LINEAR ALGEBRA IV - LINEAR MODELS AND DECISIONS

FOR $\langle V_1, +_{V_1} \rangle$ AND $\langle V_2, +_{V_2} \rangle$ BOTH GROUPS,

$f: V_1 \longrightarrow V_2$ IS A GROUP MORPHISM iff $f(x +_{V_1} y) = f(x) +_{V_2} f(y)$, $x, y \in V_1$

$f: V_1 \rightarrow V_2$ IS A GROUP ISOMORPHISM iff f IS A 1 to 1 CORRESPONDENCE AND A GROUP MORPHISM

IF V_1 AND V_2 FORM VECTOR SPACES WITH THE FIELD F

$f: V_1 \rightarrow V_2$ IS A VECTOR SPACE MORPHISM iff $f(x+y) = f(x) + f(y)$ AND $f(\alpha x) = \alpha f(x)$

THAT IS, IF $f(\alpha_1 v_1 + \dots + \alpha_N v_N) = \alpha_1 f(v_1) + \dots + \alpha_N f(v_N)$

$f: V_1 \rightarrow V_2$ IS A VECTOR SPACE ISOMORPHISM iff f IS A 1 to 1 CORRESPONDENCE AND A VECTOR SPACE MORPHISM

IN GENERAL, MORPHISMS PRESERVE STRUCTURE AND ISOMORPHISMS PRESERVE BOTH SIZE AND STRUCTURE

$\therefore$ $f: V \rightarrow \left\{ \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} \middle| \alpha_1, \dots, \alpha_N \in F \right\}$ IS A VECTOR SPACE ISOMORPHISM FOR ANY VECTOR SPACE V WITH $\dim V = N$ AND A FIXED BASIS SET

THUS, ALL VECTORS CAN ALWAYS BE REPRESENTED AND MANIPULATED IN TERMS OF THEIR COLUMN VECTOR REPRESENTATIONS

INNER PRODUCTS - F IS RESTRICTED TO THE REALS

$\langle -, - \rangle : V \times V \rightarrow F$ [i.e., FOR $x, y \in V$, $\langle x, y \rangle \in F$ - NOTE THE SAME NOTATION AS FOR AN ORDERED PAIR; USUALLY LITTLE CHANCE FOR CONFUSION, HOWEVER] IS AN INNER PRODUCT ON V iff

1) $\langle x, y \rangle = \langle y, x \rangle$
2) $\langle x, x \rangle \geq 0$
3) $\langle x, x \rangle = 0$ iff $x = 0$
4) $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$ FOR $x, y, z \in V$

A VECTOR SPACE V WITH AN INNER PRODUCT DEFINED ON IT IS CALLED AN INNER PRODUCT SPACE