LINEAR ALGEBRA – I   LINEAR MODELS AND DECISIONS

$\circ$ IS A BINARY MAP iff $\circ: S \times S \to T$ FOR SOME SETS S AND T

$\circ$ IS A BINARY MAP CLOSED ON S iff $\circ: S \times S \to S$

FOR $x, y \in S$, $\circ(x, y) \equiv x \circ y$

$\circ: S \times S \to S$ IS ASSOCIATIVE iff FOR $x, y, z \in S$, $(x \circ y) \circ z = x \circ (y \circ z)$

$\circ: S \times S \to S$ HAS AN IDENTITY iff $\exists\, e \in S \ni (x)(x \in S \Rightarrow x \circ e = e \circ x = x)$

$\circ: S \times S \to S$ HAS INVERSES iff $x \in S \Rightarrow \exists\, x^{-1} \in S \ni x \circ x^{-1} = x^{-1} \circ x = e$

FOR $e \in S$ AN IDENTITY

$\circ: S \times S \to S$ IS COMMUTATIVE iff $x, y \in S \Rightarrow x \circ y = y \circ x$

$\langle S, \circ \rangle$ IS A SEMIGROUP iff $\circ: S \times S \to S$ IS ASSOCIATIVE

$\langle S, \circ \rangle$ IS A MONOID iff $\circ: S \times S \to S$ IS ASSOCIATIVE AND HAS AN IDENTITY

$\langle S, \circ \rangle$ IS A GROUP iff $\circ: S \times S \to S$ IS ASSOCIATIVE WITH IDENTITY AND INVERSES

FOR A GROUP, $\langle S, \circ \rangle$ IS OFTEN WRITTEN $\langle G, + \rangle$ OR $\langle G, \cdot \rangle$

$\langle S, \circ \rangle$ IS AN ABELIAN (OR COMMUTATIVE) GROUP iff $\langle S, \circ \rangle$ IS A GROUP AND $\circ: S \times S \to S$ IS COMMUTATIVE

$\langle S, +, \cdot \rangle$ IS A RING iff

1) $\langle S, + \rangle$ IS AN ABELIAN GROUP
2) $\langle S, \cdot \rangle$ IS A SEMI-GROUP
3) $x \cdot (y + z) = x \cdot y + x \cdot z$ AND $(y + z) \cdot x = y \cdot x + z \cdot x$ FOR $\forall\, x, y, z \in S$

$\langle S, +, \cdot \rangle$ IS A FIELD iff

1) $\langle S, + \rangle$ IS AN ABELIAN GROUP
2) $\langle S - \{e_+\}, \cdot \rangle$ IS AN ABELIAN GROUP
3) $x \cdot (y + z) = x \cdot y + x \cdot z$ AND $(y + z) \cdot x = y \cdot x + z \cdot x$ FOR $\forall\, x, y, z \in S$