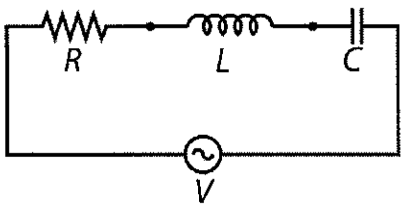
Physics 21
Fall, 2011

# Solution to Hour Exam #2

If you want to discuss the grading, you must speak with the grader by Dec. 7.
1: Malenda  2: Tupa  3: Faust  4: Beels  5: Glueckstein

**Problem 1.** For the following circuit:




(a) (8 pts.) For $R = 65.0\,\Omega$, $L = 100.0\,\text{mH}$, $C = 8.00\,\mu\text{F}$, and $\omega = 1200.0\,\text{rad/s}$, calculate the inductive reactance $X_L$ and the capacitive reactance $X_C$, and then draw a phasor diagram that is approximately to scale. Include a phasor for the ac voltage $V$. Draw a mark on your diagram to indicate the phase angle $\phi$. **Neatness counts.**

(b) (3 pts.) Evaluate the rms value of the current $I$ if the rms voltage supplied by the power supply is 6.00 V. Show all work.

(c) (3 pts.) Evaluate the phase angle $\phi$. Does the voltage across the power supply lead or lag the current?

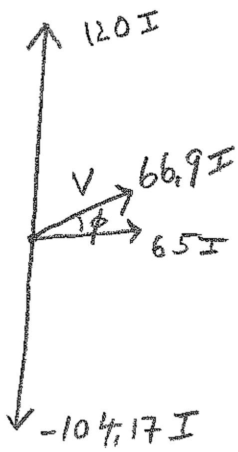
(d) (3 pts.) What is the average power delivered to the circuit by the power supply?

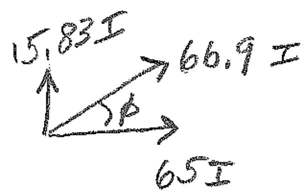
(e) (3 pts.) What is the time interval between a maximum in voltage across the power supply and the next maximum in voltage across the capacitor?

(a) $\omega L = 1200 \times 100 \times 10^{-3} = 120\,\Omega$

$\frac{1}{\omega C} = (1200 \times 8 \times 10^{-6})^{-1} = 104.17\,\Omega$

$R = 65\,\Omega$




↑ add vertical components first!

$\sqrt{65^2 + 15.83^2} = 66.9$

$V = ZI = 66.9\,I$

(b) $I_{rms} = \frac{V_{rms}}{Z} = \frac{6}{66.9} = 89.7\,\text{mA}$

(c) $\tan\phi = \frac{15.83}{65} = 0.2435$

$\phi = 13.7°$

(d) $P = I_{rms} V_{rms} \cos\phi$

$= 0.0897 \times 6 \times \cos 13.7°$

$= 0.523\ \text{W}$

(e) The angle between V (power supply) and $V_c$ (capacitor) is

$90° + 13.7° = 103.7°$

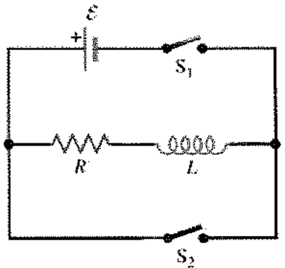
The period of the current oscillation is

$\frac{2\pi}{\omega} = \frac{2\pi}{1200} = 5.234\ \text{ms}$

Therefore interval is

$\frac{103.7}{360} \times 5.234 = 1.51\ \text{ms}$

**Problem 2.** In the figure below, $\mathcal{E} = 16.0$ V, the inductance is $L = 0.480$ H, and the resistance is $R = 32.0\,\Omega$. Both switches ($S_1$ and $S_2$) are initially open as shown.




(a) (5 pts.) At $t = 0$ switch $S_1$ is closed. What is the time constant of this circuit? What is the functional form of the current $i(t)$ through the resistor? Make an accurate plot of $i(t)$ as a function of time (for $t \geq 0$) in the box below.

(b) (5 pts.) What will be the energy stored in the inductor after a long time?

(c) (5 pts.) After the circuit has reached its steady state value, switch $S_1$ is opened and $S_2$ is simultaneously closed. What is the functional form of the current $i(t)$ through the resistor (starting from the time the switches are flipped)? Make an accurate plot of $i(t)$ as a function of time in the box below.

(d) (5 pts.) How long will it take before the energy in the inductor will decrease to 40% of its original value? At that time, what will be the current through the resistor?

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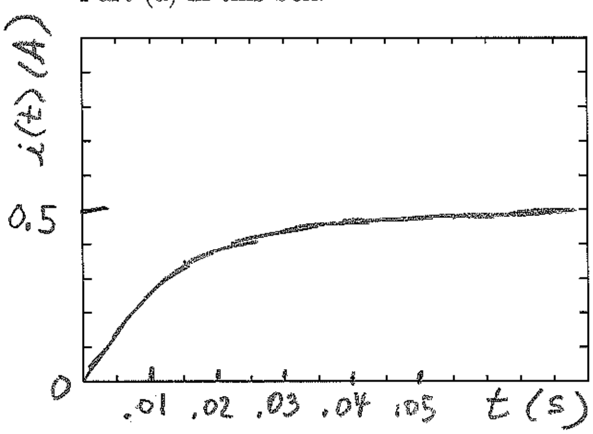
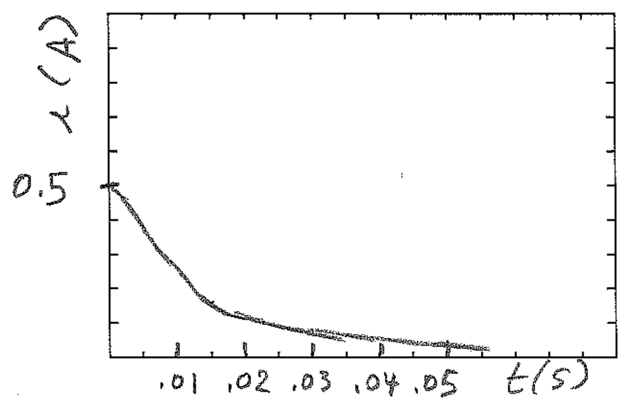
**Draw the graphs for parts (a) and (c) in the boxes below. Fill in numerical values in seconds and amperes for several tic marks on the horizontal and vertical axes to set reasonable scales for these axes. Neatness counts.**

Part (a) in this box:




Part (c) in this box:




(a) $\tau = \frac{L}{R} = \frac{.480}{32} = 0.015\,s$

SS current $= I = \frac{16\,V}{32\,\Omega} = 0.5\,A$

$i(t) = 0.5A\left(1 - e^{-t/.015s}\right)$

(b) $U_{ind} = \frac{1}{2} L I^2$

$= \frac{1}{2} \times .480\,(0.5)^2$

$= 0.060\,J$

(c) $i(t) = 0.5A\, e^{-t/.015s}$

(d) $U_{ind} = \frac{1}{2} L\, i(t)^2 \propto e^{-2t/\tau}$

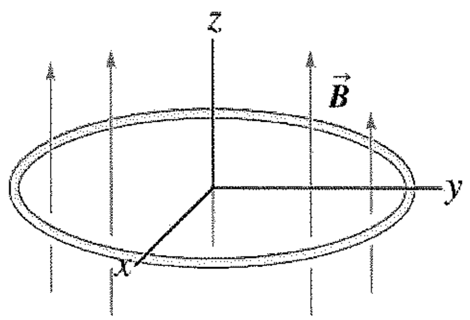
Solve $0.4 = e^{-2t/\tau}$

$t = \frac{1}{2}\tau \ln 2.5 = 6.87\,ms$

$i(t) = 0.5A\, e^{-\frac{6.87ms}{15ms}}$

$= 0.316\,A$

**Problem 3.** A circular loop of wire with radius $r$ is located in the $xy$ plane in a region where the magnetic field is spatially uniform and varies with time according to $\mathbf{B}(t) = B\exp(-at)\,\hat{\mathbf{k}}$.




(a) (3 pts.) Write Faraday's Law.

(b) (7 pts.) Find an expression for the magnitude $\mathcal{E}$ of the emf induced in the circuit. Your result should depend on $B$, $a$, $r$, $t$, and physical constants.

(c) (5 pts.) If $r = 20.0$ cm, $B = 0.140$ T, $a = 3.00\,\mathrm{s}^{-1}$, evaluate $\mathcal{E}$ at $t = 0.500$ s. If the resistance of the ring is $0.010\,\Omega$, what is the current through the ring at that time?

(d) (5 pts.) If you look down on the wire loop from above, in what direction will the induced current flow (clockwise or counterclockwise)? Explain your reasoning. [Assume the values of $r$, $B$, $a$, and $t$ given in part (c)].

(a) $\mathcal{E} = \int \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$

(b) $\mathcal{E} = -\frac{d}{dt}\left[B e^{-at}\,\pi r^2\right]$

$= -\pi r^2 B \frac{d}{dt} e^{-at}$

$\mathcal{E} = \pi r^2 a B e^{-at}$

(c) $\mathcal{E} = \pi (0.2\,\mathrm{m})^2 (3\,\mathrm{s}^{-1})\, 0.14\,\mathrm{T} \times e^{-3\times 0.5}$

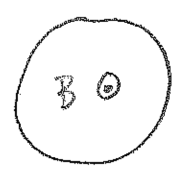
$= 0.0118\ \mathrm{V} = 11.8\ \mathrm{mV}$

$\mathcal{E} = IR \Rightarrow$

$I = \frac{\mathcal{E}}{R} = \frac{.0118\,\mathrm{V}}{.01\,\Omega}$

$= 1.18\ \mathrm{A}$

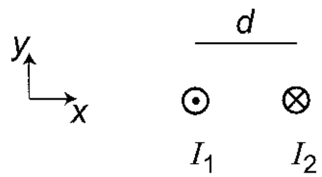
(d) As you look down on the loop, the field points towards you and is decreasing.




Therefore the induced current must cause a field that is also towards you, to oppose the decreasing B.

By right hand rule, I must be counterclockwise.

**Problem 4.** Two long, parallel wires separated by a distance $d$ carry currents $I_1$ and $I_2$ in opposite directions as shown in the end-on view below:




(a) (6 pts.) Find an expression for the vector force $\mathbf{F}_{12}$ that wire 1 exerts on a length $L$ of wire 2. Draw the appropriate fields and use the right hand rule to determine whether the force is attractive or repulsive.

(b) (4 pts.) Compare the force that wire 1 exerts on a length $L$ of wire 2 with the force that wire 2 exerts on a length $L$ of wire 1. Are the vector forces the same?

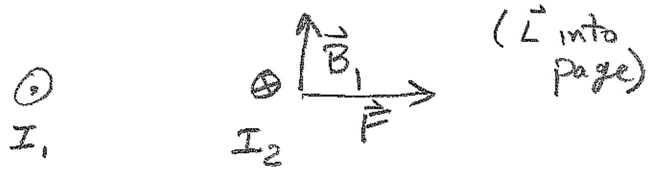
(c) (4 pts.) If $d = 12.0$ cm, $I_1 = 15.0$ A, and $I_2 = 45.0$ A, then find the magnitude of the force per meter that wire 1 exerts on wire 2.

(d) (6 pts.) For the values given in part (c), specify clearly all the locations where the total magnetic field near the wires is zero.

(a) $B_1$ of wire 1 at distance $d$ is $\frac{\mu_0 I_1}{2\pi d}\hat{j}$. Force on length $L$ of wire 2 is

$$\vec{F} = I_2 \vec{L} \times \vec{B} = I_2 L \frac{\mu_0 I_1}{2\pi d}\hat{i}$$

$$= \frac{\mu_0}{2\pi}\frac{I_1 I_2 L}{d}\hat{i}$$




$\vec{F}$ is repulsive

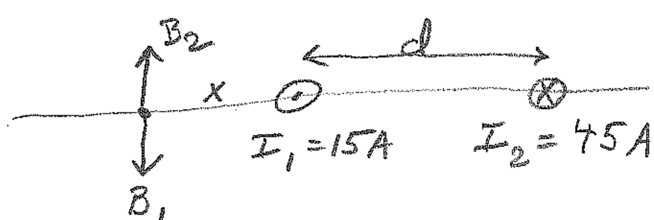
(b) By same argument we could find the other force has same magnitude, opposite direction (also repulsive). (So forces are not the same.)

(c) $$\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

$$= 2\times10^{-7}\ \frac{15\times45}{0.12}$$

$$= 1.125\times10^{-3}\ \frac{N}{m}$$

(d) The field can be zero a distance $x$ to the left of wire 1.




Since $B \propto \frac{I}{r}$,

$$\frac{I_1}{x} = \frac{I_2}{x+d}$$

$$\Rightarrow x = \frac{I_1}{I_2 - I_1}\, d$$

$$= \frac{15}{45-15}\ 12\text{ cm}$$

$$= 6\text{ cm}$$

**Problem 5.** A 300 nF capacitor is charged by a 24.0 V power supply, then disconnected from the power supply and connected in series with a 370 mH inductor. The inductor and the capacitor make a complete circuit. For the following questions labelled (a), (b), and (c), assume there are no losses in the circuit.

(a) (5 pts.) Calculate the oscillation frequency of the circuit.

(b) (5 pts.) Calculate the energy stored in the capacitor at time $t = 0$ (the moment of connection with the inductor).

(c) (5 pts.) What is the earliest time $t > 0$ for which all the energy will be contained in the magnetic field of the inductor?

(d) (5 pts.) If the resistance in the wires of the circuit is $5.00\,\mathrm{m}\Omega$, calculate how long it will take for the energy in the circuit to diminish to 30% of its original value.

(a) $\omega^2 = \frac{1}{LC} = \frac{1}{.370 \times 300 \times 10^{-7}}$

$= 3.00 \times 10^3$ rad/s

(b) $U_{cap} = \frac{1}{2} CV^2$

$= \frac{1}{2} \times 300 \times 10^{-9} (24)^2$

$= 8.64 \times 10^{-5}$ J

(c) $\frac{1}{4}$ cycle

$\omega = 3000$ rad/s $= \frac{2\pi}{T}$

$\Rightarrow T = 2.09$ ms

$\frac{T}{4} = 5.23 \times 10^{-4}$ s

(d) The eq. sheet tells us that charge $q(t)$ of damped RLC circuit decays as $e^{-Rt/2L}$.

$i(t) = \frac{dq}{dt}$ will also decay as $e^{-Rt/2L}$.

Since energy $U$ is proportional to $q(t)^2/C$ or $Li(t)^2$,

$U \propto e^{-Rt/L}$,

So solve

$e^{-Rt/L} = 0.3$

$\Rightarrow t = \frac{L}{R} \ln \frac{10}{3}$

$= \frac{0.370\,H}{.005\,\Omega} \ln \frac{10}{3}$

$= 89.1$ s