LINEAR ALGEBRA – II LINEAR MODELS AND DECISIONS

$\langle\langle V, +_V\rangle, \langle F, +_F, \cdot_F\rangle, \cdot_V\rangle$ IS A VECTOR SPACE iff

1) $\langle V, +_V\rangle$ IS AN ABELIAN GROUP
2) $\langle F, +_F, \cdot_F\rangle$ IS A FIELD
3) $\cdot_V : F \times V \longrightarrow V \ni$ FOR $\alpha, \beta \in F$, $v, w \in V$
   A) $\alpha\cdot(v+w) = \alpha\cdot v + \alpha\cdot w$
   B) $(\alpha+\beta)\cdot v = \alpha\cdot v + \beta\cdot v$
   C) $\alpha\cdot(\beta\cdot v) = (\alpha\cdot\beta)\cdot v$
4) $1\cdot v = v$ $\qquad \{1 = e_{\cdot_F}\}$

WRITE $A\cdot B$ AS AB

$\alpha_1 v_1 + \ldots + \alpha_N v_N$ FOR $\alpha_1, \ldots, \alpha_N \in F$, $\overset{\text{NOT } \forall \alpha_i = 0,}{\uparrow}$ $v_1, \ldots, v_N \in V$ IS A LINEAR COMBINATION OF THE VECTORS $v_1, \ldots, v_N$

IF $\exists$ SCALARS $\alpha_1, \ldots, \alpha_N \in F$ $\overset{\text{NOT } \forall \alpha_i = 0}{\uparrow}$ $\ni \alpha_1 v_1 + \ldots + \alpha_N v_N = 0$ FOR $v_1, \ldots, v_N, 0 \in V$ $\{0 = e_{+_V}\}$
THEN $\{v_1, \ldots, v_N\}$ IS A LINEARLY DEPENDENT SET OF VECTORS

$\{v_1, \ldots, v_N\}$ IS LINEARLY INDEPENDENT SET OF VECTORS iff IT IS NOT LINEARLY DEPENDENT
i.e. $\nexists$ A SET OF SCALARS FOR WHICH THE LINEAR COMBINATION $= 0$

IF V' FORMS A VECTOR SPACE OVER F AND $V' \subset V$, THEN V' IS A SUBSPACE OF V
SINCE $V' \subset V$, THE OPERATIONS (MAPS) ARE ALREADY DEFINED – AS IN V –; THE QUESTION, THEN, IS WHETHER OR NOT V' IS CLOSED UNDER THOSE OPERATIONS, i.e., CLOSED UNDER LINEAR COMBINATIONS

THE SPAN OF $\{v_1, \ldots, v_N\} = \{\alpha_1 v_1 + \ldots + \alpha_N v_N \mid \alpha_1, \ldots, \alpha_N \in F\} = \{\forall_{\text{LINEAR COMBINATIONS OF } v_1, \ldots, v_N}\}$

THE SPAN OF $\{v_1, \ldots, v_N\}$ IS A VECTOR SPACE
PR: 1) THE OPERATIONS HOLD SINCE $\{v_1, \ldots, v_N\} \subset V$
2) CLOSURE HOLDS BY THE DEFINITION OF A SPAN

FOR ANY VECTOR SPACE V, IF $\{v_1, \ldots, v_N\} \subset V$ SPANS V [i.e., SPAN$\{v_1, \ldots, v_N\} = V$] AND $\{v_1, \ldots, v_N\}$ IS LINEARLY INDEPENDENT, THEN $\{v_1, \ldots, v_N\}$ IS A BASIS FOR V
A BASIS IN THE SENSE THAT LINEAR COMBINATIONS OF $\{v_1, \ldots, v_N\}$ GENERATE ALL OF V

TH: ALL BASES OF A VECTOR SPACE V HAVE THE SAME NUMBER OF ELEMENTS
THAT NUMBER IS CALLED THE DIMENSION OF THE SPACE V