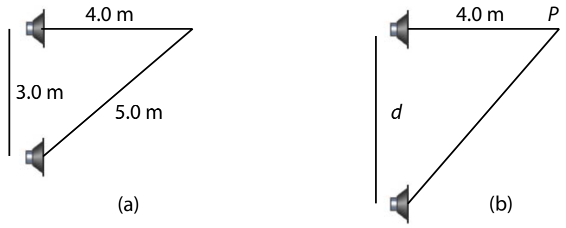
Physics 19 Fall, 2004 — Solution, Hour Exam

The graders for the problems were:
1 Lowe, 2,3 Hickman
For questions about the grading, see the grader by Nov. 12.

Problem 1. Two identical loudspeakers are connected to the same amplifier so that they are in phase. The speed of sound in air is 345 m/s.



(a) What are the two lowest frequencies f_1 and f_2 at which the speakers in panel (a) can be driven to produce maximum intensity at the point P?

Sound from one speaker travels $5 - 4 = 1$ m further; that distance must be λ for the lowest f and 2λ for the next lowest f. Using $\lambda f = v$,

$$f_1 = v/\lambda_1 = \frac{345\,\text{m/s}}{1\,\text{m}} = 345\,\text{Hz}$$
$$f_2 = v/\lambda_2 = \frac{345\,\text{m/s}}{0.5\,\text{m}} = 690\,\text{Hz}$$

(b) The wires to one speaker are now reversed so that the loudspeakers are exactly out of phase, and the distance between the speakers is increased to d as shown in panel (b). (The two speakers and the point P again form a right triangle.) Find the smallest value of $d > 3.0$ m such that intensity maxima for the frequency f_1 determined in part (a) will again be heard at P. What will be heard for frequency f_2?

For an intensity maximum, we must have

$$\sqrt{4^2 + d^2} - 4 = (n + \tfrac{1}{2})\lambda.$$

For $\lambda_1 = 1$ m, $n = 0$ leads to

$$\sqrt{4^2 + d^2} - 4 = \tfrac{1}{2} \Rightarrow d = 2.06\,\text{m}.$$

This value is too small; we specified $d > 3.0$ m, so we take $n = 1$:

$$\sqrt{4^2 + d^2} - 4 = (1\tfrac{1}{2}\,\text{m}) \Rightarrow d = 3.77\,\text{m}.$$

The path difference is 1.5 m, which for frequency f_2 is exactly three wavelengths. So at f_2, there will be destructive interference.

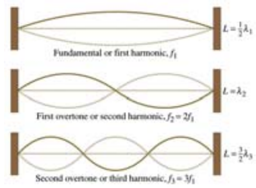
Problem 2. A violin string of length 0.33 m is tuned to a fundamental frequency of 444 Hz.

(a) What is the velocity of a wave on this string?

The fundamental wavelength λ_1 is twice the string length, so $\lambda_1 = 0.66\,\text{m}$. Then

$$v = \lambda f = (0.66\,\text{m})(444\,\text{Hz}) = 293\,\text{m/s}$$

(b) What are the frequency and wavelength of the next two possible standing waves? Sketch the form of the first three possible standing waves.



$$\lambda_2 = 0.33\,\text{m} \quad f_2 = \frac{v}{\lambda_2} = \frac{293}{0.33} = 888\,\text{Hz}$$
$$\lambda_3 = \frac{2}{3} \times 0.33 = 0.22\,\text{m} \quad f_3 = \frac{v}{\lambda_3} = 1332\,\text{Hz}$$

(c) By what fractional amount should the tension in the string be increased to shift the fundamental frequency to 450 Hz? (Neglect the change in mass density when the tension is increased.)

$f \propto v = \sqrt{T/\rho}$. For a fractional increase in f of 450/444, T must increase by $(450/444)^2 = 1.0272$, a 2.7% increase.

Problem 3. A traveling wave has the form
$D(x,t) = A\sin(\omega t - kx)$.

(a) Write the wave equation and show $D(x,t)$ satisfies it.

$$\frac{\partial^2 D}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} = 0$$
$$\frac{\partial^2}{\partial x^2} A\sin(\omega t - kx) = -k^2 A\sin(\omega t - kx)$$
$$\frac{\partial^2}{\partial t^2} A\sin(\omega t - kx) = -\omega^2 A\sin(\omega t - kx)$$

Substituting shows that D satisfies the wave equation as long as $k^2 = \omega^2/v^2$, or $k = \omega/v$, which is equivalent to $v = \omega/k = (2\pi f)/(2\pi/\lambda) = f\lambda$.

(b) Find the values of A, ω, and k if the frequency (not angular frequency) is 60 Hz, the amplitude is 0.6 m, and the wavelength is 0.1 m.

$$A = 0.6\,\text{m} \quad \omega = 2\pi(60\,\text{Hz}) = 377\,\text{rad/s}$$
$$k = 2\pi/\lambda = 2\pi/0.1 = 20\pi = 62.8\,\text{m}$$

(c) What is the velocity of the wave?

$$v = \lambda f = (0.1\,\text{m})(60\,\text{Hz}) = 6\,\text{m/s}$$

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