Physics 21
Fall, 2011

Solution to HW-28

33-31 Unpolarized light of intensity 26.0 $\mathrm{W/cm^2}$ is incident on two polarizing filters. The axis of the first filter is at an angle of 24.8° counterclockwise from the vertical (viewed in the direction the light is traveling) and the axis of the second filter is at 65.0° counterclockwise from the vertical. What is the intensity of the light after it has passed through the second polarizer?

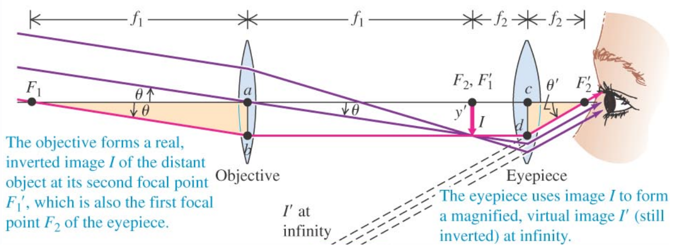
When the unpolarized light (intensity I_0) passes through the first polarizer, its intensity decreases by half and it will be polarized in the same direction as the axis of the first polarizer. Therefore the intensity after the first polarizer is $I_1 = I_0/2 = 13\ \mathrm{W/cm^2}$. The light is now polarized, so when it passes through the second polarizer its intensity decreases by another factor of $\cos^2\phi$, according to Malus' Law:

$$I_2 = I_1 \cos^2(\phi),$$

where I_2 is the intensity after the second polarizer, and ϕ is the angle between the polarization of the light and the axis of the polarizer. Since the axis of the first polarizer is at 24.8°, and the axis of the second polarizer is at 65.0° in the same direction, the angle between them is the difference, or 40.2°. Then

$$I_2 = (13\,\mathrm{W/cm^2})\cos^2(40.2°) = 7.58\,\mathrm{W/cm^2}.$$

34-58 Saturn is viewed through the Lick Observatory refracting telescope (objective focal length 18 m). If the diameter of the image of Saturn produced by the objective is 1.7 mm, what angle does Saturn subtend from when viewed from earth?



In the figure above (taken from page 1193 in the textbook), it is shown that a refracting telescope is made of two lenses, the objective and the eyepiece. Note that in this problem, we are only dealing with the objective lens. Saturn's average distance from the earth is 1.433×10^{12} m, which is very large compared to the other distances in this problem, so we can use $d_\mathrm{o} = \infty$. This allows us to calculate $d_\mathrm{i} = f = 18$ m, or as shown in the figure, the objective lens creates a real image at its focal point.

Note that the angle subtended by the object is the same as the angle subtended by the image. This fact follows by considering two straight rays: one from the base of the object to the base of the image, and the other from the tip of the object to the tip of the image. Both rays pass through the center of the lens and are not bent; they define the angles subtended by the object and by the image. Both angles are marked as θ in the figure.

We now have all the relationships needed to answer the question. We can find θ by using the right triangle formed by the center of the objective lens, the base of the image, and the tip of the image.

$$\tan\theta = \frac{h_\mathrm{i}}{f_\mathrm{o}} = \frac{1.7\,\mathrm{mm}}{18\,\mathrm{m}} = 9.44 \times 10^{-5}$$
$$\theta = \tan^{-1}\left(9.44 \times 10^{-5}\right) = 9.44 \times 10^{-5}\,\mathrm{rad}$$

Note that we could have used the small angle approximation, $\tan\theta \approx \theta$. Mastering Physics is expecting an answer in degrees and to two significant figures, so we enter 5.4×10^{-3}.

35-21 Coherent light with wavelength 450 nm passes through narrow slits with a separation of 0.350 mm. At a distance from the slits which is large compared to their separation, what is the phase difference (in radians) in the light from the two slits at an angle of 22.6° from the centerline?

Since the light source is coherent, the light that is emitted from each slit is in phase with each other. When the light waves arrive at the same point far from the slits, they will be out of phase by an amount proportional to the difference in their path lengths, $(r_2 - r_1)$. We call the phase difference between these arriving waves ϕ. For instance, when the path difference is one wavelength, one wave has gone through one complete cycle more than the other, and $\phi = 2\pi$ radians. When the path difference is $\lambda/2$, $\phi = \pi$ radians, and so on. We can express this proportionality in the equation

$$\phi = 2\pi\frac{r_2 - r_1}{\lambda}$$

If the point where the waves meet is far from the slits in comparison to their separation d, the path difference is given by $r_2 - r_1 = d\sin\theta$ where θ is the angle measured from the centerline. Combining these equations gives

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi\,(0.350\,\mathrm{mm})}{450\,\mathrm{nm}}\sin 22.6° = 1878\,\mathrm{rad}$$

YF 36-24 An interference pattern is produced by two identical parallel slits of width a and separation (between centers) $d = 3a$. Give the number m of the first interference maxima that will be missing in the pattern.

Diffraction minima occur when $a\sin\theta = n\lambda$, and interference maxima occur when $d\sin\theta = m\lambda$. If these coincide at the same θ, then

$$\sin\theta = \frac{n\lambda}{a} = \frac{m\lambda}{d} = \frac{m\lambda}{3a} \Rightarrow \frac{n\lambda}{a} = \frac{m\lambda}{3a} \Rightarrow n = \frac{m}{3}.$$

The lowest solution for m and n is $m = 3$ and $n = 1$.

LEHIGH UNIVERSITY

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36-9 Sound with a frequency of 1250 Hz leaves a room through a doorway with a width of 1.00 m. At what minimum angle relative to the centerline perpendicular to the doorway will someone outside the room hear no sound? Use 344 m/s for the speed of sound in air and assume that the source and listener are both far enough from the doorway for Fraunhofer diffraction to apply. You can ignore effects of reflections.

Even though this example involves sound waves, the basic phenomenon is still diffraction from a single slit. The condition for a dark fringe, or in this case a region with no sound, is given by

$$\sin\theta = \frac{m\lambda}{a},$$

where m is the order of the region of no sound ($m = \pm1, \pm2, \pm3, ...$), and a is the width of the door. The wavelength of the sound can be found from the stated frequency and speed of sound:

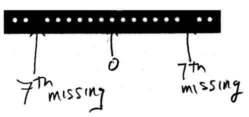
$$\lambda = \frac{v_{\text{sound}}}{f} = \frac{344\,\text{m/s}}{1250\,\text{Hz}} = 0.2752\,\text{m}.$$

To find the angles, we solve the first equation above for θ, without making small angle approximations:

$$\begin{aligned}\theta &= \sin^{-1}\frac{m\lambda}{a}\\ &= \sin^{-1}(1\times 0.2752) = 16.0^\circ \quad \text{for } m=1\\ &= \sin^{-1}(2\times 0.2752) = 33.4^\circ \quad \text{for } m=2\\ &= \sin^{-1}(3\times 0.2752) = 55.6^\circ \quad \text{for } m=3\end{aligned}$$

Had we used the small angle approximation $\sin^{-1}\theta \sim \theta$, and converted radians to degrees, we would have obtained 15.8°, 31.5°, and 47.3°, for m = 1, 2, and 3, respectively. Note that the maximum possible value of m is 3; otherwise the sine becomes greater than one.

36-26 A diffraction experiment involving two thin parallel slits yields the pattern of closely spaced bright and dark fringes shown in the figure. Only the central portion of the pattern is shown in the figure. The bright spots are equally spaced at 1.53 mm center to center (except for the missing spots) on a screen 2.40 m from the slits. The light source was a He-Ne laser producing a wavelength of 632.8 nm. (a) How far apart are the two slits? (b) How wide is each one?



(A) $\Delta\theta = \frac{\lambda}{d}$ (from class notes) $\Rightarrow d = \frac{\lambda}{\Delta\theta} = \frac{632.8\text{ nm}}{(1.53\text{mm}/2.40\text{m})}$

$= 0.993\times10^{-3}\text{m} = 0.993$ mm

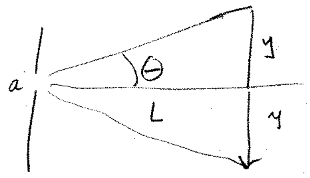
(B) $a\sin\theta = \lambda$ for $\theta = \frac{7\times 1.53\text{mm}}{2.40\text{ m}}$

$a \simeq \frac{\lambda}{\theta} = \frac{632.8\text{ nm}}{(7\times1.53\text{mm})/2.40\text{m}} = \frac{d}{7} = 0.142$ mm

K 22-19 The opening to a cave is a tall crack 43.0 cm wide. A bat that is preparing to leave the cave emits a 31.0 kHz ultrasonic chirp. How wide is the "sound beam" 110 m outside the cave opening? Use exact formulas; don't make small angle approximations. Use $v_{\text{sound}} = 340$ m/s.

$f = 31$ kHz

$\lambda = \frac{v}{f}$



$a\sin\theta = \lambda \Rightarrow \sin\theta = \frac{\lambda}{a} = \frac{v}{af}$

$= \frac{340\text{ m/s}}{0.43\text{m}\quad 31\times10^3\text{ s}^{-1}}$

$\sin\theta = 0.0255$

width of beam $= 2y = 2L\tan\theta$

$2y = 2L\tan\left(\sin^{-1}\frac{v}{af}\right) = 5.61$ m

Small angle approx would be ok in this case

36-30 If a diffraction grating produces a third-order bright spot for red light (of wavelength 700 nm) at 67.0° from the central maximum, at what angle will the second-order bright spot be for violet light (of wavelength 430 nm)?

(a) The bright spots are caused by constructive interference. From the optics handout, the condition for constructive interference is

$$d\sin\theta = m\lambda$$

From the information given in the problem, it is possible to determine d, the slit spacing of the diffraction grating. Setting $m = 3$ leads to

$$d = \frac{m\lambda}{\sin\theta} = \frac{3\times700\,\text{nm}}{\sin 67.0^\circ} = 2281.4\,\text{nm}$$

Then it is possible to find the angle for a second-order bright spot for violet light. Here $m = 2$, and

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2\times430\,\text{nm}}{2281.4\,\text{nm}}\right) = 22.1^\circ$$