Physics 21
Fall, 2004

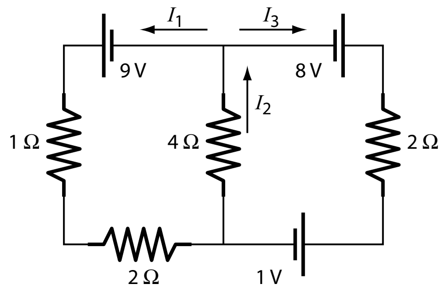
Hour Exam #1

1______ 2______ 3______ 4______ 5______ Total______ Name: ________________________

Recitation Time ______ Recitation Leader ____________ Sept. 22, 2004

This exam is closed notes and closed book. You must show enough work on all problems to convince the grader you understand how to solve the problem. You may use a calculator, but you must show a full solution to simultaneous algebraic equations. An equation sheet is on the last page. There are five problems; each counts 20 points.

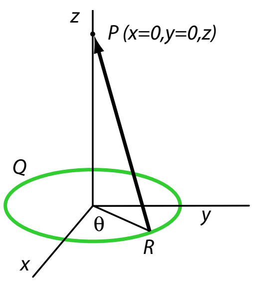
Problem 1.



(a) Write the loop and node equations needed to determine the currents I_1, I_2, and I_3 in the circuit shown. Indicate clearly the loop used to determine each loop equation.

(b) Determine the currents by explicit solution of the equations. You must show your work.

Problem 2. The circular ring of charge shown in the diagram is in the xy plane centered at the origin and has a radius R. A total charge of Q is spread uniformly around the ring.



(a) Find an expression for the linear charge density λ on the ring.

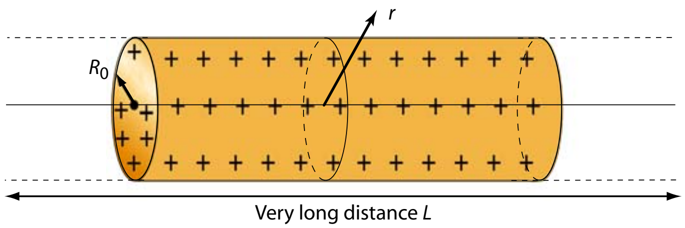
(b) Give the components of the vector shown in the diagram from the point on the ring at angle θ (with respect to the x axis) to the point P.

(c) Determine the electric field $d\mathbf{E}$ at the point P due to the element dQ of charge at the point θ on the ring.

(d) Integrate to find the electric field at the point P due to the ring of charge. Your answer should be in terms of Q, R, and z.

(e) Work out the value of the electric field numerically if $R = 0.3$ m, $Q = 3\times10^{-6}$ C, and $z = 0.4$ m.

Problem 3. A very long nonconducting cylinder of radius R_0 and length L ($R_0 \ll L$) possesses a uniform volume charge density ρ. For this problem, you are to use Gauss's Law to determine the electric field at radial distances r from the long axis of the cylinder. Assume that you are far from the ends of the cylinder. For parts (a) and (b) of the problem, you must indicate clearly the Gaussian surface you use.

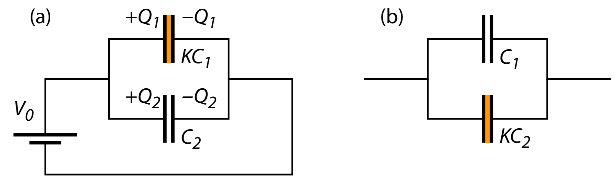


(a) Using Gauss's Law, determine the magnitude of the electric field at points $r < R_0$ inside the cylinder. What is the direction of the field?

(b) For points $r > R_0$ (and $r \ll L$) outside the cylinder, what is the direction of the field? Use Gauss's Law to find the magnitude.

(c) If $R_0 = 0.2$ m and $\rho = 8 \times 10^{-6}\,\mathrm{C/m^3}$, work out the numerical value of the magnitude of the field at the surface of the cylinder.

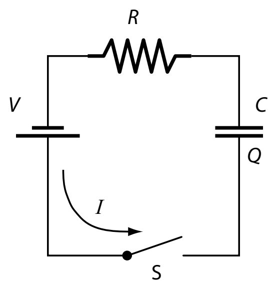
Problem 4. Two capacitors (1 and 2) are connected in parallel as shown in panel (a) of the diagram. Capacitor 1 has an insulator (dielectric constant K) that fills the space between the plates, and its capacitance is KC_1; capacitor 2 has only air between its plates. Each capacitor acquires a charge when connected to the voltage V_0. The voltage is disconnected, then the dielectric is removed from the first capacitor, and then a dielectric (also dielectric constant K) is inserted to fill the space between the plates of the second capacitor. This final setup is shown in panel (b).



(a) What are the charges Q_1 and Q_2 on each capacitor for the initial setup shown in panel (a)? Your answers will depend on C_1, C_2, V_0 and K.

(b) If $C_1 = 1.0\,\mu\text{F}$, $C_2 = 3.0\,\mu\text{F}$, $V_0 = 2.0\,\text{V}$, and $K = 5.0$, find the charge on each capacitor for the final setup shown in panel (b).

Problem 5. In the following RC circuit, the capacitor C is uncharged until the switch S is closed at $t = 0$:



(a) Write the loop equation for this circuit for times $t \geq 0$.

(b) What is the relation between I and Q?

(c) Show that the following expressions for the current I and the charge Q on the capacitor satisfy the loop equation:

$$I = \frac{V}{R}\exp\left(-\frac{t}{RC}\right), \quad Q = CV\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$$

(d) By evaluating the time integral of the battery's power output, $\int_0^\infty P(t)dt$, find the total energy E_{bat} drawn from the battery during the course of charging the capacitor. Compare this value to the energy E_{cap} stored in the capacitor. Where did the rest of the energy go?

Physics 21 Fall, 2004 — Equation Sheet for Exam-1

Physical Constants:

speed of light *in vacuo*	c	3.00×10^{8} m/s
Gravitational constant	G	6.67×10^{-11} N m^2/kg^2
Avogadro's Number	N_A	6.02×10^{23} mol^{-1}
Gas constant	R	8.315 J/mol K
Boltzmann's constant	k_{B}	1.38×10^{-23} J/K
charge on electron	e	1.60×10^{-19} C
free space permittivity	ϵ_0	8.85×10^{-12} C^2/N m^2
free space permeability	μ_0	$4\pi \times 10^{-7}$ T m/A
Planck's constant	h	6.63×10^{-34} J s
electron rest mass	m_{e}	9.11×10^{-31} kg
proton rest mass	m_{p}	1.6726×10^{-27} kg
neutron rest mass	m_{n}	1.6749×10^{-27} kg
atomic mass unit	u	1.6605×10^{-27} kg
$1/(4\pi\epsilon_0)$	k	8.99×10^{9} N m^2/C^2

Various and Sundry Equations:

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{21}^3}(\mathbf{r}_1 - \mathbf{r}_2) \qquad \text{(Coulomb's Law)}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^3}\mathbf{r} \qquad \text{(field of point charge)}$$

$$\mathbf{F} = q\mathbf{E}$$

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$$

$$\mathbf{E} = -\nabla V$$

$$\mathbf{E} = -\left(\hat{\mathbf{i}}\frac{\partial V}{\partial x} + \hat{\mathbf{j}}\frac{\partial V}{\partial y} + \hat{\mathbf{k}}\frac{\partial V}{\partial z}\right)$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$Q = CV$$

$$U = \tfrac{1}{2}CV^2$$

$$U = \tfrac{1}{2}Q^2/C$$

$$u = \tfrac{1}{2}\epsilon_0 E^2$$

$$C = \epsilon_0 K\frac{A}{d} = \epsilon\frac{A}{d}$$

$$V = IR \quad \text{or} \quad V - IR = 0$$

$$R = \rho\frac{l}{A}$$

$$\sigma = \frac{1}{\rho}$$

$$P = IV$$

$$P = I^2 R$$

$$I = \frac{dQ}{dt}$$

C's in series or R's in parallel:

$$\frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2}, \qquad \frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

C's in parallel or R's in series:

$$C_{\text{eff}} = C_1 + C_2, \qquad R_{\text{eff}} = R_1 + R_2$$

Useful Integrals:

$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left(u + \sqrt{a^2+u^2}\right)$$

$$\int \frac{u\,du}{\sqrt{a^2+u^2}} = \sqrt{a^2+u^2}$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$$

$$\int \frac{u\,du}{a^2+u^2} = \tfrac{1}{2}\ln\left(a^2+u^2\right)$$

$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}}$$

$$\int \frac{u\,du}{(a^2+u^2)^{3/2}} = -\frac{1}{\sqrt{a^2+u^2}}$$

$$\int e^{au}\,du = \frac{1}{a}e^{au}$$

$$\int \sin\theta\,d\theta = -\cos\theta$$

$$\int \cos\theta\,d\theta = \sin\theta$$

$$\int \ln u\,du = u\ln u - u$$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1}$$

$$\int \frac{du}{a+bu} = \frac{1}{b}\ln(a+bu)$$

$$\int \frac{du}{u} = \ln u$$

Lengths, Areas and Volumes:

$C = 2\pi r$	circumference of circle
$C = \pi d$	circumference of circle
$A = \pi r^2$	area of circle
$A = 4\pi r^2$	surface area of sphere
$V = \frac{4}{3}\pi r^3$	volume of sphere

Quadratic Formula:

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$