Physics 21
Fall, 2010

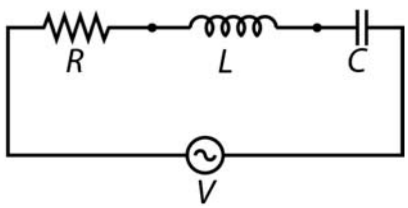
Hour Exam #2

1______ 2______ 3______ 4______ 5______ Total______ Name: ________________________

Recitation Time ______ Recitation Leader ____________ November 10, 2010

This exam is closed notes and closed book. You must show enough work on each problem to convince the grader you understand how to solve the problem. You may use a calculator, but **show every number that you use to get numerical results. The penalty for arithmetic errors is small if the grader can tell what you intended to do.** Give units for all final answers. There is an equation sheet on the last page. All problems count 20 points.

Problem 1. For the following circuit:



(a) (8 pts.) If $R = 22.0\,\Omega$, $L = 25.0\,\text{mH}$, $C = 10.0\,\mu\text{F}$, and $\omega = 800.0\,\text{rad/s}$, draw a phasor diagram that is approximately to scale. Include a phasor for the ac voltage V, and give the length of each phasor. Draw a mark on your diagram to indicate the phase angle ϕ.

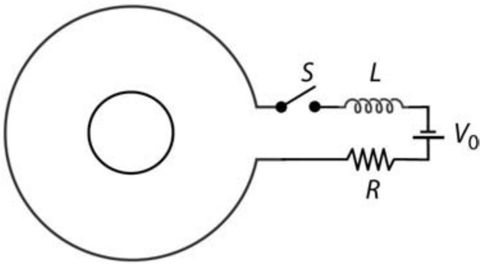
(b) (3 pts.) Evaluate the peak value of the current I if the peak voltage supplied by the power supply is 8.00 V. Show all work.

(c) (3 pts.) Evaluate the phase angle ϕ. Does the voltage lead or lag the current?

(d) (3 pts.) What is the average power delivered to the circuit by the power supply?

(e) (3 pts.) What is the time interval between a maximum in voltage across the inductor and the next maximum in voltage across the power supply?

Problem 2. A small circular wire loop is inside a larger loop that is connected to the circuit as shown. The values of the circuit elements are $L = 15.0\,\text{H}$, $R = 30.0\,\Omega$, and $V_0 = 12.0$ V. At $t = 0$ the switch S is closed to complete the circuit.



(a) (5 pts.) What is the time constant of this circuit?

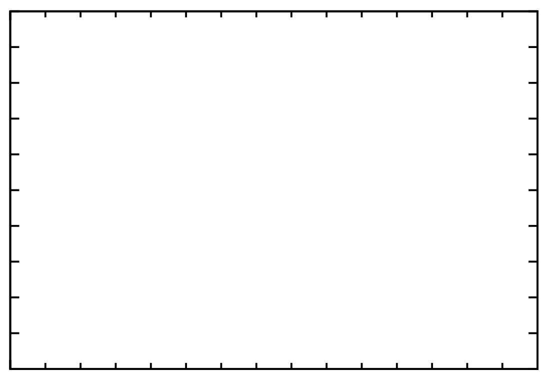
(b) (5 pts.) What is the current in the large loop a long time after the switch has been closed?

(c) (5 pts.) Using the box printed below, carefully draw a plot that shows the current $i(t)$ in the large loop as a function of time, starting at $t = 0$. Fill in numerical values in seconds and amperes for several tic marks on the horizontal and vertical axes to set reasonable scales for these axes.

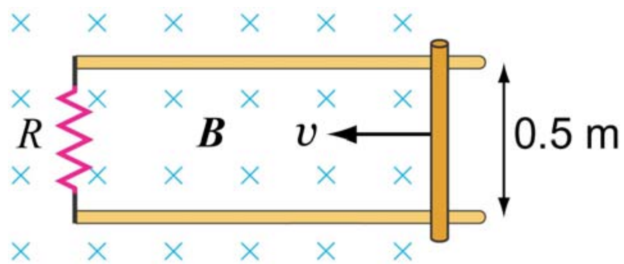
(d) (3 pts.) Is the direction of the induced current in the small circular wire loop soon after the switch is closed **clockwise** or **counterclockwise**?

(e) (2 pts.) Now this experiment is repeated. The large loop remains the same, but the small wire inner loop is replaced with one that has half the diameter. When the switch S is closed, would the induced emf in the wire loop be smaller or larger than it was in the first experiment? Briefly explain your answer.

Draw the graph for part (c) in this box:



Problem 3. A metal bar moves to the left with constant speed $v = 8.00\,\mathrm{m/s}$ through a uniform magnetic field of magnitude $B = 1.5\,\mathrm{T}$ as shown in the diagram. The distance between the rails is $0.5\,\mathrm{m}$. The only resistance in the circuit may be taken to be the resistance $R = 24\,\Omega$ shown.



(a) (4 pts.) Give Faraday's Law (as an equation).

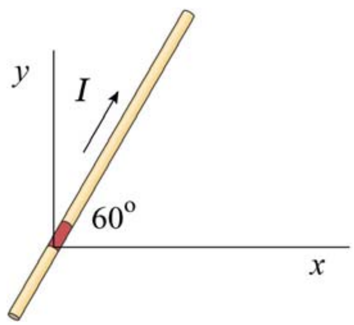
(b) (4 pts.) What is the magnitude of the emf induced in the circuit (before the metal bar hits the resistor)?

(c) (4 pts.) Is the direction of the current induced in the circuit **clockwise** or **counterclockwise**?

(d) (4 pts.) Calculate the current through the resistor.

(e) (4 pts.) Because of the induced current in the circuit, the magnetic field exerts a force on the moving metal bar. Find the magnitude and direction of that force.

Problem 4. The long wire shown in the diagram lies in the xy plane and carries a current $I = 0.500\,\text{A}$. The positive z axis points out of the page.



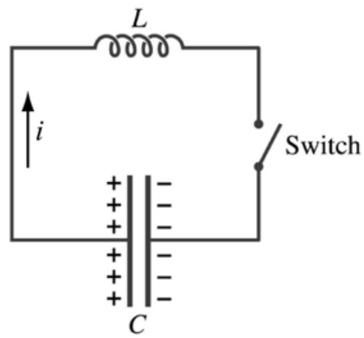
For this problem you are to find the contribution $d\mathbf{B}$ to the magnetic field at several points due to the dark segment $d\mathbf{l}$ of the wire centered at the origin. The dark segment has a length 3.0 mm and makes an angle of 60° with the x axis.

(a) (6 pts.) Write a vector expression for the current element $I\,d\mathbf{l}$. (Use the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.)

(b) (7 pts.) Find $d\mathbf{B}$ at $x = 0$, $y = 0$, $z = 3.0$ m

(c) (7 pts.) Find $d\mathbf{B}$ at $x = 4.2$ m, $y = 7.3$ m, $z = 0$

Problem 5. For the following circuit, $C = 25\,\mu$F and $L = 32$ mH. Just before the switch is closed at time $t = 0$, the charge on the capacitor is $Q_0 = 1.25$ mC.



(a) (4 pts.) What is the total energy stored in the circuit before the switch is closed?

(b) (3 pts.) Give the equation that relates the current i shown on the diagram and the charge q on the capacitor. (i gives the direction of positive current flow just after the switch is closed.)

(c) (6 pts.) Write the loop equation for this circuit and convert it to a differential equation for the charge on the capacitor, $q(t)$. Verify that the solution to the differential equation is

$$q(t) = Q_0 \cos \omega t.$$

(d) (4 pts.) Calculate the value of ω for this circuit.

(e) (3 pts.) What is the energy stored in the electric field of the capacitor at an instant when the magnitude of the **magnetic field** in the inductor is 60% of its maximum value?

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Equation Sheet

speed of light *in vacuo*	c	3.00×10^8 m/s	Planck's constant	h	6.626×10^{-34} J s
Gravitational constant	G	6.67×10^{-11} N m^2/kg^2	Planck's constant/(2π)	$\hbar = h/2\pi$	1.055×10^{-34} J s
Avogadro's Number	N_A	6.02×10^{23} mol^{-1}	electron rest mass	m_e	9.11×10^{-31} kg
Boltzmann's constant	k_B	1.38×10^{-23} J/K	proton rest mass	m_p	1.6726×10^{-27} kg
charge on electron	e	1.60×10^{-19} C	neutron rest mass	m_n	1.6749×10^{-27} kg
free space permittivity	ϵ_0	8.85×10^{-12} C^2/(N m^2)	atomic mass unit	u	1.6605×10^{-27} kg
free space permeability	μ_0	$4\pi \times 10^{-7}$ T m/A	$1/(4\pi\epsilon_0)$	k	8.99×10^9 N m^2/C^2
gravitational acceleration	g	9.807 m/s^2			

$$\mathbf{F}_{\mathbf{2\,on\,1}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2(\mathbf{r}_1 - \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3}$$

$$\mathbf{F} = q\mathbf{E}$$

$$d\mathbf{E}\,(\text{at } \mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{dQ\,(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

$$\mathbf{E} = -\nabla V = -\left(\hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}\right)$$

$$V_f - V_i = -\int_i^f \mathbf{E}\cdot d\mathbf{l}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r};\ dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{|\mathbf{r} - \mathbf{r}'|}$$

$$u_{\text{elec}} = \tfrac{1}{2}\epsilon_0 E^2,\ u_{\text{mag}} = \frac{1}{2\mu_0}B^2$$

$$\text{Work} = \int \mathbf{F}\cdot d\mathbf{l}$$

$$E_{\text{line}} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r};\ E_{\text{plane}} = \frac{\sigma}{2\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} \quad \text{for } \parallel \text{ plate capacitor}$$

$$Q = CV;\ C = \epsilon_0 K\frac{A}{d} = \epsilon\frac{A}{d}$$

$$U_{\text{cap}} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\frac{Q^2}{C}$$

$$U_{\text{ind}} = \tfrac{1}{2}LI^2$$

$$V = IR \qquad R = \frac{\rho L}{A}$$

$$P = IV \qquad P = I^2 R$$

$$R = mv_\perp/(qB) \quad \text{circ. orbit}$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\xi_i = C_i$ (series) or R_i (parallel):

$$\frac{1}{\xi_{\text{effective}}} = \frac{1}{\xi_1} + \frac{1}{\xi_2}$$

$\xi_i = C_i$ (parallel) or R_i (series):

$$\xi_{\text{effective}} = \xi_1 + \xi_2$$

$$X_R = R,\ X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

RC time constant $= RC$

LR time constant $= L/R$

$Q(t)$ for RLC circuit

$$Q_0 \exp(-Rt/2L)\cos\omega t$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B};\ d\mathbf{F} = I d\mathbf{l}\times\mathbf{B}$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{l}\times(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

long wire: $B = \dfrac{\mu_0 I}{2\pi R}$

center loop: $B = \mu_0 I/2R$

$$I = \frac{dQ}{dt} \qquad I = -neAv_d$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}, \qquad \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$\chi_m = \frac{\mu}{\mu_0} - 1$$

$$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B} \qquad \boldsymbol{\mu} = I\mathbf{A}$$

solenoid $B = \mu_0 n I$

solenoid $L = \mu_0 N^2 A/l$

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A}$$

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I + \mu_0\epsilon_0\frac{d}{dt}\int \mathbf{E}\cdot d\mathbf{A}$$

$$\sin(a \pm b) = \sin a\cos b \pm \cos a \sin b$$

$$\sin(\theta \pm \tfrac{\pi}{2}) = \sin\theta\cos\tfrac{\pi}{2} \pm \cos\theta\sin\tfrac{\pi}{2} = \pm\cos\theta$$

$$\cos(a \pm b) = \cos a\cos b \mp \sin a \sin b$$

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$C = 2\pi r$	circumference of circle
$C = \pi d$	circumference of circle
$A = \pi r^2$	area of circle
$A = 4\pi r^2$	surface area of sphere
$V = \frac{4}{3}\pi r^3$	volume of sphere

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\int \frac{du}{\sqrt{a^2 + u^2}} = \ln\left(u + \sqrt{a^2 + u^2}\right)$$

$$\int \frac{u\,du}{\sqrt{a^2 + u^2}} = \sqrt{a^2 + u^2}$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$$

$$\int \frac{u\,du}{a^2 + u^2} = \tfrac{1}{2}\ln\left(a^2 + u^2\right)$$

$$\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 + u^2}}$$

$$\int \frac{u\,du}{(a^2 + u^2)^{3/2}} = -\frac{1}{\sqrt{a^2 + u^2}}$$

$$\int e^{au}\,du = \frac{1}{a}e^{au}$$

$$\int \ln u\,du = u\ln u - u$$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1}$$

$$\int \frac{du}{a + bu} = \frac{1}{b}\ln(a + bu)$$

$$\int \frac{du}{u} = \ln u$$

$$\int_0^{2\pi}\cos^2\theta\,d\theta = \int_0^{2\pi}\sin^2\theta\,d\theta = \pi$$

$$v = \sqrt{T/\rho} \quad (T\text{=tension})$$

$$v = (347.4\,\text{m/s})\sqrt{T/300}$$

$$v = \lambda f = \omega/k$$

$$\omega = 2\pi f \qquad k = 2\pi/\lambda$$

$$T = 1/f \quad (T\text{=period})$$

$$\langle P\rangle = \tfrac{1}{2}\rho A^2\omega^2 v$$

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$$

$$\overline{S} = \tfrac{1}{2}\epsilon_0 c E_0^2 = \tfrac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0} = \frac{E_{\text{rms}}B_{\text{rms}}}{\mu_0}$$

$$c = 1/\sqrt{\epsilon_0\mu_0}$$

$$\mathbf{E}\times\mathbf{B} \propto \mathbf{v} \quad \text{(plane wave)}$$

$$\Delta x\Delta p \gtrsim \hbar \quad (\hbar = h/2\pi)$$

$$\lambda = h/p \quad \text{(de Broglie)}$$

$$-\frac{\hbar^2}{2M}\frac{\partial^2\psi}{\partial x^2} = i\hbar\frac{\partial\psi}{\partial t}$$

$$\text{KE} = p^2/(2M)$$

$$p = \hbar k \qquad E = \hbar\omega = hf$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

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