LINEAR ALGEBRA V - LINEAR MODELS AND DECISIONS

FOR V AN INNER PRODUCT SPACE AND $v \in V$, THE LENGTH OF v =

$$\|v\| = \sqrt{\langle v, v\rangle}$$

FOR $x, y \in V$, THE ANGLE θ BETWEEN x AND y IS GIVEN BY

$$\langle x, y\rangle = \|x\| \, \|y\| \cos\theta$$

THE DISTANCE FROM x TO y IS $\|x-y\|$

x IS ORTHOGONAL TO y iff $\langle x, y\rangle = 0$

$\{v_1, \dots, v_n\}$ IS AN ORTHOGONAL SET OF VECTORS iff $\langle v_i, v_j\rangle = 0$ FOR $i \neq j$

$\{v_1, \dots, v_n\}$ IS AN ORTHONORMAL SET iff IT IS ORTHOGONAL AND $\|v_i\| = 1$ FOR $\forall i$

EVERY FINITE DIMENSIONAL INNER PRODUCT SPACE HAS AN ORTHONORMAL BASIS

FOR $x = \begin{pmatrix}\alpha_1 \\ \vdots \\ \alpha_n\end{pmatrix}$, $y = \begin{pmatrix}\beta_1 \\ \vdots \\ \beta_n\end{pmatrix}$, $\langle x, y\rangle = \alpha_1\beta_1 + \dots + \alpha_n\beta_n$ IS CALLED THE EUCLIDEAN INNER PRODUCT

GEOMETRIC INTERPRETATION OF LENGTH

W A SUBSPACE OF $V \Rightarrow$ THE ORTHOGONAL COMPLEMENT OF W, $W^\perp$, IS DEFINED BY

$$W^\perp = \{x \in V \mid \langle x, w\rangle = 0 \text{ FOR } \forall w \in W\}$$

$W^\perp$ IS A SUBSPACE OF V

$(W^\perp)^\perp = W$