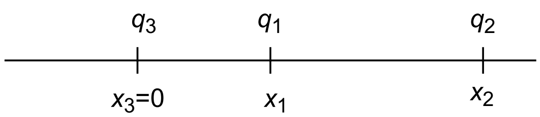
Physics 21
Fall, 2011

Solution to HW-2

21-13 Three point charges are arranged on a line. Charge $q_3 = +5.00\,\mathrm{nC}$ and is at the origin. Charge $q_2 = -3.00\,\mathrm{nC}$ and is at $x_2 = 4.50\,\mathrm{cm}$. Charge q_1 is at $x_1 = 1.00\,\mathrm{cm}$. What is q_1 (magnitude and sign) if the net force on q_3 is zero?



We can work this problem without using vectors by thinking it through. Since q_2 and q_3 have opposite sign, the force on q_3 exerted by q_2 is attractive (towards the right). If the total force on q_3 is to be zero, the force exerted by q_1 must be repulsive (toward the left). Thus q_1 and q_3 must have the same sign, and q_1 must be positive.

We can find the magnitude of q_1 by equating the magnitude of the forces on q_3 exerted by q_1 and q_2:

$$\frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{x_1^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{x_2^2}$$

Cancelling like terms on both sides of the equation, we find

$$|q_1| = \left(\frac{x_1}{x_2}\right)^2 |q_2| = \left(\frac{1.0\,\mathrm{cm}}{4.5\,\mathrm{cm}}\right)^2 (3\,\mathrm{nC}) = 0.148\,\mathrm{nC}.$$

We already concluded that q_1 was positive.

A more general way to solve this problem is to use the vector expressions for the Coulomb force. We want

$$0 = \mathbf{F}_{1\,\mathrm{on}\,3} + \mathbf{F}_{2\,\mathrm{on}\,3},$$

where

$$0 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3(\mathbf{r}_3 - \mathbf{r}_1)}{|\mathbf{r}_3 - \mathbf{r}_1|^3} + \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3(\mathbf{r}_3 - \mathbf{r}_1)}{|\mathbf{r}_3 - \mathbf{r}_2|^3}.$$

and we can evaluate the forces using the locations of the charges. Because q_1 is at the origin, $\mathbf{r}_3 = 0$. Also, $\mathbf{r}_1 = x_1\hat{\mathbf{i}}$ and $\mathbf{r}_2 = x_2\hat{\mathbf{i}}$. Substituting for the vectors gives

$$\begin{aligned} 0 &= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3(0 - x_1)\hat{\mathbf{i}}}{|0 - x_1|^3} + \frac{q_2 q_3(0 - x_2)\hat{\mathbf{i}}}{|x_2|^3}\right] \\ &= \frac{-q_3}{4\pi\epsilon_0}\left[\frac{q_1 x_1}{|x_1|^2} + \frac{q_2 x_2}{|x_2|^2}\right]\hat{\mathbf{i}} \end{aligned}$$

Note that the terms in the denominators are lengths and must be positive. The quantity in brackets must be zero, so we obtain

$$q_1 = -\frac{x_2}{x_1}\left|\frac{x_1}{x_2}\right|^3 q_2 = -\frac{4.5}{1.0}\left|\frac{1.0}{4.5}\right|^2 (-3.0\,\mathrm{nC}) = 0.148\,\mathrm{nC}$$

This formula agrees with the previous result. Because we were careful with the signs, the formula gives the correct answer for any combination of signs of the three charges and for the two vector components x_1 and x_2. (We took $x_3 = 0$.)

21-15 Three point charges are located on the positive x axis of a coordinate system. Charge $q_1 = 1.0\,\mathrm{nC}$ is 2.0 cm from the origin, charge $q_2 = -4.0\,\mathrm{nC}$ is 4.0 cm from the origin and charge $q_3 = 6.0\,\mathrm{nC}$ is located at the origin. What is the net force (magnitude and direction) on charge $q_1 = 1.0\,\mathrm{nC}$ exerted by the other two charges?

This problem is very similar to 21-13, and the same diagram applies. Here we need the sum $\mathbf{F}$ of $\mathbf{F}_{2\,\mathrm{on}\,1}$ and $\mathbf{F}_{3\,\mathrm{on}\,1}$, which is

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2(\mathbf{r}_1 - \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3} + \frac{q_1 q_3(\mathbf{r}_1 - \mathbf{r}_3)}{|\mathbf{r}_1 - \mathbf{r}_3|^3}\right],$$

where, as before, $\mathbf{r}_3 = 0$, $\mathbf{r}_1 = x_1\hat{\mathbf{i}}$, and $\mathbf{r}_2 = x_2\hat{\mathbf{i}}$. Then

$$\begin{aligned} \mathbf{F} &= \frac{1}{4\pi\epsilon_0}q_1\left[\frac{q_2(x_1 - x_2)\hat{\mathbf{i}}}{|x_1 - x_2|^3} + \frac{q_3(x_1 - x_3)\hat{\mathbf{i}}}{|x_1 - x_3|^3}\right] \\ &= \frac{1}{4\pi\epsilon_0}q_1\left[\frac{q_2(-0.02\,\mathrm{m})}{(0.02\,\mathrm{m})^3} + \frac{q_3(0.02\,\mathrm{m})}{(0.02\,\mathrm{m})^3}\right]\hat{\mathbf{i}} \end{aligned}$$

Substituting the other numbers leads to

$$\begin{aligned} \mathbf{F} &= (9\times10^9)(1\,\mathrm{nC})\left[\frac{-4\,\mathrm{nC}(-.02\,\mathrm{m})}{(.02\,\mathrm{m})^3} + \frac{6\,\mathrm{nC}(.02\,\mathrm{m})}{(.02\,\mathrm{m})^3}\right]\hat{\mathbf{i}} \\ &= 2.25\times10^{-4}\,\mathrm{N}\,\hat{\mathbf{i}} \end{aligned}$$

21-11 In an experiment in space, one proton is held fixed and another proton is released from rest a distance d away. What is the initial acceleration of the proton after it is released?

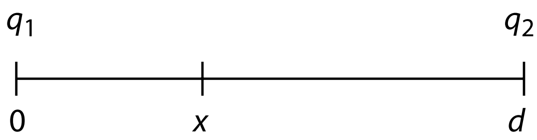
From Physics 11 you know that $\mathbf{F} = m\mathbf{a}$, or $\mathbf{a} = \mathbf{F}/m$. So just find the electrostatic force on one proton due to the other proton, and then divide by the mass. We'll drop the vector notation and just find the magnitude:

$$\begin{aligned} a &= \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_\mathrm{p} d^2} \\ &= \left(9\times10^9\,\mathrm{N\,m^2/C^2}\right)\frac{\left(1.602\times10^{-19}\,\mathrm{C}\right)^2}{(1.67\times10^{-26}\,\mathrm{kg})\,d^2} \end{aligned}$$

When you substitute for d, don't forget to convert to meters. For $d = 3\,\mathrm{mm} = 0.003\,\mathrm{m}$, the result is

$$a = 1.54\times10^4\,\mathrm{m/s^2}.$$

21-46 Two particles having charges $q_1 = 0.600$ nC and $q_2 = 5.00$ nC are separated by a distance of $d = 1.60$ m. At what point along the line connecting the two charges is the total electric field due to the two charges equal to zero?



Since both charges are positive, it's easy to keep track of the direction of the electric field. The field at x from q_1 points to the right, and the one from q_2 points to the left. These two fields must be equal in magnitude for their vector sum to be zero. Therefore

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(x-d)^2}$$

Cancelling the common factor of $1/(4\pi\epsilon_0)$, we can rewrite the above equation as

$$\frac{(d-x)^2}{x^2} = \frac{q_2}{q_1} \quad \Rightarrow \quad \frac{d-x}{x} = \sqrt{\frac{q_2}{q_1}}$$

Solving for x, we find

$$x = \frac{d}{1+\sqrt{q_2/q_1}}$$

Substituting the specific numbers given above leads to

$$x = 0.412\,\text{m}.$$

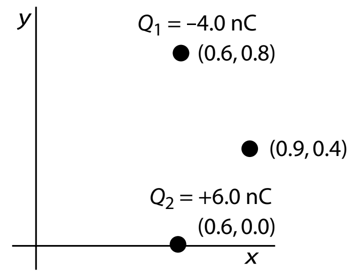
Note that instead of taking the square root and solving a linear equation for x, one could also set up a quadratic equation. One must identify the correct root of the quadratic equation, but the result is the same.

YF 21-50 mod A point charge $q_1 = -4.00$ nC is at the point $x = 0.60$ m, $y = 0.80$ m, and a second point charge $q_2 = +6.00$ nC is at the point $x = 0.60$ m, $y = 0$. (a,b) Calculate the x and y components of the net electric field at the origin due to these two point charges. (c,d) Calculate the x and y components of the net electric field at the point $x = 0.90$ m, $y = 0.40$ m due to these two point charges.

Use the vector expression given in class for the field $\mathbf{E}$ at $\mathbf{r}$ due to a charge Q at point $\mathbf{r}'$. Apply this formula to get the field at $\mathbf{r}$ due to Q_1; apply it again to get the field at $\mathbf{r}$ due to Q_2, and then add the results (superposition).

$$\mathbf{E}\,(\text{at}\,\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q\,(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

Remember, $\mathbf{r}$ = field point; $\mathbf{r}'$ = charge point.



(a,b) Find $\mathbf{E}$ at origin, $\mathbf{r} = 0\hat{\mathbf{i}}+0\hat{\mathbf{j}}$. For Q_1, $\mathbf{r}' = 0.6\hat{\mathbf{i}}+0.8\hat{\mathbf{j}}$, so $\mathbf{r}-\mathbf{r}' = -0.6\hat{\mathbf{i}}-0.8\hat{\mathbf{j}}$ and $|\mathbf{r}-\mathbf{r}'| = 1.0$ m. For Q_2, $\mathbf{r}' = 0.6\hat{\mathbf{i}}$, so $\mathbf{r}-\mathbf{r}' = -0.6\hat{\mathbf{i}}$ and $|\mathbf{r}-\mathbf{r}'| = 0.6$ m.

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\left[\frac{-4\text{ nC}(-.6\,\hat{\mathbf{i}}-.8\,\hat{\mathbf{j}})\text{m}}{(1.0\text{ m})^3} + \frac{6\text{ nC}(-.6\,\hat{\mathbf{i}})\text{m}}{(.6\text{ m})^3}\right] \\
&= \frac{1}{4\pi\epsilon_0}\left[\hat{\mathbf{i}}\left(\frac{2.4}{1^3}-\frac{3.6}{(.6)^3}\right)+\hat{\mathbf{j}}\left(\frac{3.2}{1^3}\right)\right]\frac{\text{nC}}{\text{m}^2} \\
&= \frac{1}{4\pi\epsilon_0}\left[-14.3\,\hat{\mathbf{i}}+3.2\,\hat{\mathbf{j}}\right]\frac{\text{nC}}{\text{m}^2} \\
&= \left(-128.3\,\hat{\mathbf{i}}+28.77\,\hat{\mathbf{j}}\right)\text{ N/C}
\end{aligned}$$

(c,d) Find $\mathbf{E}$ at point $\mathbf{r} = 0.9\,\hat{\mathbf{i}}+0.4\,\hat{\mathbf{j}}$. For Q_1, $\mathbf{r}' = 0.6\hat{\mathbf{i}}+0.8\hat{\mathbf{j}}$, so $\mathbf{r}-\mathbf{r}' = 0.3\,\hat{\mathbf{i}}-0.4\,\hat{\mathbf{j}}$ and $|\mathbf{r}-\mathbf{r}'| = 0.5$ m. For Q_2, $\mathbf{r}' = 0.6\hat{\mathbf{i}}$, so $\mathbf{r}-\mathbf{r}' = 0.3\,\hat{\mathbf{i}}+0.4\,\hat{\mathbf{j}}$, and $|\mathbf{r}-\mathbf{r}'| = 0.5$ m.

$$\begin{aligned}
\mathbf{E} &= \frac{1}{4\pi\epsilon_0}\left[\frac{-4\text{ nC}(.3\,\hat{\mathbf{i}}-.4\,\hat{\mathbf{j}})\text{m}}{(0.5\text{ m})^3} + \frac{6\text{ nC}(.3\,\hat{\mathbf{i}}+.4\,\hat{\mathbf{j}})\text{m}}{(0.5\text{ m})^3}\right] \\
&= \frac{1}{4\pi\epsilon_0}\left[\frac{\hat{\mathbf{i}}(-1.2+1.8)+\hat{\mathbf{j}}(1.6+2.4)}{0.125}\right]\frac{\text{nC}}{\text{m}^2} \\
&= \frac{1}{4\pi\epsilon_0}\left[4.8\,\hat{\mathbf{i}}+32\,\hat{\mathbf{j}}\right]\frac{\text{nC}}{\text{m}^2} \\
&= \left(43.2\,\hat{\mathbf{i}}+287.7\,\hat{\mathbf{j}}\right)\text{ N/C}
\end{aligned}$$