29-34 A dielectric of permittivity 3.3×10^{-11} F/m completely fills the volume between two capacitor plates. For $t > 0$ the electric flux through the dielectric is $(7800 \text{ V m/s}^3)t^3$. The dielectric is ideal and nonmagnetic; the conduction current in the dielectric is zero. At what time does the displacement current in the dielectric equal 23 μA?

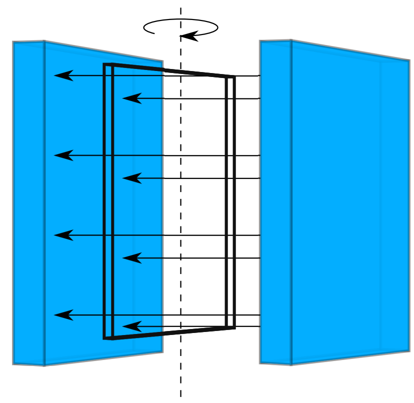
For a charging capacitor we have the equation for displacement current $i_\text{D} = \epsilon_0 \frac{d\Phi_E}{dt}$. When a dielectric is between the capacitor plates, we multiply ϵ_0 by the dielectric constant K to get the permittivity of the material $\epsilon = K\epsilon_0$. Thus we have $i_\text{D} = \epsilon \frac{d\Phi_E}{dt}$. The function for the electric flux can then be substituted:

$$\begin{aligned} i_\text{D} &= \epsilon \frac{d}{dt}\left((7800 \text{ V m/s}^3)t^3\right) = \epsilon(7800 \text{ V m/s}^3)\frac{d\left(t^3\right)}{dt} \\ &= \epsilon(7800 \text{ V m/s}^3)(3)t^2 \end{aligned}$$

Solving this for time:

$$\begin{aligned} t &= \sqrt{\frac{i_\text{D}}{\epsilon(7800 \text{ V m/s}^3)(3)}} \\ &= \sqrt{\frac{23 \times 10^{-6}}{3.3 \times 10^{-11}(7800 \text{ V m/s}^3)(3)}} = 5.5 \text{ s} \end{aligned}$$

29-51 As a new electrical engineer for the local power company, you are assigned the project of designing a generator of sinusoidal ac voltage with a maximum voltage of 120 V. Besides plenty of wire, you have two strong magnets that can produce a constant uniform magnetic field of 1.8 T over a square area with a length of 10.4 cm on a side when the magnets are separated by a distance of 12.1 cm. The basic design should consist of a square coil turning in the uniform magnetic field. To have an acceptable coil resistance, the coil can have at most 450 loops.



We can use Faraday's law to calculate the induced EMF.

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A} = -\frac{d\Phi}{dt},$$

The flux through a square loop perpendicular to the magnetic field with the maximum possible dimensions and N loops of wire will be $\Phi_B = NBA$, where A is the area. If the loop is rotated within the magnetic field, then the flux through the loop will change because number of field lines going through the loop will be changing. Rotating at an angular frequency of ω, the flux will change with time:

$$\Phi_B(t) = NBA\cos(\omega t)$$

The time derivative of this gives the induced EMF:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = NBA\omega\sin(\omega t)$$

The amplitude is then $NBA\omega$. Solving for ω

$$\omega = \frac{\mathcal{E}}{NBA} = \frac{120\,\text{V}}{450 \times 1.8 \times .104^2} = 13.7\,\text{rad/sec}$$

Mastering physics wants an answer in revolutions per minute, so the conversion is:

$$13.7\frac{\text{rad}}{\text{sec}} \cdot \frac{60\,\text{sec}}{1\,\text{min}} \cdot \frac{1\,\text{rev}}{2\pi\,\text{rad}} = 130.8\,\text{rpm}$$

31-6 A capacitance C and an inductance L are operated at the same angular frequency. (a) At what angular frequency will they have the same reactance? (b) If $L = 4.80$ mH and $C = 3.70\mu$F, what is the numerical value of the angular frequency in part (a)? (c) What is the reactance of each element?

(a) Equate the reactances and solve for ω:

$$\frac{1}{\omega C} = \omega L \quad \Rightarrow \quad 1 = \omega^2 LC \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}.$$

(b) Substituting numbers,

$$\omega = \frac{1}{\sqrt{(4.80 \times 10^{-3})(3.70 \times 10^{-6})}} = 7.50 \times 10^3 \text{ rad/s}.$$

(c) Again substituting numbers,

$$\frac{1}{\omega C} = \omega L = 36.0\,\Omega.$$

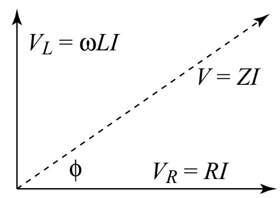
31-10 You want the current amplitude through an inductor with an inductance of 5.00 mH (part of the circuitry for a radio receiver) to be 2.10 mA when a sinusoidal voltage with an amplitude of 12.0 V is applied across the inductor. (a) What frequency is required?

(a) The frequency f is related to angular frequency ω by the equation $\omega = 2\pi f$. The amplitude of the voltage across the inductor is $V_L = IL\omega = IL2\pi f$. Thus the frequency is

$$f = \frac{V_L}{2\pi IL} = \frac{12.0V}{2\pi(2.10 \times 10^{-3}\,\text{A})(5.00 \times 10^{-3}\,\text{H})} = 182\,\text{kHz}$$

31-14 You have a 180 Ω resistor and a 0.430 H inductor. Suppose you take the resistor and inductor and make a series circuit with a voltage source that has a voltage amplitude of 28.0 V and an angular frequency of 250 rad/s. (a) What is the impedance of the circuit? (b) What is the current amplitude? (c) What is the voltage amplitude across the resistor? (d) What is the voltage amplitudes across the inductor? (e) What is the phase angle ϕ of the source voltage with respect to the current? (f) Does the source voltage lag or lead the current? (g) Construct a phasor diagram.

(a) The impedance relates the peak values of the voltage and current. We can find it by drawing the phasor diagram:



The diagram is simpler since there is no capacitor. The voltage V of the power supply, shown by the dashed line, is

$$V = \sqrt{(\omega LI)^2 + (RI)^2} = \sqrt{(\omega L)^2 + R^2}I = ZI.$$

Therefore the impedance Z is given by

$$Z = \sqrt{(180\,\Omega)^2 + (250\,\text{rad/s} \times 0.430\,\text{H})^2} = 210\,\Omega.$$

(b) We've already written the relation between V and I, so

$$I = \frac{V}{Z} = \frac{28.0\,\text{V}}{210\,\Omega} = 0.134\,\text{A} = 134\,\text{mA}.$$

(c) To find the voltage amplitude across the resistor we use Ohm's law and the current from part (b):

$$V_{\text{R}} = IR = (0.134\,\text{A})(200\,\Omega) = 24.0\,\text{V}$$

(d) The voltage amplitude across the inductor is the inductive reactance X_L times the current:

$$V_L = \omega LI = (250\,\text{rad/s})(0.430\,\text{H})(.134\,\text{A}) = 14.4\,\text{V}$$

(e) Using the values of V_L and V_R just calculated, we can easily find the phase angle from the diagram above:

$$\phi = \arctan\frac{V_L}{V_R} = \arctan\frac{14.4V}{24.0V} = 30.8°.$$

(f) From the phasor diagram above it is clear that the power supply voltage V leads the current. (Remember the current is in phase with V_R).

(g) The diagram is shown above.