r}_1^{(1)}(pc_i)$ if $\overline{pc}_i^e > \overline{pc}_i^n$ then $\overline{dd}_i^e < \overline{dd}_i^n$ should hold. This is a contradiction. Furthermore when $r_1^{(1)}(pc_i)$ is positive and it is less than 1, if $\overline{pc}_i^e > \overline{pc}_i^n$ then $\overline{dd}_i^e > \overline{dd}_i^n$ and thus $\overline{dd}_i^e - \overline{dd}_i^n < \overline{pc}_i^e - \overline{pc}_i^n$ must be true. However this causes another contradiction. Hence, given the foregoing assumptions the equilibrium for $\overline{\Omega}$ is unique.

Let (dd_i^e, pc_i^e) be the strategy pair at the equilibrium for Ω. With the same assumptions, if at equilibrium in $\overline{\Omega}$, $\overline{dd}_i^e \geq cd_i$ and $\overline{pc}_i^e \leq pc_o$, then $dd_i^e = \overline{dd}_i^e$ and $pc_i^e = \overline{pc}_i^e$. The equilibrium will be unique since the all foregoing conditions are also valid for game Ω for $dd_i \geq cd_i$ and $pc_i \leq pc_o$ (*i.e.* the cost functions are continuous and convex, $0 < r_2^{(1)}(dd_i) < 1$ and $-\infty < r_1^{(1)}(pc_i) < 1$ etc.).

If $\overline{dd}_i^e < cd_i$, from Lemma 3, $\overline{pc}_i^e > cd_i$. Let $b = cd_i - \overline{dd}_i^e$ and δ, η some parameters such that $0 < \delta < 1$ and $0 < \eta < 1$. Since dd_i is convex in $r_1(pc_i)$ and constrained by cd_i in the original game, marketing will have to increase her decision by b units to cd_i. As a response, manufacturing will increase pc_i by δb. The best response of marketing to this action is to move her decision either to $\overline{dd}_i^e + \delta\eta b$ or to some value that is less than $\overline{dd}_i^e$. Either points are less than cd_i and therefore marketing can't move. If marketing doesn't move, the manufacturing will not move either. Consequently at the only equilibrium, $dd_i^e = cd_i$ and $pc_i^e = \overline{r}_2(cd_i)$. By using the same approach it can be shown that if $\overline{pc}_i^e > pc_o$ then $pc_i^e = pc_o$ and $dd_i^e = \overline{r}_1(pc_o)$.

Finally, we conclude that if the equilibrium for $\overline{\Omega}$ is unique under the foregoing assumptions then there is a unique equilibrium for Ω as well. □

Next, we show that the total cost of the system in Ω is higher than the system's optimal solution.

THEOREM 3. *Assuming $\gamma < 1$, system's optimal solution is never a Nash equilibrium in Ω*

PROOF. Note that G_o is strictly convex. Hence, the only due date values that minimize G_o are dd_i^o and pc_i^o. If these values are equal to dd_i^e and pc_i^e respectively, then the system's solution is an equilibrium. Due to the fact that dd_i^o is strictly greater than cd_i and pc_i^o is strictly less than pc_o, if the equilibrium in Ω is observed at boundaries (*i.e.,* $dd_i^e = cd_i$ or $pc_i^e = pc_o$), the system optimal solution can't be the equilibrium. For the case where equilibrium resides within the boundaries the following equalities should hold,

$$dd_i^o = dd_i^e = cd_i + \frac{a_i}{g_i} E[[\overline{c}_i - dd_i^o]^+] = \frac{g_i cd_i - n_i pc_i^o + a_i E[[\overline{c}_i - dd_i^o]^+]}{g_i - n_i} \tag{7}$$

$$pc_i^o = pc_i^e = pc_o - \frac{a_i}{2m_i} \frac{\partial E[([\overline{c}_i - dd_i^o]^+)^2]}{\partial pc_i} = \frac{2m_i pc_o + 2n_i dd_i^o - \gamma a_i K(pc_i^*)}{2(m_i + n_i)}$$

Verify that equality (7) can only hold when $dd_i^o = pc_i^o$. Let $x = E[[\overline{c}_i - pc_i^o]^+]$ and $y = \frac{\partial E[([\overline{c}_i - pc_i^o]^+)^2]}{\partial pc_i}$. The second equality above can be rewritten as follows;

$$pc_i^o = pc_o - \frac{a_i}{2m_i} y = pc_o - \gamma \frac{a_i}{2m_i}(y - 2x)$$

Thus, $y = \gamma y - 2\gamma x$.

In this equation x is the expected tardiness and y is the derivative of the expected value of squared tardiness. We know that both terms are non-negative. Since $\gamma < 1$, foregoing equation is impossible to hold implying that $pc_i^o \neq pc_i^e$ and thus, $dd_i^o \neq dd_i^e$. □

The system optimal solution may be a Nash equilibrium in Ω only if manufacturing is charged a_i for unit tardiness from pc_i (i.e., $\gamma = 1$) and the expected tardiness is zero(!).

5. Coordinating Marketing and Manufacturing Decisions

In Theorem 3, we show that competition degrades system efficiency in the due-date quotation game. A coordination mechanism that gives proper incentives for the players to cooperate can lead to overall lower costs. Thus, the goal of the firm is to specify a profit sharing scheme that distributes additional revenue across the marketing and manufacturing divisions in such a way that it obliterates any incentives to deviate from the system's optimal solution. To achieve this, the share of each department in total cost can be devised based on all cost components in the system including the internal and external due-dates quoted by the players and their deviation, the deviation between the external due-date and the customer's preferences, and the expected tardiness for both divisions. Suppose the center distributes a fixed proportion of the revenue gained from the sale of order i among marketing and manufacturing. Let L denote this amount, $L_1 - T$ the marketing's share and $L_2 + T$ the manufacturing's share. Moreover, suppose L_1 and L_2 are constants and their summation is L while T is a function defined as follows;

$$T = \beta_1(dd_i - cd_i)^2 + \beta_2 \int_{pc_i}^{\infty} (x - pc_i)^2 \phi(x, pc_i)dx + \beta_3 \int_{dd_i}^{\infty} (x - dd_i)^2 \phi(x, pc_i)dx + \beta_4(pc_o - pc_i)^2 + \beta_5(pc_i - dd_i)^2$$

T can be looked as a transfer payment to manufacturing stipulated for marketing. We must determine the set of contracts, (i.e., the *value ranges* for the coefficients in T) such that the Nash equilibrium solution coincides with the optimal solution. No sign restrictions are set for the coefficients and a negative value for a coefficient represents a payment from manufacturing to marketing. After the transfer payments, cost functions of the players become as follows:

$T_1 = H_1 + T$ and $T_2 = H_2 - T$

First, we assume that T_1 is convex in dd_i for a given pc_i and T_2 is convex in pc_i for a given dd_i. Next we determine the allotments in which dd_i^o satisfies marketing's first order condition and pc_i^o satisfies manufacturing's first order condition. Afterwards, we need to determine the subset of these allotments that also satisfy the convexity conditions. We first write the first order conditions for the players' cost functions that have been refined with the transfer payments. Then determine the coefficient values with which the first order conditions are met at dd_i^o and pc_i^o.

$$\frac{\partial T_1(dd_i^o, pc_i^o)}{\partial dd_i} = 2(g_i + \beta_1)(dd_i^o - cd_i) - 2(a_i + \beta_3)E[[\overline{c}_i - dd_i^o]^+] + 2(n_i - \beta_5)(pc_i - dd_i^o) = 0 \tag{8}$$

$$\frac{\partial T_2(dd_i^o, pc_i^o)}{\partial pc_i} = 2(\beta_4 - m_i)(pc_o - pc_i^o) + (\gamma a_i - \beta_2)K(pc_i^o) - \beta_3 \frac{\partial E[([\overline{c}_i - dd_i^o]^+)^2]}{\partial pc_i} + 2(n_i - \beta_5)(pc_i^o - dd_i^o) = 0 \tag{9}$$

Solving (2), (3), (8) and (9) for new cost coefficients yields the following equations in three unknowns:

(i) $\quad \beta_1 = \dfrac{g_i}{a_i}\beta_3$ (10)

(ii) $\quad \beta_2 = \gamma a_i$ (11)

(iii) $\quad \beta_4 = m_i + \dfrac{m_i}{a_i}\beta_3$ (12)

(iv) $\quad \beta_5 = n_i$ (13)

As a last step we need to ensure that cost functions of the players are still strictly convex.

THEOREM 4. *Assuming the equations (10-13) there exists a Nash equilibrium corresponding to the optimal solution for the new cost settings if the following inequality holds*
$-a_i < \beta_3 < 0$

PROOF. Following is the second derivative of T_2 with respect to pc_i for any contract that satisfies equations (10-13)

$$\frac{\partial^2 T_2(dd_i, pc_i)}{\partial pc_i^2} = -\beta_3\left(2\frac{m_i}{a_i} + \frac{\partial^2 E[([\overline{c}_i - dd_i]^+)^2]}{\partial pc_i^2}\right)$$

The term inside the parenthesis is positive since the squared expected tardiness is convex in pc_i. To have a convex cost function for manufacturing, β_3 should be negative. Given this, lets also write the second derivative of the marketing's cost function with respect to dd_i

$$\frac{\partial^2 T_1(dd_i, pc_i)}{\partial dd_i^2} = 2(a_i + \beta_3)\left(\frac{g_i}{a_i} + 1 - \Phi(dd_i, pc_i)\right)$$

To have a strictly convex T_1, β_3 needs to be greater than $-a_i$. Hence, in order to guarantee the existence of a Nash equilibrium $-a_i < \beta_3 < 0$ should hold. This inequality implies that $-g_i < \beta_1 < 0$ and $0 < \beta_4 < m_i$. Thus, the conditions for the existence of a Nash equilibrium is satisfied. □

These contracts bring about the following results;

$$\underline{r}_1^{(1)}(pc_i) = -\left(\frac{\partial^2 T_1}{\partial dd_i \partial pc_i} / \frac{\partial^2 T_1}{\partial dd_i^2}\right) = \frac{\frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i}}{g_i/a_i + \overline{\Phi}(dd_i, pc_i)} \tag{14}$$

$$\underline{r}_2^{(1)}(dd_i) = -\left(\frac{\partial^2 T_2}{\partial pc_i \partial dd_i} / \frac{\partial^2 T_2}{\partial pc_i^2}\right) = \frac{2\frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i}}{2m_i/a_i + \frac{\partial^2 E[([\overline{c}_i - dd_i]^+)^2]}{\partial pc_i^2}} \tag{15}$$

If it is guaranteed that for any positive values of dd_i and pc_i $0 < \underline{r}_1^{(1)}(pc_i) < 1$ and $0 < \underline{r}_2^{(1)}(dd_i) < 1$, from Theorem 2, it can be concluded that optimal solution is the unique equilibrium.

With these contracts the cost of deviation between division due-dates are eliminated. Also, instead of paying a penalty for his own tardiness, manufacturing shares the whole tardiness penalty with the marketing specified by the selection of the coefficients according to equations (10-13). According to these contracts both parties share all the cost. Since β_3 cannot be equal to 0 or $-a_i$ (hence, $\beta_1 \neq 0$ or $-g_i$ and $\beta_4 \neq 0$ or m_i) no cost entry is solely charged to one division. Specifically, the proportions of the total external negotiation cost and the total capacity increment cost that are allocated to a division must be equal. If $\beta_3 = -a_i/2$, cost for each entry is equally shared by the departments. Moreover, if $L_1 = L_2$ then the revenue is also shared evenly and as a result departments make the same profit at equilibrium. However, as β_3 increases, the marketing's share in cost increases and the opposite occurs as β_3 decreases.

These contracts achieve the coordination for any belief function that has a convex expected tardiness function. In order for these contracts to be useful in coordination, the game doesn't need to have a unique equilibrium. Even if there exist additional Nash equilibria, the one coinciding the optimal solution Pareto dominates the other. It has been observed by previous researchers that players tend to choose to coordinate on a Pareto dominant equilibrium (Cooper *et al.* 1989). As a result of implementing one of these contracts one division's cost may increase although total cost declines and that division may not be willing to participate in the contract. To overcome this complication, a fixed fee that is independent of all other costs and actions can be paid to this division or one can also look for contracts such that each department's cost is no greater than in the original Nash equilibrium.

6. A Case Study: When Weibull(α,λ) Distribution for Belief Function

In practice, capturing the flowtime distribution (the believe function) may be difficult especially in complex production and/or service environments. Forecasting method may be employed and it can be as straightforward as calculating the mean and some higher moments based on the estimations provided by seasoned production managers, or schedulers. In such cases, the believe function can be fit to well-known distributions such as Normal, Lognormal, Erlang, Weibull, etc. In the following, we illustrate our main results using a Weibull belief function. This provide insights for broader cases since Weibull includes the Exponential and the Rayleigh distributions as special cases. For the shape parameter in the neighborhood of 3.6, it is similar in shape to a Normal distribution, and with the shape parameter greater than that value for some skewness value ranges it closely resembles Pearson Type VI and lognormal distributions (Johnson *et. al.* 1994).

6.1. Modeling the Capacity Adjustment

We assume that ϕ and Φ are the Weibull density and distribution functions regarding the belief of the completion time. Particularly we assume any Weibull distribution with a shape parameter, $\alpha > 1$. Let λ be the inverse of the scale parameter at the beginning for the (nominal) belief function and z the fractional increase in λ's value representing the increment in capacity utilization. Consequently, with the increase in z, which implies a decrease in the expected completion time, the resulting scale parameter will be $1/(\lambda(1+z))$. By solving $\Phi(pc_i, pc_i) = \theta$ for $1/\lambda(1+z)$ we find the following equality:

$$1/\lambda(1+z) = \frac{pc_i}{(-ln(1-\theta))^{1/\alpha}}$$

Hence, we can use the right hand side of the equation as the scale parameter of the Weibull distribution in our calculations. Consequently, the mean flow time $\mu(pc_i)$ can be written as follows;

$$\mu(pc_i) = \frac{pc_i}{\alpha(-ln(1-\theta))^{1/\alpha}}\Gamma\left(\frac{1}{\alpha}\right) \tag{16}$$

Observe that, in this setting, the mean flow time is linearly increasing in internal due-date. At the beginning of the game where $z = 0$, the nominal θ-service-level date, pc_o ($pc_o = (-ln(1-\theta))^{1/\alpha}/\lambda$), is observed *a priory* as stated before.

Next, based on the foregoing model for the capacity adjustment we analyze the behaviors of the divisions under equilibria and investigate circumstances guaranteeing a unique Nash equilibrium.

6.2. Analysis of the Game with Weibull Belief Functions

As pointed out earlier, all parties in the firm share the same belief function regarding the completion date of an order including the central management. First we show that the system's cost function is convex with Weibull belief function.

PROPOSITION 1. *Assuming Weibull belief function, system's cost function is convex.*

PROOF. See the appendix. □

This proposition leads us to notice the following consequence;

COROLLARY. *Assuming Weibull belief function, from lemmas 1 and 2 and Theorem 2 cost functions of the marketing and manufacturing divisions are convex and there exists at least one Nash equilibrium in the game.*

One of the most useful results of assuming Weibull distribution is that the best response function of manufacturing in $\overline{\Omega}$ has a closed form definition due to the fact that $K(pc_i)$, is linear in pc_i (i.e., $K(pc_i) = k{*}pc_i$) for any shape and scale parameters, it is increasing in pc_i and takes non-negative values if $pc_i \geq 0$. k is a positive constant whose value depends on the shape parameter, α, and the service level, θ. With k being constant and positive, tardiness penalty for manufacturing increases in pc_i. In other words, manufacturing's expected tardiness cost decreases as the capacity increases. This implies that even though the probability of the order being tardy with respect to the internal due-date is the same for any given pc_i, the expected squared tardiness is up-sloping in pc_i. Note that the expected squared tardiness in pc_i with respect to pc_i is convex and thus, the foregoing observation is intuitive. We can rewrite the best response function of manufacturing as follows;

$$\overline{r}_2(dd_i) = \frac{2m_i pc_o + 2n_i dd_i}{2m_i + 2n_i + \gamma a_i k} \tag{17}$$

The next proposition states the conditions for guaranteeing unique Nash equilibrium in both games. For the following proposition let

$$l_1 = \mu^{'}(pc_i) - \overline{\Phi}(pc_i, pc_i) = \frac{\Gamma\left(\frac{1}{\alpha}\right)}{\alpha(-ln(1-\theta))^{1/\alpha}} - (1-\theta)$$

PROPOSITION 2. *Assuming Weibull distribution for the belief function and,* $g_i/a_i > l_1$, *there exists a unique Nash equilibrium for* Ω.

PROOF. See the appendix. □

Note that if $\alpha \geq 1$ and $\theta \geq 1 - e^{-1}$, then $l_1 < 1$ (e.g. for $\alpha = 1$ and $\theta = 1 - e^{-1}$, $l_1 = 1 - e^{-1}$) . In Theorem 3 and 4 it has been shown that the Nash equilibrium solution does not coincide with the system optimal, while the proposed transfer payment T can achieve coordination. Next proposition puts forward the relationships between game parameters that are sufficient to guarantee that the equilibrium is unique after the coordination. Let

$$l_2 = \frac{1}{\alpha(-ln(1-\theta))^{1/\alpha}}\left(\Gamma\left(\frac{1}{\alpha}\right) - \frac{2\Gamma\left(\frac{2}{\alpha}\right)}{(-ln(1-\theta))^{1/\alpha}}\right) \tag{18}$$

PROPOSITION 3. *Assuming any Weibull distribution,* $g_i/a_i > l_1$ *and* $m_i/a_i > l_2$ *and employing equations (10-13) there exists a unique Nash equilibrium after the coordination for*

$$-a_i < \beta_3 < 0$$

PROOF. See the appendix. □

7.3. Numerical Results

To further illustrate the above analysis, we applied our model to numerical examples that are slight variations of each other. We consider three special cases of Weibull Belief Function which are namely, Exponential and Rayleigh distributions, and Weibull Distribution for $\alpha = 3$. Following parameter values are employed for all cases.

$$\begin{array}{ll} cd_i = 7 & \theta = 0.85 \\ pc_o = 15 & \gamma = \{0, 0.5, 1\} \\ a_i = 10 & m_i = 8 \\ g_i = 6 & n_i = \{1 - 6\} \end{array}$$

In exponential case, the optimal external and internal due-dates values that minimize the system cost function are 9.6 and 10.7 time units, respectively, and the corresponding optimal cost is 401.3. We have investigated the equilibrium behaviors of the players employing different values for the marginal internal negotiation cost, $(n_i = 1 - 5)$ and the proportion of tardiness cost share of manufacturing division ($\gamma = 0, 0.5, 1$). Notice that these parameters don't take part in system's cost function however effect the local cost structure of the players. Hence, the equilibrium decisions, and thus the deviation from the system optimal vary for different values of these parameters. In this setting, $l_1 = 0.43$ and $l_2 = -0.16$. Therefore, there exists unique equilibrium both for pre-coordination and post-coordination games. Some of the outcome of both the relaxed version and the original games for the example setting, including the decision values at equilibrium, the system costs corresponding to this values and their percentage deviations from the system optimum are summarized in Tables 1 and 2.

Table 1. Equilibrium Values for $\alpha = 1$ and $n_i = 1 - 3$

	$n_i = 1$			$n_i = 2$			$n_i = 3$		
	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
Game	Ω	Ω	Ω	Ω	Ω	Ω	Ω	Ω	Ω
dd_i^e	10.9	10.5	10.1	10.3	10	9.7	9.6	9.4	9.2
pc_i^e	14.5	13.3	12.3	14.1	13	12	13.5	12.6	11.7
$G_o(dd_i^e, pc_i^e)$	591.3	489.5	433.7	550.7	470.1	426.0	517.2	455.1	421.1
Percentage Dev.	47	22	8	37	17	6	29	13	5

Table 2. Equilibrium Values for $\alpha = 1$ and $n_i = 4 - 6$

	$n_i = 4$			$n_i = 5$			$n_i = 6$		
	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
Game	Ω	Ω	Ω	Ω	Ω	Ω	Ω	Ω	Ω
dd_i^e	8.8	8.7	8.7	7.9	8	8.1	7	7.2	7.4
pc_i^e	12.9	12.1	11.4	12.3	11.6	11	11.6	11	10.7
$G_o(dd_i^e, pc_i^e)$	493.8	446.5	420.5	483.7	447.1	426.2	491.1	460.4	441.2
Percentage Dev.	23	11	5	21	11	6	22	15	10

The results indicate that the both the internal negotiation cost and the cost sharing effect the outcomes in certain ways. In the example, it is observed that the gap between the game's outcome and the system optimal decreases significantly as manufacturing's share in tardiness cost grows. This pattern is intuitive and consistent with what the coordination mechanism suggests. As explained before, it simply indicates that to achieve the coordination, all cost components in the system should be shared across the departments. Basically, if manufacturing is not responsible for the tardiness penalty there is not much incentive to improve the capacity which leads the system cost to deviate from the optimal. As the share increases and since the expected tardiness cost increases in pc_i, manufacturing is forced to shorten the internal due-date. However it looks like that the effect of cost sharing is not significant on the marketing's decision.

The decisions of the players and thus, the percentage deviation from optimum also noticeably vary with different internal negotiation costs. In general, upto a certain point (i.e. $n_i = 5$) both dd_i and pc_i values at equilibrium along with the percentage deviation of the system's cost in the game from the optimal decrease in n_i. Also note that dd_i increases in pc_i (i.e. $0 < \overline{r}_1^{(1)}(pc_i) < 1$). After this point, where $\overline{r}_1^{(1)}(pc_i)$ becomes negative, pc_i continues to decrease, however, dd_i and the deviation from optimal cost increases in n_i. Basically, beyond that point the effect of internal negotiation cost dominates the other determinants in the local cost structure of the marketing so that this division counter intuitively quotes earlier external due-dates as pc_i increases so as to amplify the payment received from manufacturing that is proportional to the deviation between dd_i and pc_i. Figures 3 and 4 show the response curves of the parties along with equilibrium and system optimal points for both cases.

The mentioned phenomenon is more obvious for Rayleigh and other Weibull distributions with higher (lower) values of the shape parameter, α, (variance). For Rayleigh case it is detected that $\overline{r}_1^{(1)}(pc_i)$ becomes negative quickly with respect to exponential distribution case. The system cost deviation starts to increase with n_i when $n_i = 2$ (Figure 5). While for $\gamma = 0.5$ and $n_i = 2$ the deviation is 14%, it jumps up to 37% and 58% when n_i is 4 and 6 respectively. The increase is more significant when $\alpha = 3$. Namely, the deviation increases from 37% to 75% when the value of n_i is increased from 2 to 4 and it is 128% when $n_i = 6$. In general, if the marginal internal negotiation cost is high enough (in the last two cases when $n_i > 2$), marketing will not negotiate with the customer and set its external due-date equal to the customer's preferred delivery date (Figure 6). Basically, by this, marketing passes the burden for the tardiness cost to manufacturing indirectly through the internal negotiation cost (see equation (4)). The solutions hint that having too small or too large internal negotiation cost increases the influence of externalities and one should expect higher gap between the equilibrium and system optimal costs. We also suspect that as the uncertainty in the system decreases the outcome of the competition is more degrading on the system performance hence the need for coordination is more obvious, however, more costly.

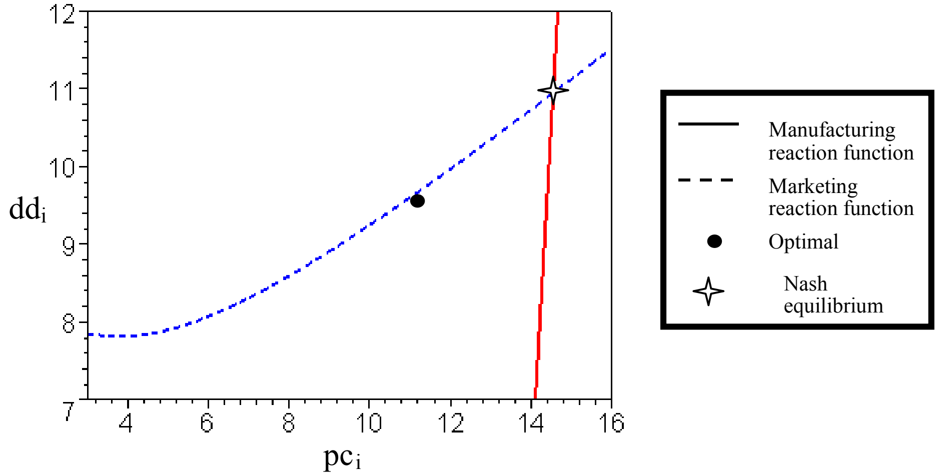

Figure 3. Reaction functions for $\alpha=1$, $n_i=1$, and $\gamma=0$

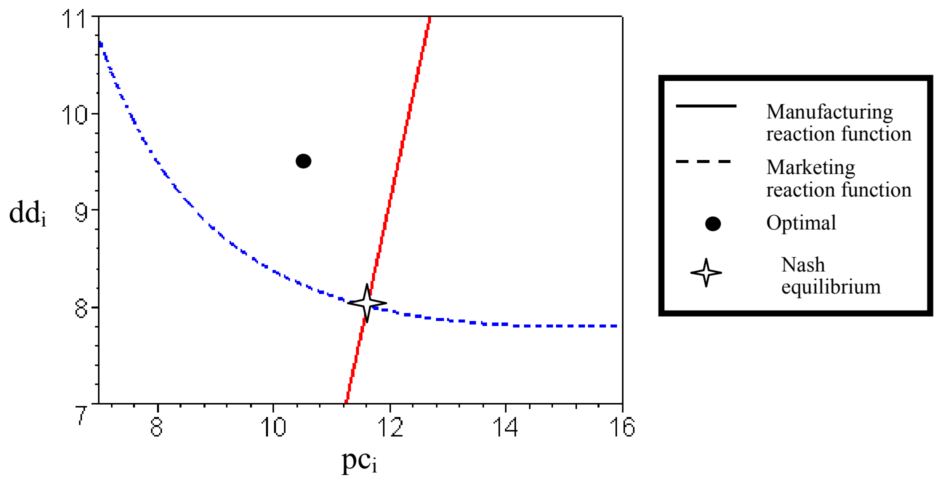

Figure 4. Reaction functions for $\alpha=1$, $n_i=5$, and $\gamma=0.5$

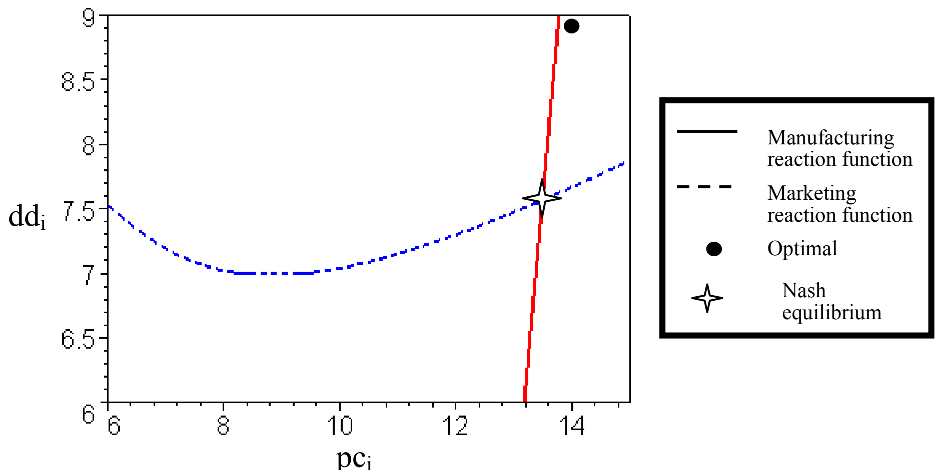

Figure 5. Reaction functions for α=*2, n_i=2,* and γ= *0.5*

To achieve coordination in this specific setting, the value of β_3 in the proposed transfer payment should range between 0 and $a_i = 10$. In particular, so as to make the coordination more appealing for the departments it is better to seek a value such that each party's cost is no greater than in the original equilibrium. Divisions' gain, S_i, through the coordination can be expressed as follows;

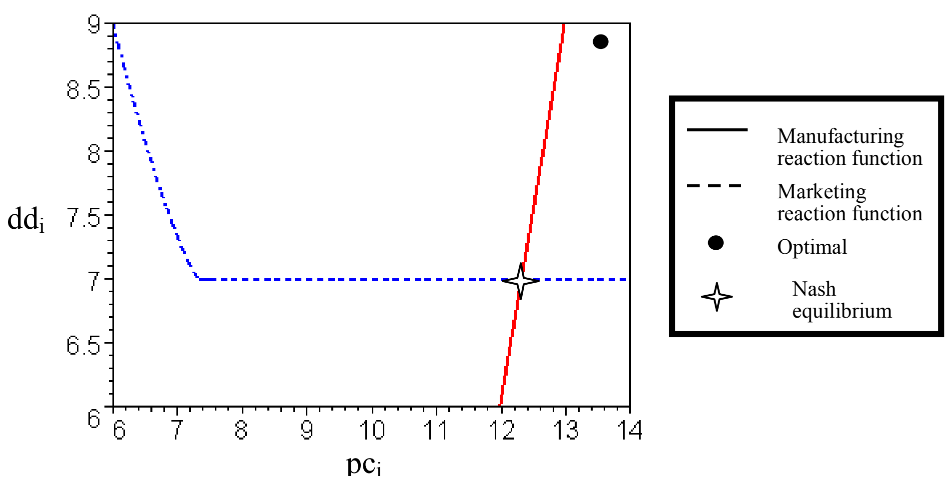

Figure 6. Reaction functions for α=*2, n_i=4,* and γ= *0.5*

$$S_i = H_i(dd_i^e, pc_i^e) - T_i(dd_i^o, pc_i^o)$$

Clearly, if S_i is positive for division i, that division is better off after the contract. Figure 7 depicts S_i values for different proposed values of β_3. Both parties are better of together only when $-4.6 > \beta_3 > -5.7$. Hence, for that specific case, the contracts embodying a value within this range should be preferred to others.

If the relative influence of the internal negotiation cost in the original game is high, coordination may become more costly. Figure 8 illustrates such an example where none of the proposed contract can decrease marketing division's cost. As underlined in chapter 5, in such cases, a fixed fee from manufacturing to marketing that is unrelated to all other costs and actions may be transferred for the willing participation of marketing in the coordination process.

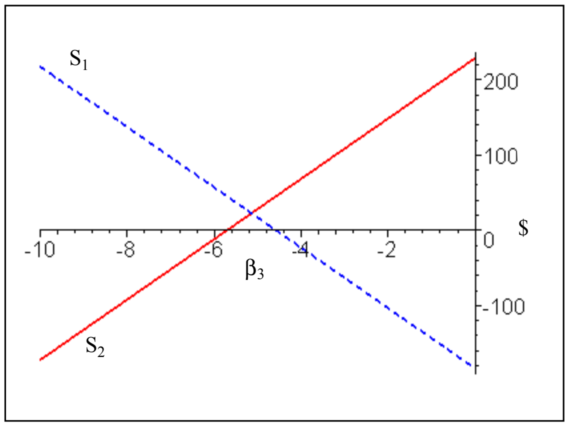

Figure 7. Gain through coordination for α=*1*, n_i=*4*, and γ= *0.5*

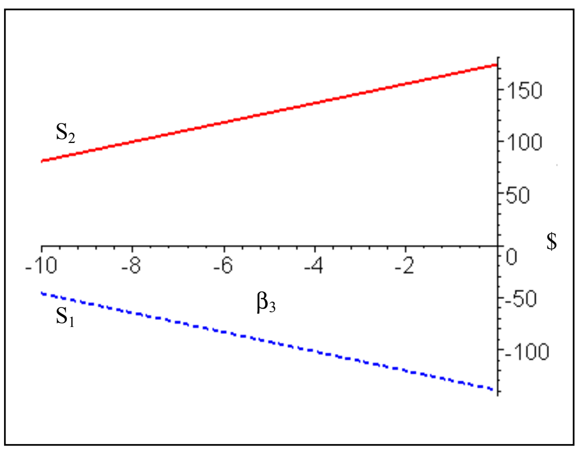

Figure 8. Gain through coordination for α=*2*, n_i=*4*, and γ= *0.5*

7. Conclusions

In this paper, we propose an incentive mechanism to coordinate due-date quotation in an internal market with players representing the marketing and manufacturing interests. We first analyze the system's model where the due-date quotation and the capacity utilization decisions are jointly given. We then investigate the decentralized case in which the manufacturing and marketing departments are considered as independent decision makers. To model the incentives for the players, we consider a Nash game in which the players announce their own decisions simultaneously based on their local cost structures. We characterize the basic properties of the player's utility and show that the Nash equilibrium decisions never optimize the firm's problem due to externalities. We propose a set of contracts regulating the allotment of the revenue that are composed of nonlinear transfer payments constructed based on cost elements within the system. By employing these transfer payments one can achieve the coordinated solution as the incentives to deviate from the system optimal are eliminated, thus, the system solution becomes the unique Nash equilibrium. With any belief function that would ensure a convex expected tardiness, the proposed coordination mechanism achieves system integration.

We illustrate our approach by a case study using a Weibull belief function. In general, the existence of equilibria can be proved so long as the capacity adjustment correspond to a convex expected tardiness function. Moreover, when the scale parameter is modeled as a linear function of pc_i, besides Weibull, there is always at least one Nash equilibrium for distributions such as uniform, gamma and Pearson Type V. However, the uniqueness of the equilibria needs to be further investigated for each distribution. Our numerical analysis with Weibull supports the conclusion of Cachon and Zipkin (1999) in that while the competition degrades the system efficiency, the extent of the efficiency loss is context specific.

In our game-theoretic analysis, the internal negotiation cost is an important factor that may lead to the detriment of system efficiency. In general, for very small and very large negotiation costs, marketing becomes more sensitive to manufacturing's decisions. Specifically, if the *internal negotiation cost* is high, marketing would lower the external due-date relative to the optimal solution. This is due to the fact that the tardiness cost to marketing is over-compensated by the payment from manufacturing. On the other hand, the larger the *external negotiation cost*, the more sensitive manufacturing would be to marketing's decision since the internal negotiation cost charged to manufacturing increases in proportional to the deviation between the external and internal due-dates. Moreover, as shown by the numerical analysis we suspect that this factor becomes potentially more dominant as the uncertainty level (variation) decreases with respect to other cost elements. Fortunately, the proposed coordination eliminates this factor through the transfer payments.

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APPENDIX

Proofs of Lemmas 1-3 and Propositions 1- 3 follow;

PROOF OF LEMMA 1 Fix pc_i and take the second derivative of $H_1(dd_i, pc_i)$ with respect to dd_i :

$$\frac{\partial^2 H_1(dd_i, pc_i)}{\partial dd_i^2} = 2g_i + 2a_i\overline{\Phi}(dd_i, pc_i) - 2n_i$$

Since $g_i > n_i$ and $\overline{\Phi}(dd_i, pc_i) \geq 0$, the second derivative will be strictly greater than 0. Therefore the foregoing function is strictly convex.

□

PROOF OF LEMMA 2. Fix dd_i and take the second derivative of $H_2(dd_i, pc_i)$ with respect to pc_i:

$$\frac{\partial^2 H_2(dd_i, pc_i)}{\partial pc_i^2} = 2m_i + \gamma a_i K^{(1)}(pc_i) + 2n_i$$

Here, m_i and n_i are strictly positive. Therefore it is sufficient to show that the second term in the forgoing equation is non-negative. Since the expected tardiness is a convex function, it is trivial to see that this term is also positive. Thus, H_2 is convex in pc_i.

□

PROOF OF LEMMA 3. In order to complete the proof we show that it is not possible that $\overline{r}_1(pc_i) < cd_i$ when $pc_i \leq cd_i$. We can rewrite the inequality as follows;

$$\overline{r}_1(pc_i) = \frac{g_i cd_i - n_i pc_i + a_i E[[\overline{c}_i - dd_i]^+]}{g_i - n_i} < cd_i$$

which can be reduced to

$a_i E[[\overline{c}_i - dd_i]^+] < n_i(pc_i - cd_i)$

It is obvious that if $pc_i \leq cd_i$, the right hand side of the inequality will be non-positive. However, since the left hand side is the expected tardiness given that the order is completed on or after dd_i and thus, is always positive, the inequality will not hold implying that $\overline{r}_1(pc_i) < cd_i$ can only be true only when $pc_i \geq cd_i$.

□

PROOF OF PROPOSITION 1. To prove convexity it is sufficient to show that the Hessian matrix for the squared expected tardiness is positive semidefinite. We first take the second derivatives of the function for both dd_i and pc_i;

$$\frac{\partial^2 E[[\overline{(c_i} - dd_i)^+]^2]}{\partial dd_i^2} = \overline{\Phi}(dd_i, pc_i)$$

The second derivative with respect to pc_i has no closed form. However, taking the derivative of that function with respect to dd_i yields the following result;

$$\frac{\partial^3 E[[\overline{(c_i} - dd_i)^+]^2]}{\partial pc_i^2 \partial \mathrm{dd}_i} = -2(dd_i/pc_i)^2 \phi(dd_i.pc_i)$$

which implies that the second derivative of squared tardiness with respect to pc_i decreases in dd_i and approaches zero when dd_i goes to infinity. Thus, it is non-negative. The determinant of the Hessian is as follows;

$$\mathrm{H} = 2\overline{\Phi}(dd_i.pc_i)\frac{\partial^2 E[[\overline{(c_i} - dd_i)^+]^2]}{\partial pc_i^2} - 4(\int_{dd_i}^{\infty} \overline{\Phi}^{(2)}(x, pc_i)dx)^2$$

The first derivative of H with respect to dd_i is as follows;

$$\mathrm{H}^{(1)}(dd_i, pc_i) = -2\phi(dd_i.pc_i)\left(\frac{\partial^2 E[[\overline{(c_i} - dd_i)^+]^2]}{\partial pc_i^2} + 2(dd_i/pc_i)^2\overline{\Phi}(dd_i.pc_i) - 4(dd_i/pc_i)\int_{dd_i}^{\infty} \overline{\Phi}^{(2)}(x, pc_i)dx\right)$$

Let $s(dd_i, pc_i)$ denote the function in the foregoing paranthesis. Taking the first derivative with respect to dd_i yields the following;

$$s^{(1)}(dd_i, pc_i) = 4dd_i\overline{\Phi}(dd_i.pc_i) - 4\int_{dd_i}^{\infty} \overline{\Phi}^{(2)}(x, pc_i)dx/pc_i$$

It is trivial to see that the foregoing function is increasing in dd_i and has negative values for all non-negative values of dd_i and pc_i. Thus, s increases in dd_i. Since $s(0, pc_i) = 0$, it is obvious that $s \geq 0$ implying that H decreases in

dd_i. H goes to zero as dd_i goes to infinity. Hence, we can conclude that H is positive and squared expected tardiness is convex. As a consequence, system cost function is convex as well.

□

PROOF OF PROPOSITION 2. It is shown in Theorem 3 that if $-\infty < \overline{r}_1^{(1)}(pc_i) < 1$ for any dd_i and pc_i then the Nash equilibrium is unique. So that it is sufficient to show that with the given parameter relations, inequality (6) is always true.

Lets first write the derivative of the right hand side of (6) with respect to dd_i;

$$\phi(x, pc_i) - \overline{\Phi}^{(2)}(x, pc_i) = \phi(x, pc_i)(1 - dd_i/pc_i)$$

Observe that the right hand side of (6) increases in dd_i until $dd_i = pc_i$ and decreases afterwards and thus, it is a unimodular function which takes its maximum value when both due-dates are equal. This value can be obtained using the following equation;

$$\int_{pc_i}^{\infty}(x - pc_i)\phi^{(2)}(x, pc_i) - (1 - \theta)$$

Also observe that;

$$\mu^{'}(pc_i) \geq \int_{pc_i}^{\infty}(x - pc_i)\phi^{(2)}(x, pc_i)$$

Therefore l_1 is an upperbound for the right hand side of (6). Hence if $g_i/a_i > l_1$ then $\overline{r}_1^{(1)}(pc_i) < 1$. This completes the proof.

□

PROOF OF PROPOSITION 3. From Theorem 4 we know that $-a_i < \beta_3 < 0$ should hold so that existence of Nash equilibria is guaranteed.

$\underline{r}_1^{(1)}$and $\underline{r}_2^{(1)}$ are given in (14) and (15) respectively. Based on Proposition 2, if we show that $0 < \underline{r}_1^{(1)}(pc_i) < 1$ and $0 < \underline{r}_2^{(1)}(dd_i) < 1$ then optimal solution is the unique equilibrium. We have already shown in Proposition 2 that if $g_i/a_i > l_1$ then the former inequality holds. If

$$m_i/a_i > \frac{\partial E[[\overline{c}_i - dd_i]^+]}{\partial pc_i} - \frac{1}{2}\frac{\partial^2 E[([\overline{c}_i - dd_i]^+)^2]}{\partial pc_i^2}$$

for any positive values of dd_i and pc_i then the latter inequality also holds. In this inequality, observe that the right hand side decreases in dd_i meaning that the maximum value it will take its maximum value when $dd_i = 0$. If $dd_i = 0$ the function returns the first derivative of expected completion time with respect to pc_i less the half of the second derivative of squared expected completion time with respect to pc_i which is equal to l_2. Hence the foregoing inequality always holds. Consequently, by Theorem 2, there exists a unique Nash equilibrium which resides at system optimal solution.

□