11 SEP 77

LINEAR ALGEBRA III - LINEAR MODELS AND DECISIONS

FOR $\dim V$ (THE DIMENSION OF V) $= N$, WE CAN CHOOSE A BASIS FOR V $\langle v_1, \dots, v_N \rangle \ni$
$v \in V \Rightarrow \exists\, \alpha_1, \dots, \alpha_N \in F \ni v = \alpha_1 v_1 + \dots + \alpha_N v_N$

THAT IS, WITH A FIXED BASIS SET, EACH $v \in V$ IS COMPLETELY SPECIFIED BY THE SET OF SCALARS $\langle \alpha_1, \dots, \alpha_N \rangle$ WHICH SPECIFY THE LINEAR COMBINATION OF THE BASIS VECTORS WHICH EQUALS THE GIVEN $v \in V$

CONVERSELY, EACH SET OF SCALARS $\langle \alpha_1, \dots, \alpha_N \rangle$ SPECIFIES A LINEAR COMBINATION $\alpha_1 v_1 + \dots + \alpha_N v_N = v \in V$

THUS, $\exists$ A 1 TO 1 CORRESPONDENCE BETWEEN V AND $\{ \langle \alpha_1, \dots, \alpha_N \rangle \mid \alpha_1, \dots, \alpha_N \in F \}$ FOR $\dim V = N$ AND A FIXED BASIS FOR V

$\therefore$ @ $v \in V$ CAN BE REPRESENTED BY ITS CORRESPONDING COEFFICIENTS OF THE APPROPRIATE LINEAR COMBINATION $\langle \alpha_1, \dots, \alpha_N \rangle$. THESE COEFFICIENTS ARE WRITTEN AS A COOLUMN OF SCALARS, $\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix}$, CALLED COLUMN VECTORS

WE WOULD NOW LIKE TO KNOW WHAT THE VECTOR SPACE OPERATIONS, $+_V$ AND $\cdot_V$, LOOK LIKE IN TERMS OF THESE COLUMN VECTOR REPRESENTATIONS.

FOR $\dim V = N$, $v_1, v_2 \in V$, $v_1 + v_2 = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix} = \alpha_1 v_1^* + \dots + \alpha_N v_N^* + \beta_1 v_1^* + \dots + \beta_N v_N^*$

$= (\alpha_1 + \beta_1) v_1^* + \dots + (\alpha_N + \beta_N) v_N^* = \begin{pmatrix} \alpha_1 + \beta_1 \\ \vdots \\ \alpha_N + \beta_N \end{pmatrix} \quad \therefore \quad \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix} = \begin{pmatrix} \alpha_1 + \beta_1 \\ \vdots \\ \alpha_N + \beta_N \end{pmatrix}$

FOR $v \in V$, $\alpha v = \alpha \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} = \alpha (\alpha_1 v_1^* + \dots + \alpha_N v_N^*) = \alpha\alpha_1 v_1^* + \dots + \alpha\alpha_N v_N^* = \begin{pmatrix} \alpha\alpha_1 \\ \vdots \\ \alpha\alpha_N \end{pmatrix}$

$\therefore \alpha \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \alpha\alpha_1 \\ \vdots \\ \alpha\alpha_N \end{pmatrix}$ $\qquad \langle v_1^*, \dots, v_N^* \rangle$ A BASIS FOR V

THUS, LINEAR COMBINATIONS OF COLUMN VECTORS ARE EQUAL TO COLUMN VECTORS OF THE CORRESPONDING LINEAR COMBINATIONS OF THE COEFFICIENTS IN THOSE VECTORS

THAT IS,

$$\beta_1 v_1 + \dots + \beta_M v_M = \beta_1 \begin{pmatrix} \alpha_{11} \\ \vdots \\ \alpha_{N1} \end{pmatrix} + \dots + \beta_M \begin{pmatrix} \alpha_{1M} \\ \vdots \\ \alpha_{NM} \end{pmatrix} = \begin{pmatrix} \beta_1 \alpha_{11} + \dots + \beta_M \alpha_{1M} \\ \vdots \\ \beta_1 \alpha_{N1} + \dots + \beta_M \alpha_{NM} \end{pmatrix}$$