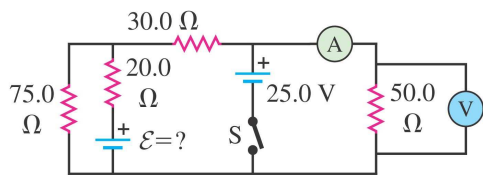
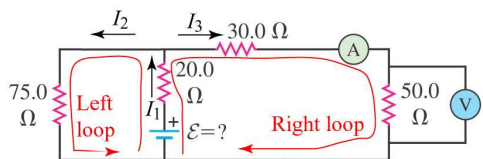


26-27 In the circuit shown in the figure the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15 V. (a) Find the emf \mathcal{E} of the battery. (b) What will the ammeter read when the switch is closed?



(a) For this part we can ignore the battery with the switch, since the switch is open and there will be no current through it. So let's draw the circuit that we are concerned with, showing also the currents in each branch and the loops we will use to write Kirchhoff's equations:



We can find I_3 since we were given the measured voltage across the $50\ \Omega$ resistor. Using Ohm's Law,

$$V = IR \implies I_3 = \frac{V}{R} = \frac{15\ \text{V}}{50\ \Omega} = 0.30\ \text{A}.$$

Now, let's write out the loop and node equations:

Node : $I_1 = I_2 + I_3$
 Left loop : $\mathcal{E} - (20\ \Omega)I_1 - (75\ \Omega)I_2 = 0$
 Right loop : $\mathcal{E} - (20\ \Omega)I_1 - (30\ \Omega)I_3 - (50\ \Omega)I_3 = 0$

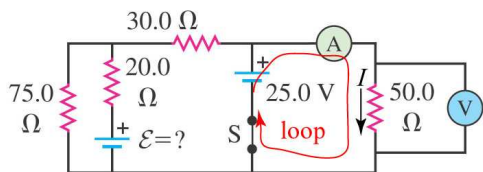
Since we already know I_3 , there are three unknowns in this system, \mathcal{E} , I_1 , and I_2 . Here are the equations after substituting for I_3 and combining some terms:

Node : $I_1 = I_2 + 0.3$
 Left loop : $\mathcal{E} - 20I_1 - 75I_2 = 0$
 Right loop : $\mathcal{E} - 20I_1 - 24 = 0$

Now we have three equations for three unknowns. Next we simplify the equations and solve for \mathcal{E} . The solution is

$$\mathcal{E} = 36.4\ \text{V}, \quad I_1 = 0.62\ \text{A}, \quad I_2 = 0.32\ \text{A}.$$

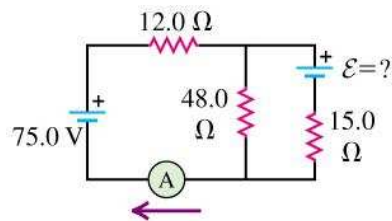
(b) The circuit is different for this part and is shown below. Conveniently, to find the new current in the ammeter, we only need to consider the one loop shown.



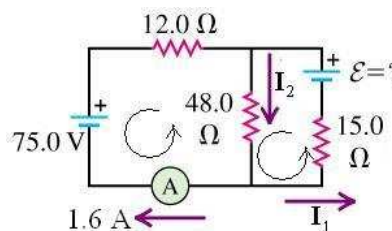
Set up the loop equation for this new loop:

$$25\ \text{V} - (50\ \Omega)I = 0 \implies I = \frac{25\ \text{V}}{50\ \Omega} = 0.50\ \text{A}$$

26-28 In the circuit shown in the figure both batteries have insignificant internal resistance and the idealized ammeter reads 1.60 A in the direction shown. (a) Find the emf \mathcal{E} of the battery. (b) Is the polarity shown correct?



(a) To solve this problem, we will use Kirchhoff's rules, but this time instead of solving for three currents, we will solve for two currents and the emf. We assign currents I_1 and I_2 to the remaining branches and loops as shown in the diagram:



The node equation is

$$I_2 = I_1 + 1.60\ \text{A}.$$

The left loop equation is

$$0 = 48.0\ \Omega(I_2) + 12\ \Omega(1.6\ \text{A}) - 75.0\ \text{V} \\ = 48.0\ \Omega(I_2) - 55.8\ \text{V},$$

and the right loop equation is

$$0 = -15.0\ \Omega(I_1) + \mathcal{E} - 48.0\ \Omega(I_2)$$

We can solve the left loop equation directly for I_2 :

$$I_2 = \frac{55.8\ \text{V}}{48.0\ \Omega} = 1.1625\ \text{A}.$$

Now substitute this I_2 into the node equation to solve for I_1 :

$$I_1 = I_2 - 1.60\ \text{A} = -.4375\ \text{A}$$

Substituting the currents found above into the right loop equations gives

$$0 = -15.0\ \Omega(-.4375\ \text{A}) + \mathcal{E} - 48.0\ \Omega(1.1625\ \text{A}) \\ \mathcal{E} = 15.0\ \Omega(-.4375\ \text{A}) + 48.0\ \Omega(1.1625\ \text{A}) = 49.24\ \text{V}.$$

(b) The polarity of the battery as shown is correct, because the \mathcal{E} we calculated was positive. If the calculated \mathcal{E} had been negative, it would imply that the assumed polarity in the drawing was incorrect.

26-40 A $12.8\mu\text{F}$ capacitor is connected through a $0.890\text{ M}\Omega$ resistor to a constant potential difference of 60.0 V . (a) Compute the charge on the capacitor at the following times after the connections are made: 0 s , 5.0 s , 10.0 s , 20.0 s , and 100.0 s . (b) Compute the charging currents at the same instants.

(a) As derived, the formula for the charge on a charging capacitor as a function of time is:

$$q(t) = Q_f \left(1 - e^{-t/RC}\right),$$

where the final charge $Q_f = CV = 7.68 \times 10^{-4}\text{ C}$. The time constant $\tau = RC = 11.392\text{ s}$. The table below gives $q(t)$ at the times specified.

(b) The relationship between charge and current is $i = dq/dt$, so we can determine the current as a function of time by differentiating the expression for $q(t)$ above:

$$i(t) = \frac{dq}{dt} = \frac{Q_f}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

where we substituted $Q_f = CV$ and noted that the initial current I_0 that flows is the battery voltage V divided by the resistance R . I_0 is $6.74 \times 10^{-5}\text{ A}$.

Here is a table of the charge and current at various times. Note that 100 s is about nine times the time constant, and at that point the capacitor is essentially fully charged, and the current from the battery is essentially zero.

t (s)	$q(t)$ (C)	$i(t)$ (A)
0	0	6.74×10^{-5}
5	2.73×10^{-4}	4.35×10^{-5}
10	4.49×10^{-4}	2.80×10^{-5}
20	6.35×10^{-4}	1.16×10^{-5}
100	7.68×10^{-4}	1.00×10^{-8}

26-43 An emf source with a magnitude of $\mathcal{E} = 120\text{ V}$, a resistor with a resistance of $R = 87.0\ \Omega$, and a capacitor with a capacitance of $C = 3.90\ \mu\text{F}$ are connected in series. As the capacitor charges, when the current in the resistor is 0.700 A , what is the magnitude of the charge on each plate of the capacitor?

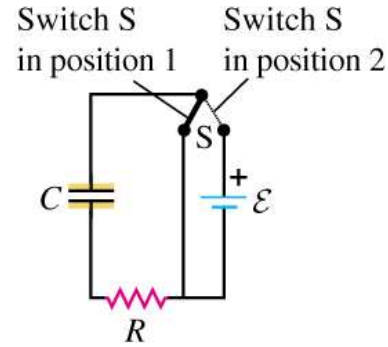
(a) The simplest way is to apply the loop equation. Let $q(t)$ and $i(t)$ be the instantaneous charge on the capacitor and current in the circuit. Then

$$\begin{aligned} \mathcal{E} - V_R - V_C &= 0 \\ \mathcal{E} - i(t)R - \frac{q(t)}{C} &= 0. \end{aligned}$$

Now we can solve for the charge on the capacitor as a function of the current:

$$\begin{aligned} q(t) &= C[\mathcal{E} - i(t)R] = 3.90\ \mu\text{C} [120\text{ V} - (0.7\text{ A})(87\ \Omega)] \\ &= 230\ \mu\text{C}. \end{aligned}$$

26-48 In the circuit shown below, $C = 5.90\ \mu\text{F}$, $\mathcal{E} = 28.0\text{ V}$, and the emf has negligible resistance. Initially the capacitor is uncharged and the switch S is in position 1. The switch is then moved to position 2, so the capacitor begins to charge. (a) What will be the charge on the capacitor a long time after the switch is moved to position 2? (b) After the switch has been moved to position 2 for 3.00 ms , the charge on the capacitor is measured to be $110\ \mu\text{C}$. What is the value of the resistance R ? (c) How long after the switch is moved to position 2 will the charge on the capacitor be equal to 99.0% of the final value found in part (a)?



(a) We know that after a long time the circuit will approach a steady state where the charge on the capacitor will be simply given by

$$Q = C\mathcal{E}.$$

Substituting $C = 5.90\ \mu\text{F}$ and $\mathcal{E} = 28.0\text{ V}$ we find the charge on the capacitor after a long time will be $Q = 165.2\ \mu\text{C}$.

(b) We know that the resistance R is part of the time constant in the function $q(t)$. We found $q(t)$ by solving the differential equation obtained from Kirchoff's loop equation.

$$q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

Since we know $q(t = 3\text{ s})$ we can rearrange this equation to solve for R .

$$R = -\frac{t}{C} \frac{1}{\ln(1 - \frac{q}{C\mathcal{E}})}$$

We insert our value for $q(t = 3\text{ s}) = 110\ \mu\text{C}$ and find the resistance $R = 464\ \Omega$.

(c) We want to find at what time t will the charge on the capacitor be 99% of its final value which we found in part (a). In other words we want to solve for t when $q = 0.99 \times C\mathcal{E}$. We rearrange our equation for $q(t)$ to get:

$$t = -RC \ln\left(1 - \frac{q}{C\mathcal{E}}\right).$$

Substituting our values of R , C , and q we find:

$$t = -(464\ \Omega)(5.9 \times 10^{-6}\text{ F}) \ln(1 - 0.99) = 0.0126\text{ s}$$