Solution to HW-1

21 Q 19 The form of Coulomb's law is very similar to that for Newton's law of universal gravitation. What are the differences between these two laws? Compare also gravitational mass and electric charge.

Both laws describe a force between two point objects 1 and 2 that depends inversely on the square of the distance r between the objects, and is directed from one object toward the other. For gravity, the magnitude of the force depends on the "mass" property of the object and is given by

$$F = G\frac{M_1 M_2}{r^2},$$

where G is the gravitational constant. For Coulombs law, the magnitude of the force depends on the "charge" property and is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2},$$

where $1/4\pi\epsilon_0$ is a different proportionality constant.

Masses are always positive, and the gravitational force is always attractive. Charges may be positive or negative, and the electrical force may be attractive or repulsive.

21 Q 12 What experimental observations mentioned in the text rule out the possibility that the numerator in Coulomb's law contains the sum $(Q_1 + Q_2)$ rather than the product $Q_1 Q_2$?

The question is why Coulomb's law could not have the form

$$\mathbf{F}_{12} = k\frac{Q_1 + Q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}?$$

A good example would be that this formula predicts that the force between two equal and opposite charges is zero, since in that case

$$Q_1 + Q_2 = 0.$$

Very simple experiments show that such a prediction is not valid.

21 P 8 A person scuffing her feet on a wool rug on a dry day accumulates a net charge of $-50\,\mathrm{mC}$. How many excess electrons does this person get, and by how much does her mass increase?

The total charge $Q = -50\,\mathrm{mC} = -50 \times 10^{-3}\,\mathrm{C}$ is equal to ne, where the charge e of the electron is 1.60×10^{-19} C. Hence

$$n = -Q/e = 3.1 \times 10^{17}.$$

Her mass increase ΔM is nm_e, which is

$$(3.1 \times 10^{17}) \times (9.11 \times 10^{-31}\,\mathrm{kg}) = 2.85 \times 10^{-13}\,\mathrm{kg}.$$

21 P 9 What is the total charge of all the electrons in 1.0 kg of CO_2?

First work out the total number of electrons in 1 kg of CO_2. Each CO_2 molecule has a mass that comes almost entirely from the nuclear masses of C (12 amu) and O (16 amu), and each molecule has 22 electrons (six from C, and eight from each O). The total mass M of one molecule is about

$$M = 44\,\mathrm{amu} = 44 \times (1.66 \times 10^{-27}\,\mathrm{kg}) = 7.3 \times 10^{-26}\,\mathrm{kg}.$$

The number n of electrons in 1 kg of CO_2 will be

$$n = \frac{1\,\mathrm{kg}}{7.3 \times 10^{-26}\,\mathrm{kg}} \times 22 = 3.0 \times 10^{26},$$

for a total electron charge of

$$Q = (3.0 \times 10^{26}) \times (-1.60 \times 10^{-19}\,\mathrm{C}) = -4.8 \times 10^{7}\,\mathrm{C}.$$