



# Previous lecture

- Wave Equation

$$\frac{\partial^2 D}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2} = 0$$

- Traveling and standing waves
- Formulas:

$$v = f\lambda, \quad \omega = 2\pi f, \quad k = 2\pi/\lambda$$
$$\sin\left(\omega\left(t - x/v\right)\right) \rightarrow \sin(\omega t - kx)$$

# Announcements

- 2<sup>nd</sup> Hour Exam Tuesday Nov. 2 at 4:10 pm in Packard 101
- Study Guide and Equation Sheet are on the web
- Practice Exam will be on the web by the end of the week

# Today

- Superposition
- Beat Frequency
- Reflection of waves
- Interference

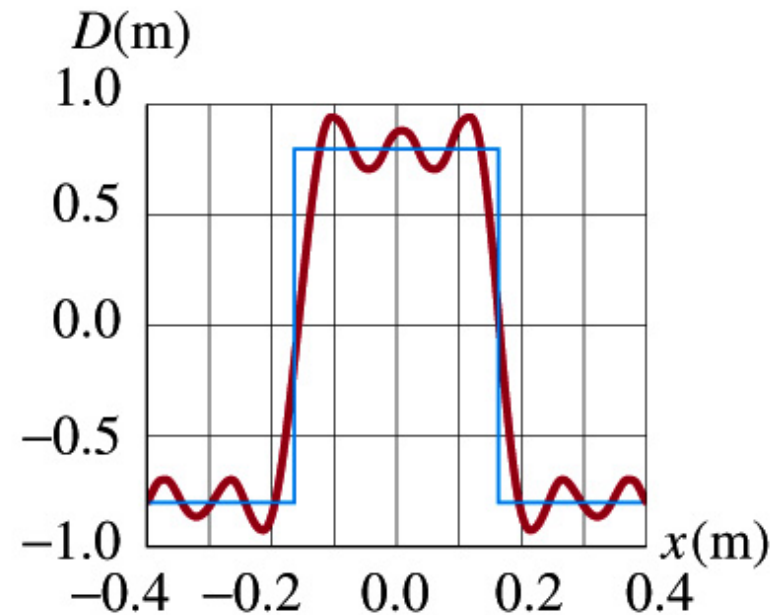
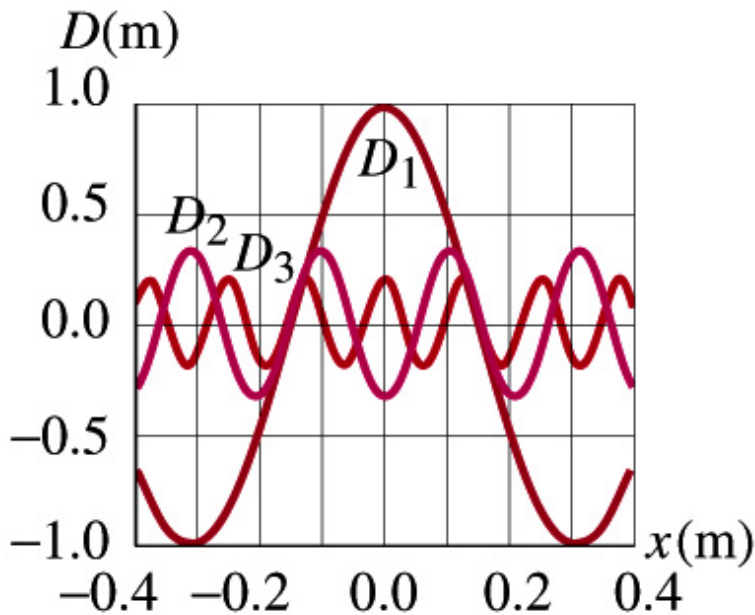
# Mathematical Note

If two waves are solutions to the wave equation, then so is their superposition (sum).

We get a lot of mileage out of superposition.

# Superposition

Any pulse can be written as a superposition of sines and cosines.



# Trigonometry

$$\sin a + \sin b =$$

$$2 \cos \left( \frac{a - b}{2} \right) \sin \left( \frac{a + b}{2} \right)$$

$$\sin a - \sin b =$$

$$2 \cos \left( \frac{a + b}{2} \right) \sin \left( \frac{a - b}{2} \right)$$

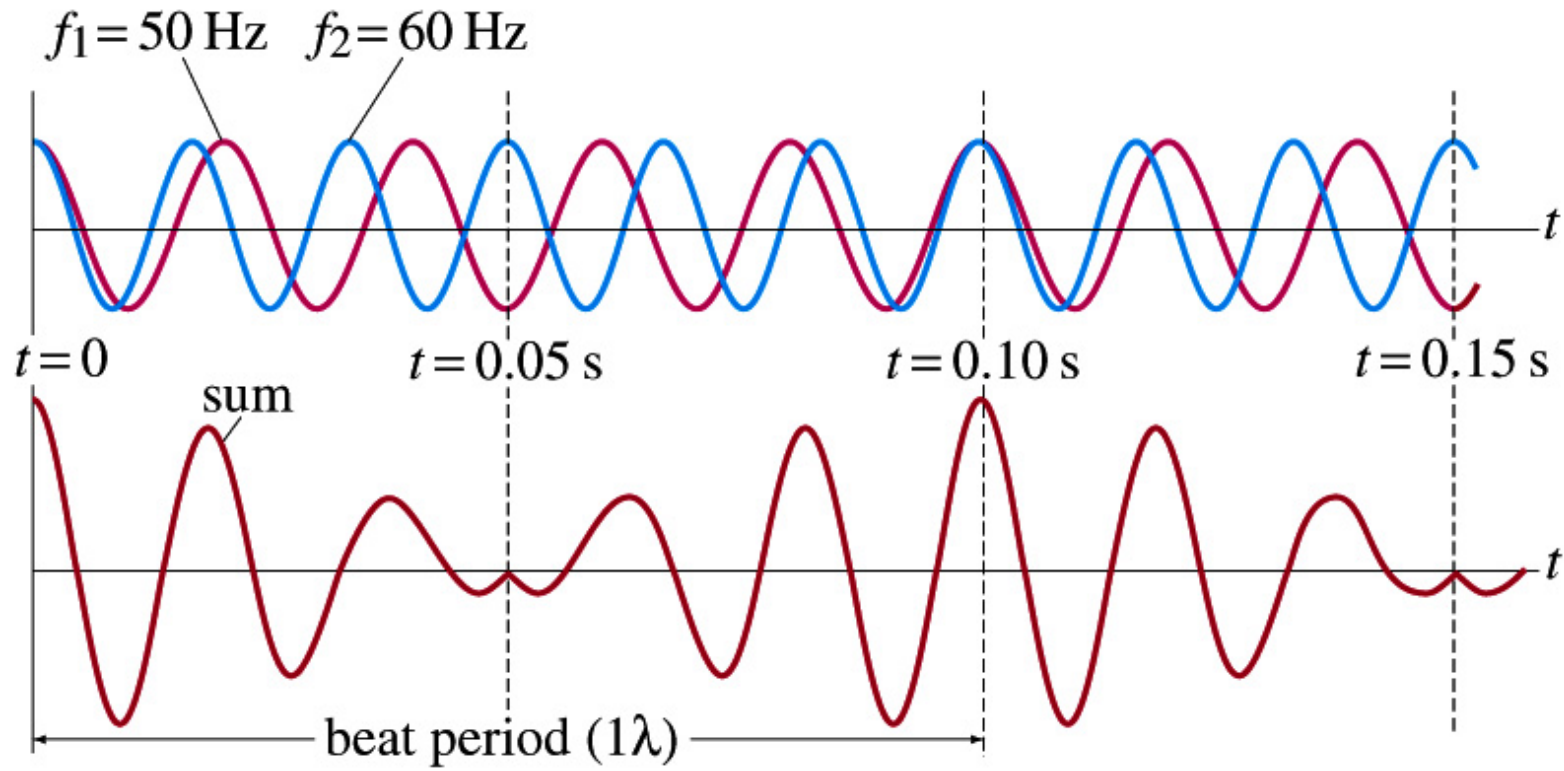
# Sum of traveling waves

The sum of waves traveling to the left and right is a standing wave.

$$\begin{aligned}\sin(\omega t + kx) + \sin(\omega t - kx) \\ &= 2 \cos\left(\frac{2kx}{2}\right) \sin\left(\frac{2\omega t}{2}\right) \\ &= 2 \cos kx \sin \omega t\end{aligned}$$



# Beats

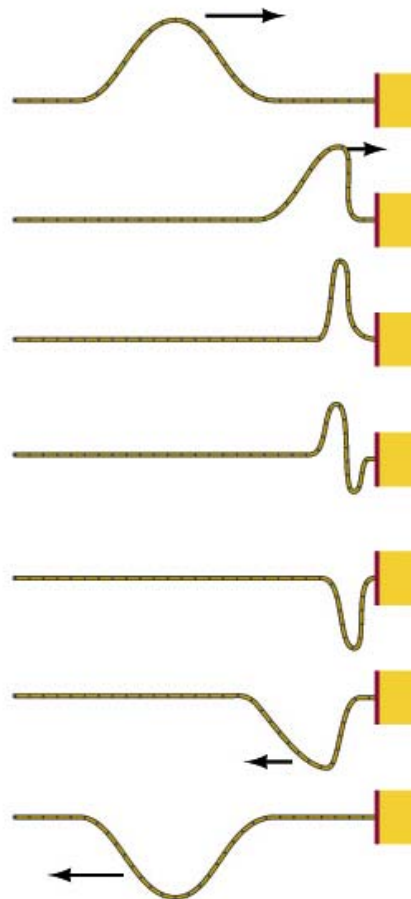




# Reflection

The end of the rope is fixed.

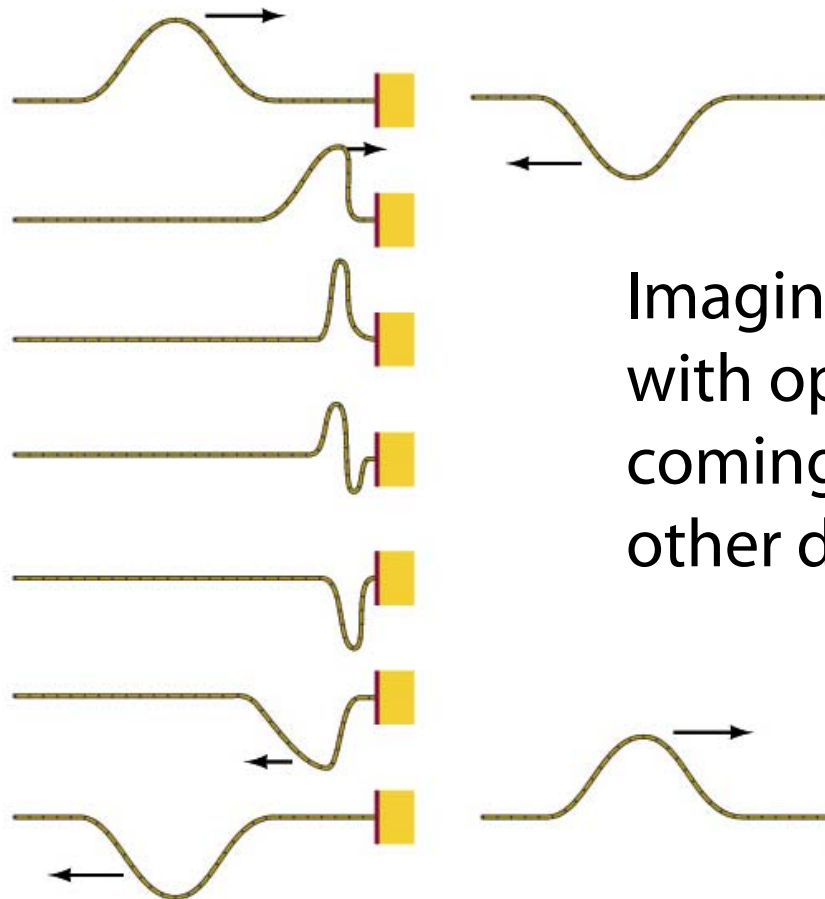
Wave is reflected with sign change.



# Reflection

The end of the rope is fixed.

Wave is reflected with sign change.

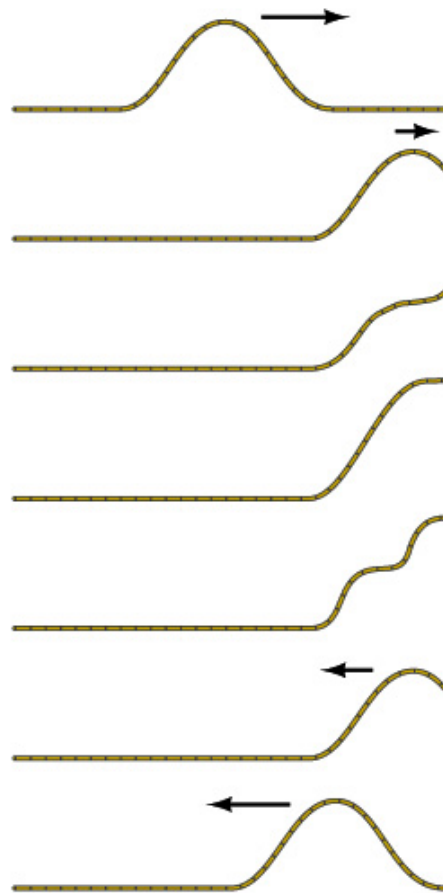


Imagine a wave with opposite sign coming from the other direction.

# Reflection

The end of the rope can move.

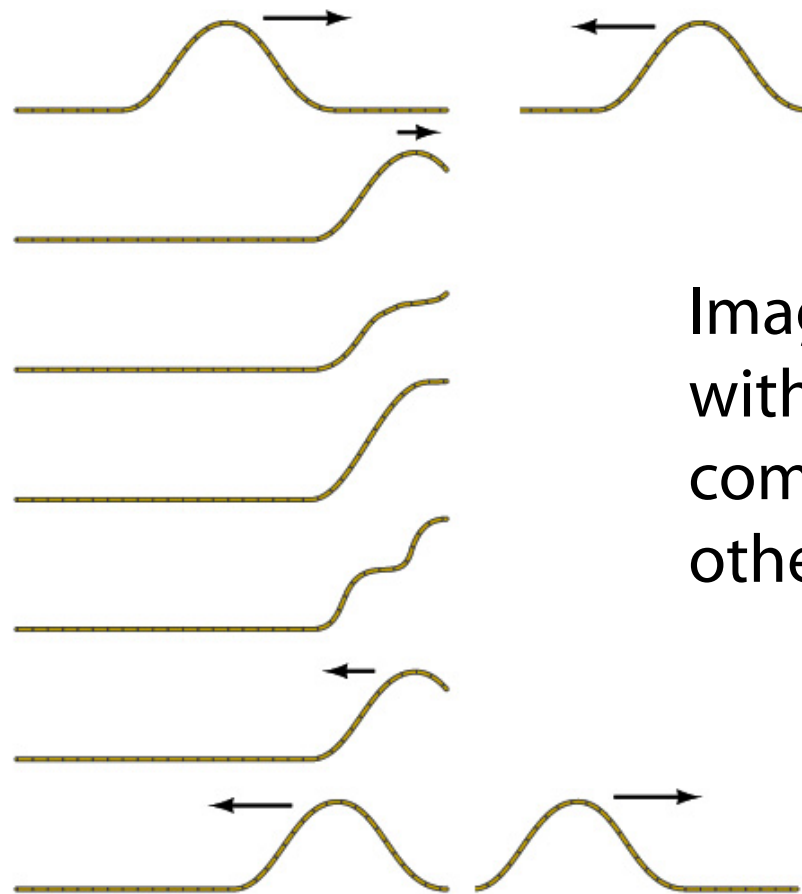
Wave is reflected without change of sign



# Reflection

The end of the rope can move.

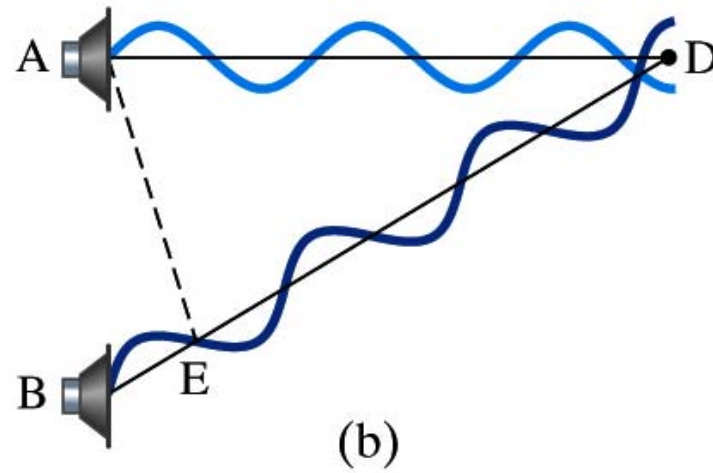
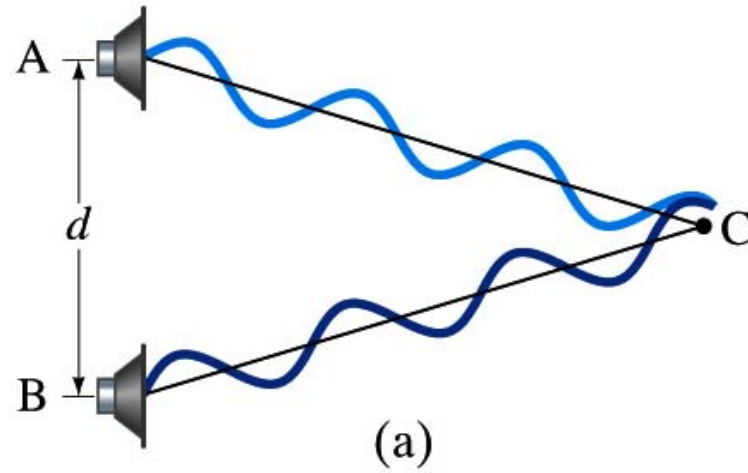
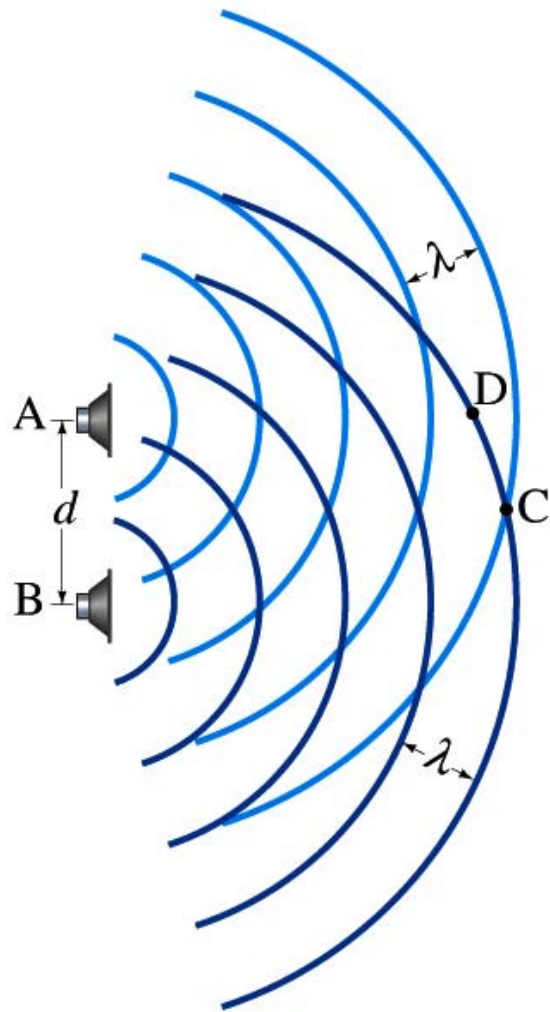
Wave is reflected without change of sign



Imagine a wave with same sign coming from the other direction.



# Interference



# Standing Wave BC

- both ends fixed (violin string)
- one end fixed, one open (organ pipe)

