Previous Lecture

Coulomb's Law (vector form)

$$\mathbf{F}_{\mathbf{2\,on\,1}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2(\mathbf{r}_1 - \mathbf{r}_2)}{\left|\mathbf{r}_1 - \mathbf{r}_2\right|^3}$$

Field of a point charge q at origin

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\,\mathbf{r}$$

Principle of Superposition

Announcement

Beginning next Tuesday, you should pick a seat and sit in it every time. We will take attendance by checking which seats are occupied.

Today

- Continuous charge distributions.
- Charge density
- Integrating to get the electric field
- Field lines

Tricks of the Trade

Tricks of the Trade

- Make approximations

Tricks of the Trade

- Make approximations
- Use symmetry

Tricks of the Trade

- Make approximations
- Use symmetry
- Solve problems algebraically

From the Equation Sheet

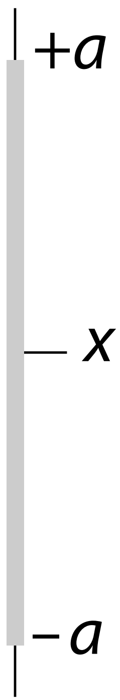
$$\mathbf{F}_{\mathbf{2\,on\,1}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2(\mathbf{r}_1 - \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3}$$

$$\mathbf{F} = q\mathbf{E}$$

$$d\mathbf{E}\,(\mathrm{at}\,\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\,\frac{dQ\,(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

Limiting Case I

Close to a long wire: $x \ll a \quad \Rightarrow \quad \sqrt{x^2 + a^2} \approx a$



$$\mathbf{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x} \frac{a}{\sqrt{x^2 + a^2}} \hat{\mathbf{i}} \approx \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x} \frac{a}{a} \hat{\mathbf{i}}$$

$$\mathbf{E} \approx \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x} \hat{\mathbf{i}}$$

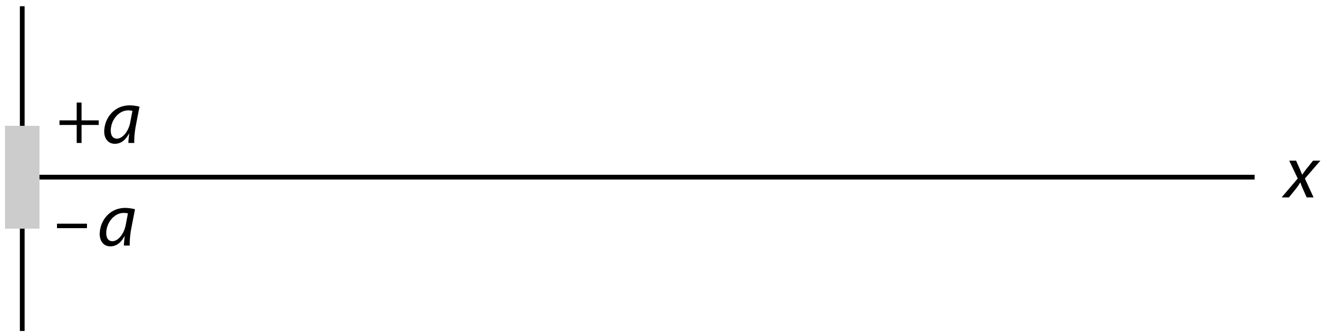
Near a long wire, $\mathbf{E}$ depends only on the $\perp$ distance to the wire and is directed $\perp$ to the wire. On the equation sheet you will find

$$E_{\text{line}} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

(You need to know the direction!)

Limiting Case II

Far from a short wire. $x \gg a \quad \Rightarrow \quad \sqrt{x^2 + a^2} \approx x$



$$\mathbf{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x}\frac{a}{\sqrt{x^2+a^2}}\,\hat{\mathbf{i}} \approx \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x}\frac{a}{x}\,\hat{\mathbf{i}} = \frac{1}{4\pi\epsilon_0}\frac{2a\lambda}{x^2}\,\hat{\mathbf{i}}$$

Since the total charge on the wire is $Q = 2a\lambda$,

$$\mathbf{E} \approx \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\,\hat{\mathbf{i}}.$$

From a distant point, the charge appears to be concentrated at the origin, and the result becomes Coulomb's law for a point charge.

Charge Density

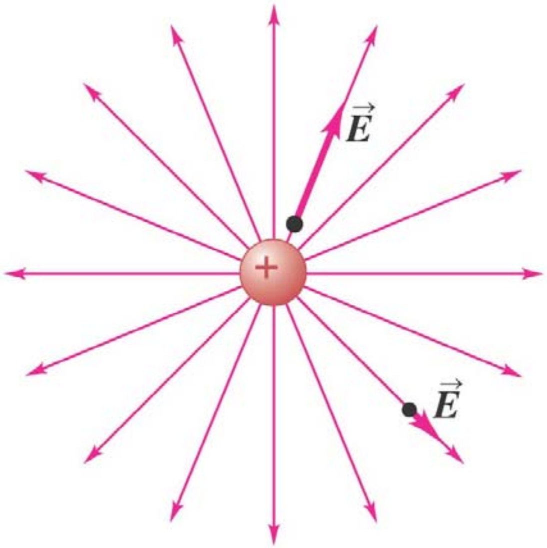
λ $\quad \dfrac{\text{charge}}{\text{length}}$ line

σ $\quad \dfrac{\text{charge}}{\text{area}}$ surface

ρ $\quad \dfrac{\text{charge}}{\text{volume}}$ volume

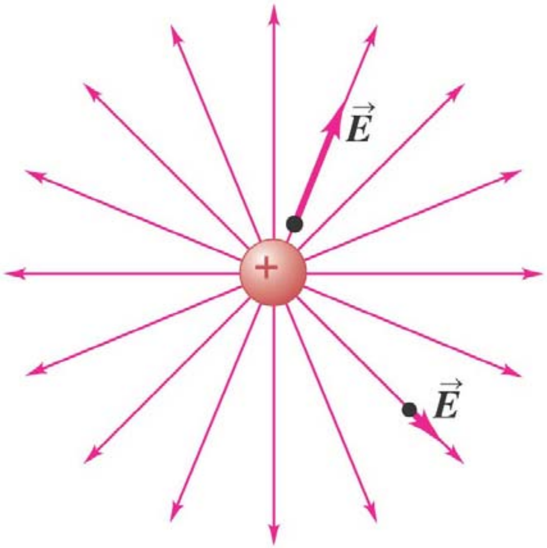
Field Lines

Field lines of a point charge



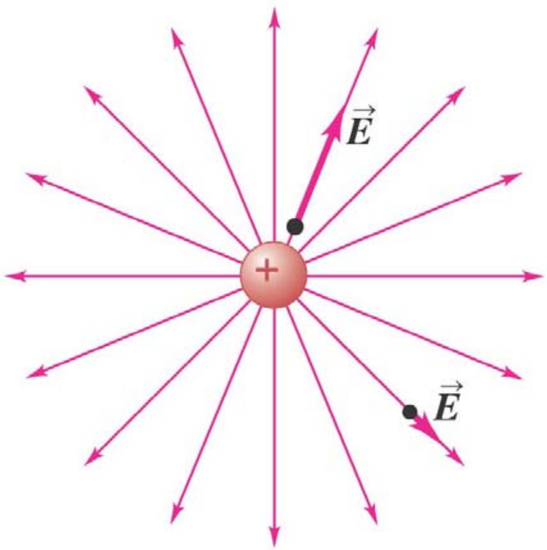
Field Lines

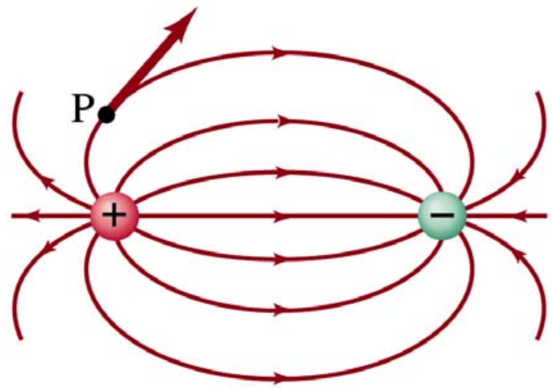
Field lines of a point charge

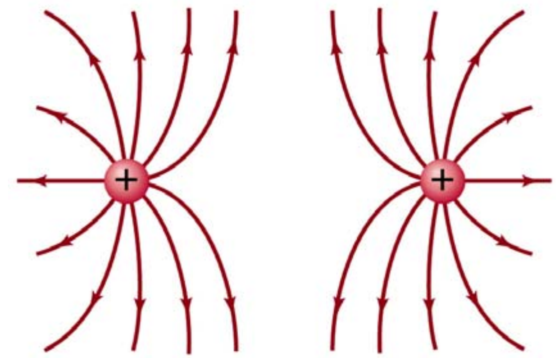


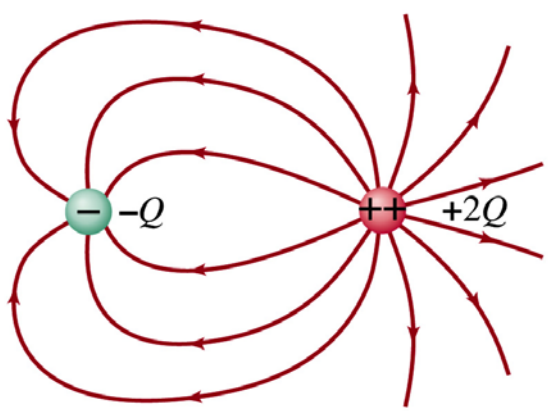
What are the field lines of a long line of charge (uniform density λ)?

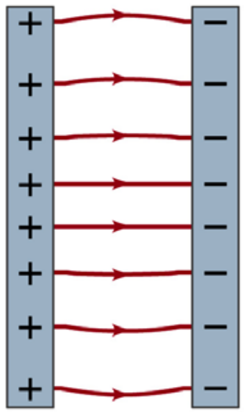
Field Lines





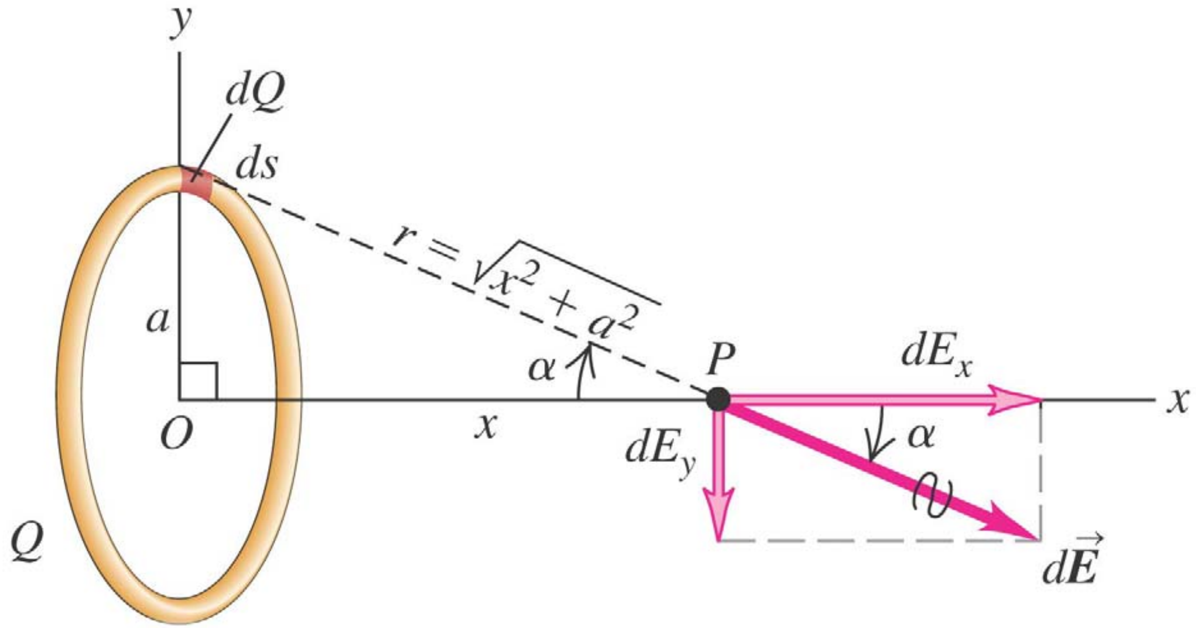






Field of a Ring of Charge

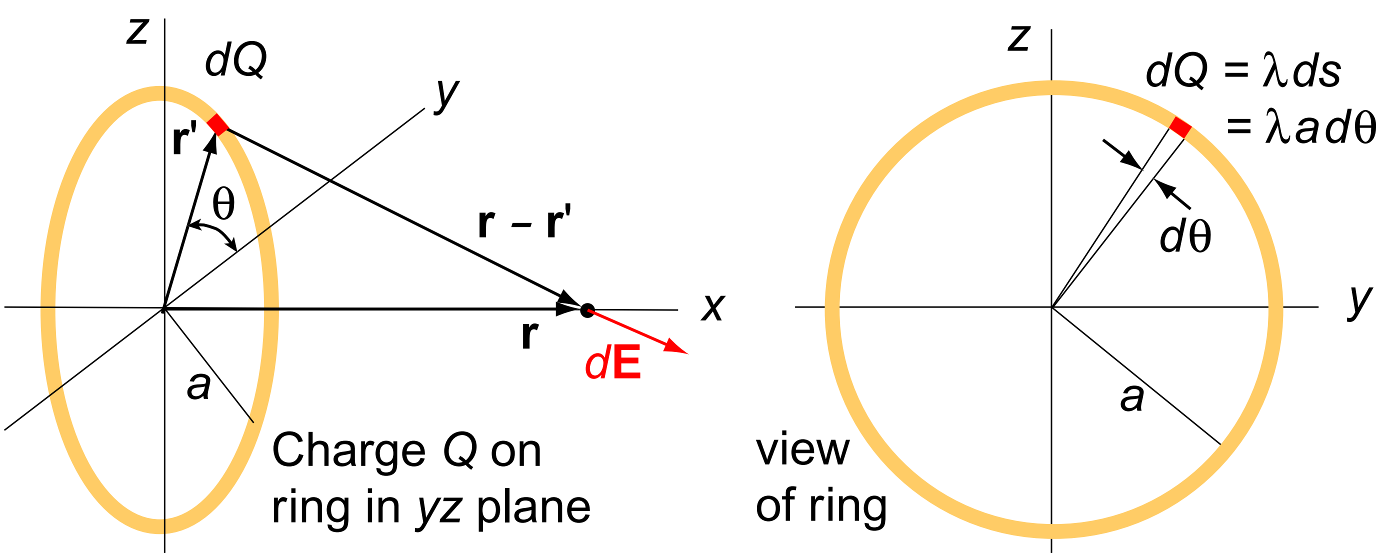
Here's Fig. 21.24 from the text:



We can relate this diagram to our general formula

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dQ\,(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}.$$

Field of a Ring of Charge



Using $\lambda = Q/(2\pi a)$, $\mathbf{r} = x\,\hat{\mathbf{i}}$, and $\mathbf{r}' = a\cos\theta\,\hat{\mathbf{j}} + a\sin\theta\,\hat{\mathbf{k}}$,

$$dQ = \lambda a\,d\theta = \frac{Q\,d\theta}{2\pi}$$

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dQ\,(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} = \frac{1}{4\pi\epsilon_0}\frac{Q\,d\theta}{2\pi}\left(\frac{x\,\hat{\mathbf{i}} - a\cos\theta\,\hat{\mathbf{j}} - a\sin\theta\,\hat{\mathbf{k}}}{(x^2+a^2)^{3/2}}\right)$$

$$\mathbf{E} = \int_{\theta=0}^{\theta=2\pi} d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}\,\hat{\mathbf{i}} \qquad (\hat{\mathbf{j}},\ \hat{\mathbf{k}}\text{ components zero})$$

Limiting Case (Field of Ring)

The field, on axis, of a ring of total charge Q is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}\,\hat{\mathbf{i}}$$

What happens if we are a long way from the ring ($x \to \infty$)?

Limiting Case (Field of Ring)

The field, on axis, of a ring of total charge Q is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}\,\hat{\mathbf{i}}$$

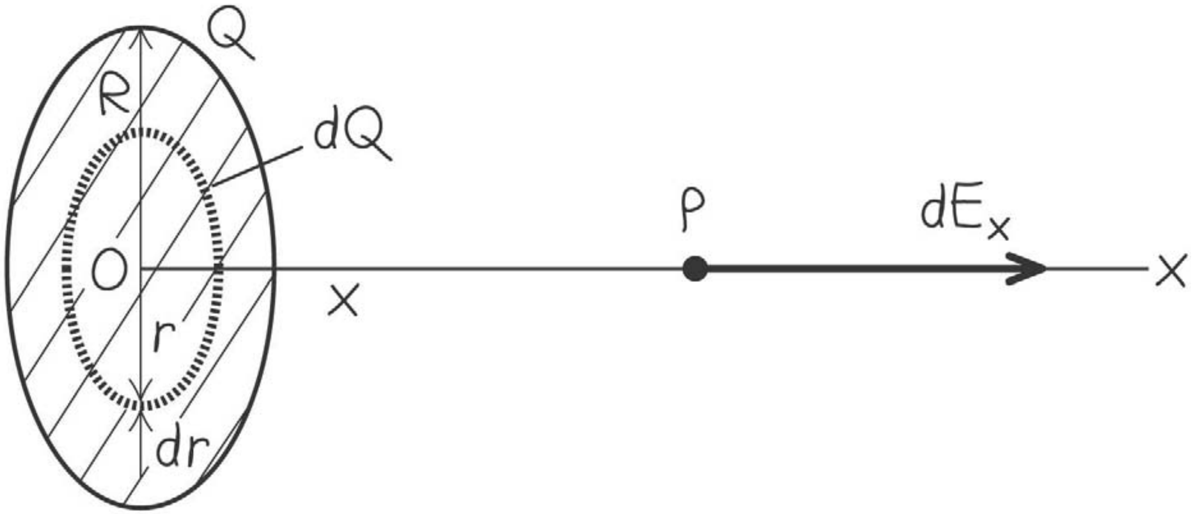
What happens if we are a long way from the ring ($x \to \infty$)?

All the charge distributed on the ring appears to be concentrated at one point, and we have the field of a point charge Q:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\,\hat{\mathbf{i}}$$

Field of a Charged Disk

Example 21.12:



We can find the field of a uniformly charged disk by integrating over a series of concentric rings of width dr from $r = 0$ to $r = R$. The result is

$$\mathbf{E} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2) + 1}}\right]\hat{\mathbf{i}}$$