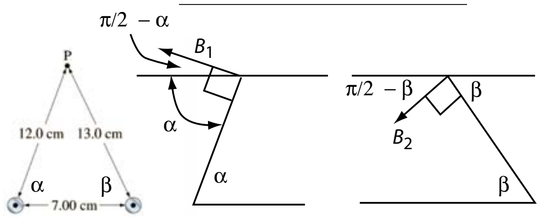
Physics 21
Fall, 2004

Solution to HW-13

28 P 18 Two long parallel wires 7.0 cm apart carry 16.5 A currents in the same direction. Determine the magnetic field strength at a point P 12.0 cm from one wire and 13.0 cm from the other.



The field $\mathbf{B}$ at the point P will be the vector sum of the fields from the two wires. We must find the components of the two fields so we can add them. We can use the Law of Cosines to solve for the angles α:

$$13^2 = 12^2 + 7^2 - 2(12)(7)\cos\alpha \;\Rightarrow\; \alpha = 81.79^\circ$$
$$12^2 = 13^2 + 7^2 - 2(13)(7)\cos\beta \;\Rightarrow\; \beta = 66.01^\circ$$

The magnitude of the fields are obtained from $B = \mu_0 I/2\pi R$, where $R_1 = 0.012$ m for the left wire and $R_2 = 0.013$ m for the right wire. The diagrams show how we can use geometry to find the components of B_1 and B_2:

$$\mathbf{B}_1 = \frac{\mu_0 I}{2\pi(0.12\,\mathrm{m})}\left(-\cos(\pi/2-\alpha)\hat{\mathbf{i}} + \sin(\pi/2-\alpha)\hat{\mathbf{j}}\right)$$
$$\mathbf{B}_2 = \frac{\mu_0 I}{2\pi(0.13\,\mathrm{m})}\left(-\cos(\pi/2-\beta)\hat{\mathbf{i}} - \sin(\pi/2-\beta)\hat{\mathbf{j}}\right)$$

Subsituting the numbers, we find

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = (-5.04\,\hat{\mathbf{i}} - 0.64\,\hat{\mathbf{j}}) \times 10^{-5}\,\mathrm{T}$$

28 P 32 A small loop of wire of radius 1.8 cm is placed at the center of a 25.0 cm wire loop. The planes of the loops are perpendicular to each other, and a 7.0 A current flows in each. Estimate the torque the large loop exerts on the smaller one. What simplifying assumptions did you make?

Assume the magnetic field $\mathbf{B}$ of the large loop is constant near its center. Then the torque on the small loop, whose magnetic moment $\boldsymbol{\mu}$ we can calculate, is $\boldsymbol{\mu} \times \mathbf{B}$.

Let the large loop ($R = 0.25$ m) be in the xy plane with the direction of current such that the field at its center (according to class notes or Ex. 28-10) is

$$\mathbf{B} = \frac{\mu_0 I}{2R}\hat{\mathbf{k}}.$$

The magnetic moment of the small loop ($r = 0.018$ m) must be in the xy plane, we take it to be

$$\boldsymbol{\mu} = I\mathbf{A} = \pi r^2 I\hat{\mathbf{j}},$$

Then

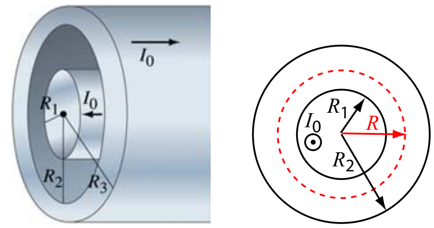
$$\boldsymbol{\mu} \times \mathbf{B} = (\pi r^2 I\hat{\mathbf{j}}) \times (\frac{\mu_0 I}{2R}\hat{\mathbf{k}}) = \frac{\pi r^2 \mu_0 I^2}{2R}\hat{\mathbf{i}}$$

Substituting numbers, we obtain

$$\boldsymbol{\mu} \times \mathbf{B} = 1.25 \times 10^{-7}\,\hat{\mathbf{j}}\,\mathrm{Nm}.$$

The magnetic moment $\boldsymbol{\mu}$ will precess in the xy plane.

28 P 27b A coaxial cable consists of a solid inner conductor of radius R_1, surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3. The conductors carry equal and opposite currents I_0 distributed uniformly across their cross sections. Determine the magnetic field at a distance R from the axis for ... (b) $R_1 < R < R_2$.

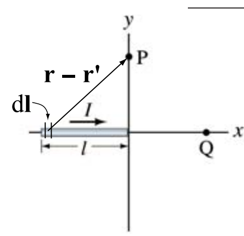


Use Ampere's law with the loop of radius R shown:

$$\oint \mathbf{B}\cdot d\mathbf{l} = 2\pi R B = \mu I_0 \;\Rightarrow\; B = \frac{\mu_0 I_0}{2\pi R}$$

The direction of $\mathbf{B}$ is given by the right hand rule.

28 P 37 A segment of wire of length l carries a current I as shown in the figure. (a) Show that for points along the positive x axis (the axis of the wire), such as point Q, the magnetic field $\mathbf{B}$ is zero. (b) Determine a formula for the field at points along the y axis, such as point P.



As usual, let $\mathbf{r}'$ be a point on the current segment, and $\mathbf{r}$ be the point where we want the field. If $\mathbf{r}$ is at Q, then the cross product in the Biot-Savart Law is zero and so the field is zero. For a point P on the y axis as shown, we have

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3},$$

where

$$d\mathbf{l} = dx'\,\hat{\mathbf{i}}, \quad \mathbf{r} - \mathbf{r}' = -x'\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}.$$

Using $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = 0$ and $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, we have

$$d\mathbf{l} \times (\mathbf{r} - \mathbf{r}') = dx'\,\hat{\mathbf{i}} \times (-x'\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}) = y\,dx'\,\hat{\mathbf{k}},$$

so that

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int_{-l}^{0}\frac{y\,dx'\,\hat{\mathbf{k}}}{(x'^2 + y^2)^{3/2}} = \frac{\mu_0 I y}{4\pi}\hat{\mathbf{k}}\int_{-l}^{0}\frac{dx'}{(x'^2 + y^2)^{3/2}}$$

Evaluating the definite integral,

$$\int_{-l}^{0}\frac{dx'}{(x'^2 + y^2)^{3/2}} = \left.\frac{x'}{y^2\sqrt{x'^2 + y^2}}\right|_{-l}^{0} = \frac{l}{y^2\sqrt{l^2 + y^2}},$$

w obtain the result

$$\mathbf{B} = \frac{\mu_0 I}{4\pi y}\frac{l}{\sqrt{l^2 + y^2}}$$