Physics 21
Fall, 2010

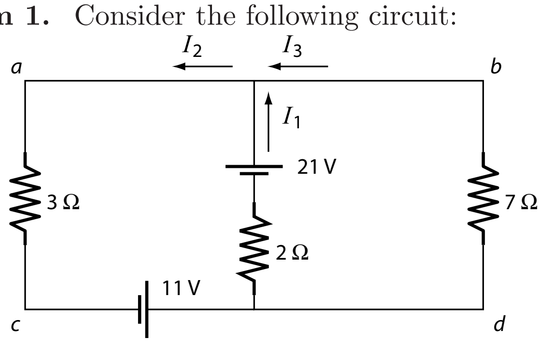
Hour Exam #1

1______ 2______ 3______ 4______ 5______ Total______ Name: ________________________

Recitation Time ______ Recitation Leader ____________ October 6, 2010

This exam is closed notes and closed book. You must show enough work on each problem to convince the grader you understand how to solve the problem. Make it easy for the grader to identify your final answers to each question or part of a question. You may use a calculator, but **show every number that you use to get numerical results. The penalty for arithmetic errors is small if the grader can tell what you intended to do.** Give units for all final answers. There is an equation sheet on the last page. All problems count 20 points.

Problem 1. Consider the following circuit:



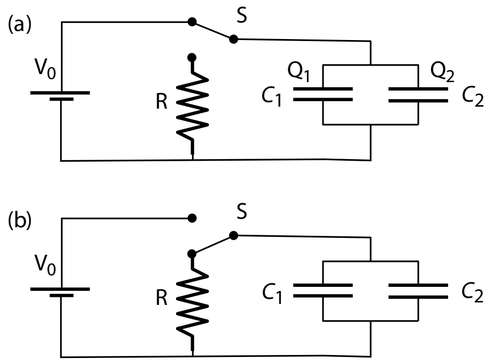
(a) (13 pts.) Write Kirchhoff's loop and junction (or node) equations needed to determine the currents I_1, I_2, and I_3. Use the currents and their directions specified by the arrows in the diagram. **You must write these equations as defined in Physics 21.** You should have three equations for three unknowns. **Draw clearly and label on the diagram the loop used to determine each loop equation.**

(b) (3 pts.) Determine the currents I_1, I_2, and I_3 (including the correct sign) by explicit solution of the equations you determined in part (a). **You must show your work.**

(c) (2 pts.) Use the currents you calculated in part (b) to find the potential difference $V_d - V_a$ between the corner points labelled d and a on the diagram, using the path through corner point b. Show your work.

(d) (2 pts.) Repeat part (c), but take the path through corner point c. Show your work, including the potential change across the resistor and battery separately. You should get the same value for $V_d - V_a$ that you found in part (c).

Problem 2. Two parallel plate capacitors are connected to a battery and a resistor as shown in the circuit. For the first capacitor, $C_1 = 1.24$ nF; for the other, $C_2 = 3.72$ nF. The voltage of the battery is $V_0 = 12$ volts, and the resistance $R = 3500\ \Omega$.



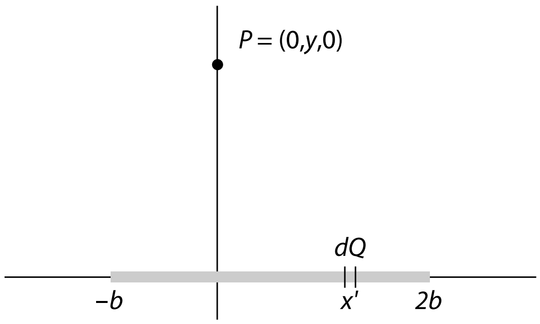
(a) (8 pts.) Assuming that the circuit in panel (a) has been connected for a very long time, what are the charges Q_1 and Q_2 on each capacitor? What is the energy stored in each capacitor? Give numerical answers.

(b) (6 pts.) At time $t = 0$ the switch S is flipped so that the circuit appears as shown in panel (b), and the capacitors discharge through the resistor. What is the time constant of the circuit?

(c) (2 pts.) What is the current that starts to flow in the circuit of panel (b) just after $t = 0$?

(d) (4 pts.) At what time t' will the total charge on the two capacitors diminish to 60% of its initial value? What fraction of the initial energy will remain at time $t = t'$?

Problem 3. For this problem you are to find the electric field at the point P on the y axis due to the charge distribution on the x axis. A total charge of $3Q$ is uniformly spread out from $x = -b$ to $x = +2b$.



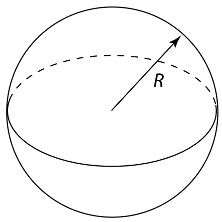
(a) (3 pts.) What is the linear charge density λ on the rod?

(b) (3 pts.) On the diagram shown, draw the vector $(\mathbf{r} - \mathbf{r}')$ (as defined in class) from the charge dQ on the rod at $x = x'$ to the point P on the y axis. Write an expression for the vector $(\mathbf{r} - \mathbf{r}')$ that indicates its components.

(c) (7 pts.) Write an expression for the electric field $d\mathbf{E}$ at the point P due to the element of charge dQ at the point x' on the rod. The expression for $d\mathbf{E}$ should be in terms of Q, b, dx', x', y, and NOT dQ.

(d) (7 pts.) Integrate to find ONLY THE y COMPONENT of the electric field $\mathbf{E}$ at the point P due to the rod. Your answer should be in terms of Q, b, and y.

Problem 4. A solid sphere of radius $R = 2.00$ mm made of an insulating material has a uniform volume charge density of $\rho = 4.80 \times 10^{-12}\,\mathrm{C/m^3}$.



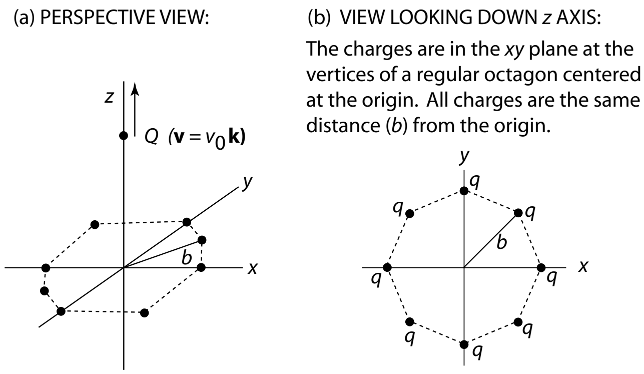
(a) (3 pts.) What is the total charge on the sphere?

(b) (3 pts.) Write Gauss's Law.

(c) (7 pts.) Use Gauss's Law to find the magnitude of the electric field at a distance $r = 1.00$ mm from the center of the sphere. Sketch the Gaussian surface that you use and explain your result.

(d) (7 pts.) Find the magnitude of the electric field at the surface of the sphere.

Problem 5. Panel (a) shows a negative ion with mass $m = 1.50 \times 10^{-26}$ kg and charge $Q = -2.00 \times 10^{-19}$ C moving along the z axis away from eight identical point charges with $q = 3.25 \times 10^{-19}$ C. These eight charges are fixed in the xy plane as shown in panel (b); all eight charges are the same distance $b = 0.5$ nm from the origin.



(a) (7 pts.) Write an expression for the potential at an arbitrary point on the z axis due to the fixed charge distribution in the xy plane. Give your answer in terms of q, z, b, and standard physical constants. *Hint:* Use symmetry.

(b) (6 pts.) At the instant shown in the figure, the z coordinate of the negative ion is 1.2 nm, and its speed is $v_0 = 15.3$ km/s. Find the potential energy, kinetic energy, and total energy of the ion. Show all work and give numerical answers in joules.

(c) (7 pts.) The negative ion will slow down and reach a point $z = z'$ where its speed is zero. Show explicitly the equation that must be solved to find z', and then solve that equation for z'. Assume that the particle is constrained to stay on the z axis.

Physics 21
Fall, 2010

Equation Sheet

speed of light *in vacuo*	c	3.00×10^{8} m/s	Planck's constant	h	6.626×10^{-34} J s
Gravitational constant	G	6.67×10^{-11} N m^2/kg^2	Planck's constant/(2π)	$\hbar=h/2\pi$	1.055×10^{-34} J s
Avogadro's Number	N_A	6.02×10^{23} mol^{-1}	electron rest mass	$m_{\rm e}$	9.11×10^{-31} kg
Boltzmann's constant	$k_{\rm B}$	1.38×10^{-23} J/K	proton rest mass	$m_{\rm p}$	1.6726×10^{-27} kg
charge on electron	e	1.60×10^{-19} C	neutron rest mass	$m_{\rm n}$	1.6749×10^{-27} kg
free space permittivity	ϵ_0	8.85×10^{-12} C^2/(N m^2)	atomic mass unit	u	1.6605×10^{-27} kg
free space permeability	μ_0	$4\pi\times10^{-7}$ T m/A	$1/(4\pi\epsilon_0)$	k	8.99×10^{9} N m^2/C^2
gravitational acceleration	g	9.807 m/s^2			

$$\mathbf{F}_{\mathbf{2\,on\,1}}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2(\mathbf{r}_1-\mathbf{r}_2)}{|\mathbf{r}_1-\mathbf{r}_2|^3}$$

$$\mathbf{F}=q\mathbf{E}$$

$$d\mathbf{E}\,(\text{at}\,\mathbf{r})=\frac{1}{4\pi\epsilon_0}\frac{dQ\,(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$\mathbf{E}=-\nabla V=-\left(\hat{\mathbf{i}}\frac{\partial V}{\partial x}+\hat{\mathbf{j}}\frac{\partial V}{\partial y}+\hat{\mathbf{k}}\frac{\partial V}{\partial z}\right)$$

$$V_f-V_i=-\int_i^f\mathbf{E}\cdot d\mathbf{l}$$

$$V=\frac{1}{4\pi\epsilon_0}\frac{Q}{r};\ dV=\frac{1}{4\pi\epsilon_0}\frac{dQ}{|\mathbf{r}-\mathbf{r}'|}$$

$$u_{\rm elec}=\tfrac12\epsilon_0E^2,\ u_{\rm mag}=\frac{1}{2\mu_0}B^2$$

$$\text{Work}=\int\mathbf{F}\cdot d\mathbf{l}$$

$$E_{\rm line}=\frac{1}{2\pi\epsilon_0}\frac{\lambda}{r};\ E_{\rm plane}=\frac{\sigma}{2\epsilon_0}$$

$$E=\frac{Q}{\epsilon_0A}=\frac{\sigma}{\epsilon_0}\quad\text{for}\parallel\text{plate capacitor}$$

$$Q=CV;\ C=\epsilon_0K\frac{A}{d}=\epsilon\frac{A}{d}$$

$$U_{\rm cap}=\tfrac12CV^2=\tfrac12\frac{Q^2}{C}$$

$$U_{\rm ind}=\tfrac12LI^2$$

$$V=IR\qquad R=\frac{\rho L}{A}$$

$$P=IV\qquad P=I^2R$$

$$R=mv_\perp/(qB)\quad\text{circ. orbit}$$

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}}&\hat{\mathbf{j}}&\hat{\mathbf{k}}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}$$

$\xi_i=C_i$ (series) or R_i (parallel):

$$\frac{1}{\xi_{\rm effective}}=\frac{1}{\xi_1}+\frac{1}{\xi_2}$$

$\xi_i=C_i$ (parallel) or R_i (series):

$$\xi_{\rm effective}=\xi_1+\xi_2$$

$$X_R=R,\ X_L=\omega L,\ X_C=\frac{1}{\omega C}$$

RC time constant $=RC$

LR time constant $=L/R$

$Q(t)$ for RLC circuit

$$Q_0\exp(-Rt/2L)\cos\omega t$$

$$\omega^2=\frac{1}{LC}-\frac{R^2}{4L^2}$$

$$\mathbf{F}=q\mathbf{v}\times\mathbf{B};\ d\mathbf{F}=Id\mathbf{l}\times\mathbf{B}$$

$$d\mathbf{B}=\frac{\mu_0}{4\pi}\frac{Id\mathbf{l}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

long wire: $B=\dfrac{\mu_0I}{2\pi R}$

center loop: $B=\mu_0I/2R$

$$I=\frac{dQ}{dt}\qquad I=-neAv_{\rm d}$$

$$\frac{V_{\rm s}}{V_{\rm p}}=\frac{N_{\rm s}}{N_{\rm p}},\qquad\frac{I_{\rm s}}{I_{\rm p}}=\frac{N_{\rm p}}{N_{\rm s}}$$

$$\chi_{\rm m}=\frac{\mu}{\mu_0}-1$$

$$\boldsymbol{\tau}=\boldsymbol{\mu}\times\mathbf{B}\qquad\boldsymbol{\mu}=I\mathbf{A}$$

solenoid $B=\mu_0nI$

solenoid $L=\mu_0N^2A/l$

$$\oint\mathbf{E}\cdot d\mathbf{A}=\frac{Q}{\epsilon_0}$$

$$\oint\mathbf{B}\cdot d\mathbf{A}=0$$

$$\oint\mathbf{E}\cdot d\mathbf{l}=-\frac{d}{dt}\int\mathbf{B}\cdot d\mathbf{A}$$

$$\oint\mathbf{B}\cdot d\mathbf{l}=\mu_0I+\mu_0\epsilon_0\frac{d}{dt}\int\mathbf{E}\cdot d\mathbf{A}$$

$$\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$$

$$\sin(\theta\pm\tfrac{\pi}{2})=\sin\theta\cos\tfrac{\pi}{2}\pm\cos\theta\sin\tfrac{\pi}{2}=\pm\cos\theta$$

$$\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$$

$$\sin a+\sin b=2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$C=2\pi r$	circumference of circle
$C=\pi d$	circumference of circle
$A=\pi r^2$	area of circle
$A=4\pi r^2$	surface area of sphere
$V=\frac43\pi r^3$	volume of sphere

$$ax^2+bx+c=0\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\int\frac{du}{\sqrt{a^2+u^2}}=\ln\left(u+\sqrt{a^2+u^2}\right)$$

$$\int\frac{u\,du}{\sqrt{a^2+u^2}}=\sqrt{a^2+u^2}$$

$$\int\frac{du}{a^2+u^2}=\frac1a\tan^{-1}\left(\frac{u}{a}\right)$$

$$\int\frac{u\,du}{a^2+u^2}=\tfrac12\ln\left(a^2+u^2\right)$$

$$\int\frac{du}{(a^2+u^2)^{3/2}}=\frac{u}{a^2\sqrt{a^2+u^2}}$$

$$\int\frac{u\,du}{(a^2+u^2)^{3/2}}=-\frac{1}{\sqrt{a^2+u^2}}$$

$$\int e^{au}\,du=\frac1ae^{au}$$

$$\int\ln u\,du=u\ln u-u$$

$$\int u^n\,du=\frac{1}{n+1}u^{n+1}$$

$$\int\frac{du}{a+bu}=\frac1b\ln(a+bu)$$

$$\int\frac{du}{u}=\ln u$$

$$\int_0^{2\pi}\cos^2\theta\,d\theta=\int_0^{2\pi}\sin^2\theta\,d\theta=\pi$$

$$v=\sqrt{T/\rho}\quad(T=\text{tension})$$

$$v=(347.4\,\text{m/s})\sqrt{T/300}$$

$$v=\lambda f=\omega/k$$

$$\omega=2\pi f\qquad k=2\pi/\lambda$$

$$T=1/f\quad(T=\text{period})$$

$$\langle P\rangle=\tfrac12\rho A^2\omega^2v$$

$$\frac{\partial^2D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2D}{\partial t^2}$$

$$\mathbf{S}=\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$$

$$\overline{S}=\tfrac12\epsilon_0cE_0^2=\tfrac12\frac{c}{\mu_0}B_0^2=\frac{E_0B_0}{2\mu_0}=\frac{E_{\rm rms}B_{\rm rms}}{\mu_0}$$

$$c=1/\sqrt{\epsilon_0\mu_0}$$

$\mathbf{E}\times\mathbf{B}\propto\mathbf{v}$ (plane wave)

$\Delta x\Delta p\gtrsim\hbar$ $\quad(\hbar=h/2\pi)$

$\lambda=h/p$ (de Broglie)

$$-\frac{\hbar^2}{2M}\frac{\partial^2\psi}{\partial x^2}=i\hbar\frac{\partial\psi}{\partial t}$$

$$\text{KE}=p^2/(2M)$$

$$p=\hbar k\qquad E=\hbar\omega=hf$$

$$e^{i\theta}=\cos\theta+i\sin\theta$$

LEHIGH UNIVERSITY

August 29, 2010