Physics 21
Fall, 2004

Solution to HW-28

38 P 31 Calculate the wavelength of a 0.21 kg ball traveling at 0.1 m/s.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.21 \times 0.1} = 3.2 \times 10^{-32}\,\mathrm{m}$$

38 P 32 What is the wavelength of a neutron ($m = 1.67 \times 10^{-27}$ kg) traveling at 5.5×10^4 m/s?

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{(1.67 \times 10^{-27})(5.5 \times 10^4)} = 7.2 \times 10^{-12}\,\mathrm{m}$$

38 P 36 Show that if an electron and a proton have the same nonrelativistic kinetic energy, the proton has the shorter wavelength.

Use the nonrelativistic relation between kinetic energy E, momentum p, and mass m:

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}.$$

Then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

For equal energies E, the larger mass of the proton will lead to a smaller de Broglie wavelength.

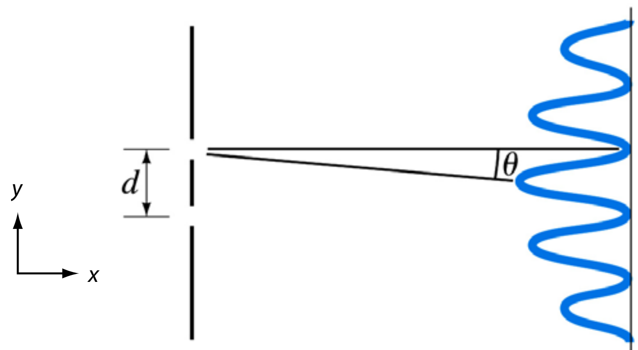
39 P 3 A proton is traveling with a speed of $(4.825 \pm 0.012) \times 10^5$ m/s. With what maximum accuracy can its position be ascertained?

The value of Δp is the range of likely values of p, and $\Delta p = m\Delta v$. We take $\Delta v = 0.024 \times 10^5$ m/s (the full range of the error bar). Then the Heisenberg Uncertainty Principle gives (with the equal sign to get the minimum uncertainty in Δx theoretically possible):

$$\Delta x = \frac{\hbar}{\Delta p} = \frac{\hbar}{m\Delta v} = \frac{10^{-34}}{(1.67 \times 10^{-27})(0.024 \times 10^5)} = 2.5 \times 10^{-11}\,\mathrm{m}$$

The value of x could be determined to $\pm\frac{1}{2}\Delta x$, or $\pm 1.2 \times 10^{-11}$ m.

39 P 11 In a double slit experiment on electrons (or photons), suppose that we use indicators to determine which slit each electron went through. The indicators must tell us the y coordinate within $d/2$, where d is the distance between slits. Use the uncertainty principle to show that the interference pattern will be destroyed. *Hint:* First show that the angle θ between maxima and minima of the interference pattern is given by $\frac{1}{2}\lambda/d$.



For this analysis we must carefully distinguish between the x and y components of momentum and position. The electrons are incident in the x direction, so their wavelength is determined by p_x. Note that the Heisenberg principle holds separately for each component (that is, $\Delta y \Delta p_y \geq \hbar$ and $\Delta x \Delta p_x \geq \hbar$).

Following the hint, we note that the angular separation θ shown in the diagram satisfies $d \sin\theta = \lambda/2$. Assuming $\sin\theta \approx \theta$ gives

$$\theta = \frac{\lambda}{2d},$$

and the wavelength λ is given by $h/p_x = 2\pi\hbar/p_x$. Hence

$$\theta = \frac{2\pi\hbar/p_x}{2d} = \frac{\pi\hbar}{p_x d}.$$

Now consider the effects on p_y when we specify Δy within $d/2$ (by measuring which slit the electron passes through). We have

$$\Delta p_y = \frac{\hbar}{d/2} = \frac{2\hbar}{d}$$

When the electrons are known to pass through one of the slits, the angle their momentum makes with the x axis after passing through the slit (p_y/p_x) is uncertain because of the uncertainty Δp_y. The uncertainty in this angle is

$$\Delta\theta = \frac{\Delta p_y}{p_x} = \frac{2\hbar}{p_x d}.$$

Compare this $\Delta\theta$ with the angular separation θ between minima and maxima in the interference pattern. Measuring which slit the electrons pass through introduces an uncertainty into their subsequent trajectory that is large enough to wash out the interference pattern.