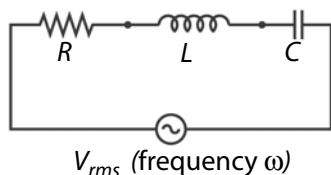


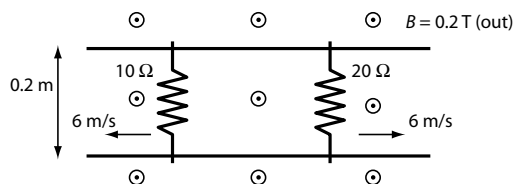
This exam is closed notes and closed book. You must show enough work on all problems to convince the grader you understand how to solve the problem. You may use a calculator. An equation sheet is on the last page. There are five problems; each counts 20 points.

Problem 1. For the following circuit:



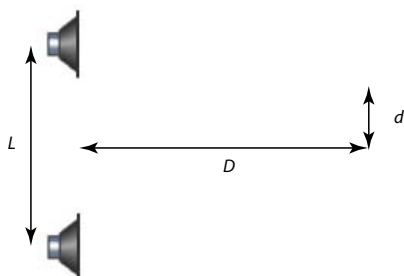
- Write the loop equation for this circuit.
- If $R = 20 \Omega$, $L = 0.025 \text{ H}$, and $C = 50 \mu\text{F}$ ($50 \times 10^{-6} \text{ F}$), find the frequency ω for which the current lags the voltage across the inductor by 30° . *Hint:* Make sure $\omega > 0$.
- If $V_{\text{rms}} = 100 \text{ V}$, what is the rms current? What is the peak current?

Problem 2. A magnetic field $B = 0.2 \text{ T}$ points out of the paper. The 10Ω resistor moves to the left with a speed of 6 m/s , and the 20Ω resistor moves to the right at 6 m/s . The top and bottom wires have no resistance, and the loop is 0.2 m high.

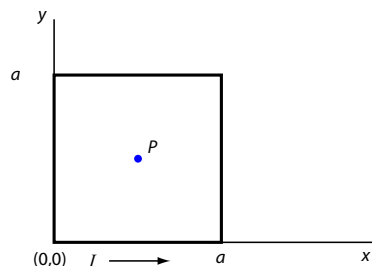


- What is the current induced in the circuit?
- What direction does it flow in?

Problem 3. Two identical speakers driven at frequency ω are separated by a distance $L = 3.0 \text{ m}$. When a microphone is centered between the speakers a distance $D = 4.0 \text{ m}$ in front of them, the sound is loudest. When the microphone is moved $d = 0.5 \text{ m}$ as shown, the first minimum in volume is heard. What is the frequency of the sound? Use 345 m/s for the speed of sound in air.

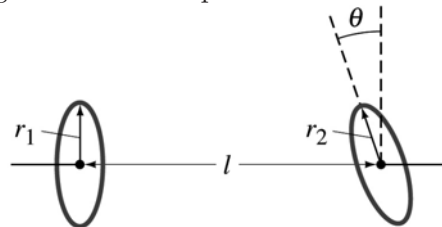


Problem 4. Consider a current loop that has the form of a square of side a :



- Explain why the current in each side of the square makes an identical contribution to the magnetic field at the point P at the center of the square.
- Find the magnetic field at P , using the Biot-Savart Law. *Hint:* When you set up the integral over the bottom side of the square, change variables to $u = x - \frac{a}{2}$ in order to transform the integral to one on the equation sheet.

Problem 5. Consider two small circular loops of radii r_1 and r_2 , which are separated by a distance l that is large compared to r_1 and r_2 . The line joining their centers is perpendicular to the plane of coil 1, and the angle between the planes of the two coils is θ .



- Show that the mutual inductance M is given approximately by

$$M = \frac{\mu_0}{2\pi} \frac{A_1 A_2 \cos \theta}{l^3},$$

where the A 's are the areas of the loops. State clearly the approximations you make. *Hint:* The magnitude of the field of a coil of radius r , on axis, a distance x from the plane of the coil, is

$$B = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}.$$

- If $r_1 = 5 \text{ mm}$, $r_2 = 8 \text{ mm}$, $\theta = 60^\circ$, and $l = 1.5 \text{ m}$, find M .

Physics 21
Fall, 2004

Equation Sheet

speed of light <i>in vacuo</i>	c	3.00×10^8 m/s	free space permeability	μ_0	$4\pi \times 10^{-7}$ T m/A
Gravitational constant	G	6.67×10^{-11} N m ² /kg ²	Planck's constant	h	6.63×10^{-34} J s
Avogadro's Number	N_A	6.02×10^{23} mol ⁻¹	electron rest mass	m_e	9.11×10^{-31} kg
Gas constant	R	8.315 J/mol K	proton rest mass	m_p	1.6726×10^{-27} kg
Boltzmann's constant	k_B	1.38×10^{-23} J/K	neutron rest mass	m_n	1.6749×10^{-27} kg
charge on electron	e	1.60×10^{-19} C	atomic mass unit	u	1.6605×10^{-27} kg
free space permittivity	ϵ_0	8.85×10^{-12} C ² /N m ²		k	8.99×10^9 N m ² /C ²

$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{21}^3} (\mathbf{r}_1 - \mathbf{r}_2)$	$Q = CV$	C 's in series or R 's in parallel:	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ or $\mathbf{l} \times \mathbf{B}$
$\mathbf{F} = q\mathbf{E}$	$C = \epsilon_0 K \frac{A}{d} = \epsilon \frac{A}{d}$	$\frac{1}{X_{\text{eff}}} = \frac{1}{X_1} + \frac{1}{X_2}$	$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times (\mathbf{r} - \mathbf{r}')}{ \mathbf{r} - \mathbf{r}' ^3}$
$d\mathbf{E} = \frac{4\pi}{\epsilon_0} \frac{dQ(\mathbf{r} - \mathbf{r}')}{ \mathbf{r} - \mathbf{r}' ^3}$	$U_{\text{cap}} = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$	C 's in parallel or R 's in series:	field of long wire: $B = \frac{\mu_0 I}{2\pi R}$
$\mathbf{E} = -\nabla V$	$U_{\text{ind}} = \frac{1}{2} LI^2$	$X_{\text{eff}} = X_1 + X_2$	$I = \frac{dQ}{dt}$
$\mathbf{E} = -\left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z}\right)$	$V = IR$ or $V - IR = 0$	$X_R = R, X_L = \omega L, X_C = \frac{1}{\omega C}$	$\frac{V_s}{V_p} = \frac{N_s}{N_p}, \frac{I_s}{I_p} = \frac{N_p}{N_s}$
$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$	$R = \rho \frac{l}{A} \quad \sigma = \frac{1}{\rho}$	RC time constant = RC	$\chi_m = \frac{\mu}{\mu_0} - 1$
$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$	$P = IV \quad P = I^2 R$	LR time constant = L/R	$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \quad \boldsymbol{\mu} = IA$
$u_{\text{elec}} = \frac{1}{2} \epsilon_0 E^2, u_{\text{mag}} = \frac{1}{2\mu_0} B^2$	$E = \frac{\sigma}{\epsilon_0}$ parallel plate capacitor	$Q(t)$ for RLC decay	solenoid $B = \mu_0 n I$
	$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$Q_0 \exp(-Rt/2L) \cos \omega t$	solenoid $L = \mu_0 N^2 A/l$
		$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$	

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$	$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$	$C = 2\pi r$ circumference of circle
$\oint \mathbf{B} \cdot d\mathbf{A} = 0$	$\sin(\theta \pm \frac{\pi}{2}) = \sin \theta \cos \frac{\pi}{2} \pm \cos \theta \sin \frac{\pi}{2} = \pm \cos \theta$	$C = \pi d$ circumference of circle
$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint \mathbf{E} \cdot d\mathbf{A}$	$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$	$A = \pi r^2$ area of circle
$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \oint \mathbf{E} \cdot d\mathbf{A}$	$\sin a + \sin b = 2 \cos \left(\frac{a-b}{2}\right) \sin \left(\frac{a+b}{2}\right)$	$A = 4\pi r^2$ surface area of sphere
		$V = \frac{4}{3} \pi r^3$ volume of sphere
		$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2})$	$\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}}$	$\int u^n du = \frac{1}{n+1} u^{n+1}$
$\int \frac{u du}{\sqrt{a^2 + u^2}} = \sqrt{a^2 + u^2}$	$\int \frac{u du}{(a^2 + u^2)^{3/2}} = -\frac{1}{\sqrt{a^2 + u^2}}$	$\int \frac{du}{a + bu} = \frac{1}{b} \ln(a + bu)$
$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \left(\frac{u}{a}\right)$	$\int e^{au} du = \frac{1}{a} e^{au}$	$\int \frac{du}{u} = \ln u$
$\int \frac{u du}{a^2 + u^2} = \frac{1}{2} \ln(a^2 + u^2)$	$\int \ln u du = u \ln u - u$	

$v = \sqrt{T/\rho}$ (T =tension)	$v = f\lambda$	$\omega = 2\pi f$	$\langle P \rangle = \frac{1}{2} \rho A^2 \omega^2 v$
$v = (348 \text{ m/s}) \sqrt{T/300}$	$T = 1/f$ (T =period)	$k = 2\pi/\lambda$	